Electrical Engineering

December
1936

Published Monthly by the
American Institute of Electrical Engineers

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This Month—

Front Cover

Circuit interrupter assembly of 2,500,000-kva 287.5-kv circuit breakers for Boulder Dam lines
Photo courtesy Westinghouse Electric & Manufacturing Company

Special Articles

AIEE Papers

—Turn to next page

1936 Index...1408

On the 21 pages beginning with page 1408 of this issue is published the annual reference index, listing some 2,400 entries, covering all text material published in ELECTRICAL ENGINEERING during 1935. Special information regarding the index is given on page 1408.

In This Issue—

DIELECTRICS are featured in 2 papers and in the news section of this issue. In one paper the accuracy of a method of predetermining the a-c characteristics of dielectrics by d-c measurements, previously developed, is discussed (*pages 1329–37*). A transient calorimetric method for measuring dielectric losses in solids at high voltage and high frequency is describe in the second paper, and experimental data obtained by this method are presented. This method is said to reduce the time of making such measurements from hours to minutes (*pages 1347–56*). In the news section appears a report of the ninth annual meeting of the National Research Council's committee on electrical insulation, which was attended by more than 100 scientists and engineers who listened to 26 papers on various phases of dielectric research (*pages 1394–5*).

RECOMMENDATIONS of importance to the engineering profession are embodied in the 1936 report of the committee on professional recognition of the Engineers' Council for Professional Development. The report was received by ECPD at its recent annual meeting, but formal action on the recommendations contained therein was deferred. Comments from members of the Institute are invited (*pages 1390–2*). Announcement recently was made of a grant of $16,000 by the Carnegie Corporation toward support of the work of ECPD (*pages 1395–6*).

METHOD of measuring the negative sequence reactance of synchronous machines proposed by the AIEE committee on electrical machinery has been shown by theoretical analysis to lead to correct measurements of this quantity for the operating conditions most frequently met in applying the method of symmetrical components, but not for the operating conditions used in the proposed AIEE test (*pages 1378–85*).

LIGHTNING research has been conducted for the past 10 years on a 220-kv line in eastern Pennsylvania. The most recent tests, reported in this issue, included measurements of line-structure currents, made by means of surge-crest ammeters, and further data on the effects of counterpoises and ground wires (*pages 1306–13*).

TO DETERMINE the lowest resistance to ground that can be obtained for a given cost requires formulas for comparing different arrangements of grounding structures. A collection of such formulas for d-c resistance has been assembled into one paper, with some discussion of their relative accuracies (*pages 1319–28*).

THE Institute's 1937 winter convention, to be held in New York, N. Y., January 25–29, will mark the resumption of a 5-day winter-convention schedule. A tentative technical program, including more than 50 papers on a wide variety of subjects, has been announced (*pages 1388–9*).

RESISTANCE spot and seam welding, which is being applied increasingly to industrial and manufacturing processes, requires extremely accurate control of welding current. Some new developments in electronic control for this type of welding are discussed in this issue (*pages 1371–8*).

THE METHOD of circular loci, which has been applied to various types of a-c circuits in previous Institute papers, has now been applied to the determination of the vector amplification of resistance-capacitance coupled electronic tube amplifiers (*pages 1364–71*).

COMPLEX vectors are said to be exceedingly useful in the study of polyphase or multibranch a-c networks. A study of some of the properties of complex vectors, with a view to their immediate application in 3-phase circuits, is presented in this issue (*pages 1356–64*).

A PROMINENT engineer urges "a quickened sense on the part of the engineer, of his responsibilities, not alone in a purely professional sense, but as a citizen of his community, of his state, of his country, of the world" (*pages 1301–03*).

CLASSED as one of the largest and most effective of the Institute's District meetings, the South West District meeting held recently in Dallas, Texas, was attended by more than 500 Institute members and guests (*pages 1392–4*).

VISUALIZATION of transients in electric circuits can be accomplished by means of the cathode-ray oscillograph with the aid of auxiliary electronic tube circuits. Several auxiliary circuits for this purpose have been devised (*pages 1314–18*).

ANNUAL report of United Engineering Trustees, Inc., the Engineering Societies Library, and The Engineering Foundation were presented at the recent annual meeting of UET (*pages 1399–1402*).

LETTERS to the Editor columns of this issue include a contribution by a noted past-president of the Institute who offers some comments on the early mechanical development of the transformer (*page 1403*).

GENERAL formulas for determining the current and potentials along leaky ground-return conductors, applicable to electric-railway track circuits, have been derived (*pages 1338–46*).

APPROPRIATION budget for the Institute for the year 1936–37 was adopted by the board of directors at its meeting on October 20, 1936 (*pages 1396–7*).

SOME novel lighting effects were achieved at the Great Lakes Exposition held during the past summer at Cleveland, Ohio (*pages 1304–05*).

1937 TRANSACTIONS!

Last Call!

The AIEE TRANSACTIONS for 1937 will be produced only for those having subscription cards on file at AIEE headquarters not later than Friday, December 18, 1936. Present production methods require determination, in advance of the printing of the January 1937 issue of ELECTRICAL ENGINEERING of the exact number of TRANSACTIONS volumes to be provided for. For the convenience of members desiring to maintain in this permanent form a file of AIEE papers and discussions, important additional information and a convenient order blank are provided on page 8 of the advertising section of ~~this~~ The November issue.

The Work of the Institute's Committees

—A Message From the President

I DOUBT if the average member of the American Institute of Electrical Engineers has a full realization of the tremendous voluntary contribution by the officers and by the chairmen and members of the AIEE committees to "the theory and practice of electrical engineering and of the allied arts and sciences, and the maintenance of a high professional standing among its members." The quotation is the purpose of our organization as expressed in our constitution.

The greater part of the creative work accomplished by the AIEE originates in, and is carried on through, its committee organization.

Underlying the Institute are the Sections—the very foundation on which the society is built. There are 62 Sections, and in each Section it may be assumed that there are on the average 6 officers and 20 members who engage in committee activity. It may be assumed further that each officer contributes at least 30 hours of his time and each committee member 20 hours of his time during the year—a voluntary contribution of 36,000 man-hours, or 4,500 man-days. This is an equivalent of 15 engineers giving their time continuously throughout the year.

It is my purpose to consider more particularly the general and technical committee activity of the whole organization, leaving to each Section and Section chairman the consideration of the committee activity of the individual Section.

The YEARBOOK of our organization lists, in addition to the executive committee, 23 general committees and 18 technical committees. All the technical committees and most general committees are required by the constitution or by-laws and cannot be dropped without a change in the constitution or by-laws. In addition, there are 26 national officers of the Institute aside from the national secretary and the headquarters staff. From personal observation, I believe that each officer contributes at least 160 hours per year on the average, and each committee chairman at least 50 hours per year. There are more than 500 members of these general and technical committees, who on the average contribute not less than 20 hours of their time each year and many of them far exceed this average. This means a total of more than 16,000 man-hours or 2,000 man-days—about the equal of 7 engineers devoting their time exclusively to general committee activity and affairs of the organization. This tremendous force is available for advancing the fundamental purposes of the Institute. The officers feel a tremendous responsibility in effectively and efficiently directing this force.

These 41 chairmen and more than 500 members of national committees are appointed by the president, and he feels a deep responsibility for the success of the committee work. This number represents a little more than 3 per cent of the entire membership and does not include any members of subcommittees not appointed by the president. Committee work offers the opportunity for the membership to participate in the work of the organization, and through these committees the board of directors hopes to receive recommendations guiding them in the conduct of Institute affairs.

The chairmen and members of the committees are selected by the President with the invaluable aid of the national secretary. Officially, the president learns of his election in June, and all committees should be appointed by the first of September. This is a tremendous task and a very grave responsibility. Very largely the committee personnel is based upon the recommendations of the past year's chairmen, and there is a decided tendency to reappoint the old members, with such additions as may be suggested. This tendency is apt to lead to too-large committees and should be guarded against. This year, in addition to asking the recommendations of the past year's chairmen, letters were written to all of the last year's officers of the Institute, to all the incoming officers, and to several officers of past years who have shown an unusual and continued interest in the organization. The letters to the officers, present and past, resulted in considerably more than 100 suggestions for committee chairmen or committee members, and these suggestions were of very great assistance.

It is very desirable that the committee activity should be fully representative of the whole Institute, and I have suggested for the coming year that the Section chairmen be asked to make recommendations for national committee appointments, and that this activity be carried on through the national secretary, starting about April 1.

A most unusual response was received from those members who were requested to accept committee chairmanship for the current year. Out of 43 so requested, 41 accepted, and the response from those who were requested to serve on the committees has almost equalled this. Of those members who thought that they must decline committee appointments, an unusually large proportion were executives of electrical companies. I wish to emphasize the great desirability of having executives on the committees. While they may not be able to devote as large a proportion of their time to committee activity, their viewpoints and experience and the prestige of their names is most desirable.

I believe it should be the aim of each committee to do those things which need to be done, and to disregard those things which do not need to be done; and one of the most important duties of a committee is to decide what it shall attempt to do during the year. If a certain committee should decide that there is no work which needs to be done, it

should recommend to the board of directors a change in the constitution or by-laws, eliminating that committee. It is very destructive to the morale of the organization to have a committee appointed which is inactive. Such a committee should be either stimulated to activity or discontinued.

It seems to me that the following are the requisites for committee membership:

1. An interest in the AIEE.
2. An interest in the type of work being handled by the committee.
3. A willingness to give some time to the work of the committee, at least 20 hours per year.
4. A suitable geographical location.

Geographical remoteness from headquarters does not necessarily constitute a bar to committee activity. It is particularly desirable that the committee personnel should represent all viewpoints in the Institute, and there have been many examples of effective committee work through correspondence. It is much more desirable to have on a committee a member from the Pacific Coast who is interested than a member from the Atlantic Coast who is not interested and who does not contribute to the activity of the committee.

Each year the Institute should consider whether there are too many committees, or whether there are too few.

Committees may be divided into 2 classes:

1. Those whose work automatically comes to them, such as the board of examiners and the finance committee.

2. Those which develop their own work, such as the committees on electrical machinery, protective devices, and research, and the membership committee.

On Monday evening, September 14, 1936, all of the committee chairmen who could be present, numbering about 30, joined with me in an evening meeting to consider the committee activity during the year 1936–37, and many very helpful suggestions were offered. Following is a summary applying to most technical committees, and in many cases to general committees; it represents the consensus of opinion as expressed at that meeting:

1. The chairman should write to each committee member asking his comments on last year's work and suggestions for the work of this year. Any member who fails to reply cannot be interested in the work of the committee.

2. If at all practicable, there should be at least one meeting of the committee each year, and if possible 2 meetings. Those members whose geographical remoteness prevents their attending the committee meeting should contribu[te] their suggestions in writing.

3. At least 2 national papers each year should be provided by eac[h] technical committee.

4. The committee for the present year should make a suggestio[n] as to the committee personnel for the coming year.

5. Each committee should consider and make recommendations a[s] to whether the work of the committee should be continued throug[h] the next year.

6. Toward the close of the year, the committee chairman shoul[d] write to each committee member asking for comments on the wor[k] of the committee and how it could be improved the following year.

7. The committee should present a report to the board of directo[rs] of the AIEE.

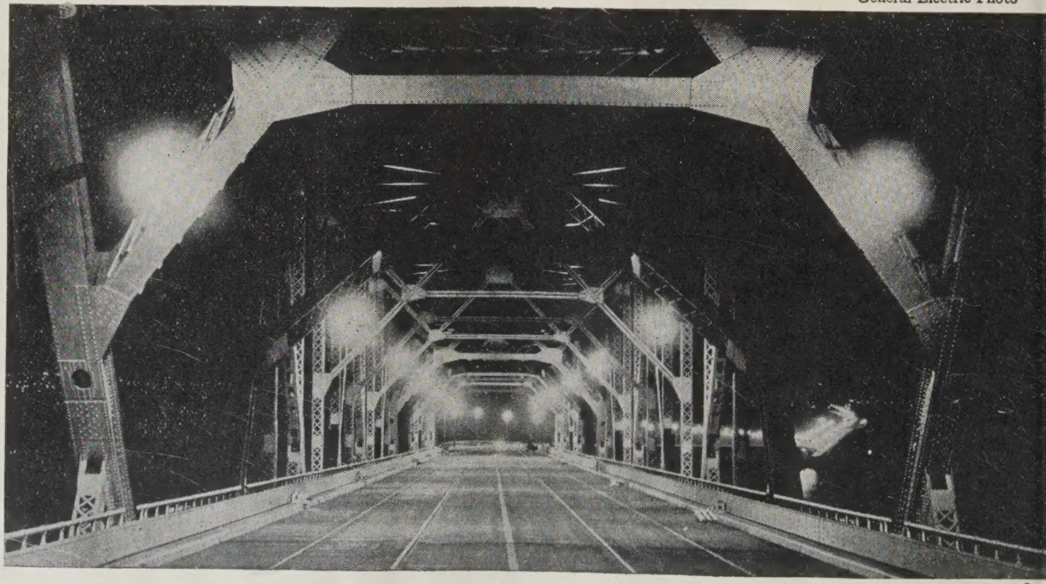
The committee organization of the Institute ma[y] be likened to an electrical network, with each com[-]mittee a generating station developing power whic[h] can be fed into the network for the purpose of carry[-]ing out the fundamental aims of the Institute an[d] each committee tied in with every other committe[e] as a co-ordinated system.

I hope that each member of the organization wil[l] consider the committee work as something in whic[h] he is vitally interested. Our product is not perfect[;] there is no organization the product of which i[s] perfect. The responsibility for our product rest[s] with ourselves—with each member of the nationa[l] organization, with each member of a committee[,] with each member of a Section. The Institute i[s] what we are. The Institute improves as we improv[e] in our organized efforts. If any of us see any faul[t] or defect in our product, should we not each as[k] ourselves whether we are doing our part, and if w[e] are satisfied that we are, suggest such changes as wil[l] greatly improve the product.

A. M. MacCutcheon

Sodium Lamps Used on San Francisco-Oakland Bay Bridge

General Electric Photo



Why the Engineer?

By William F. Durand

Chairman, Third World Power Conference

The new conditions of life brought about by the material advances of recent times, which have been achieved primarily as a result of the work of scientists and engineers, have brought with them new and pressing problems—social, economic, political, and international. In this address,* Doctor Durand states that "we (engineers) cannot evade the responsibility which rests upon us to take our due share, even the lead, in the study of the problems which our own activities have, in a large measure, developed." He urges "a quickened sense on the part of the engineer, of his responsibilities, not alone in a purely professional sense, but as a citizen of his community, of his state, of his country, of the world."

A SOMEWHAT trifling answer might be found in the reply, because the world has always had the engineer and cannot get along without him. Regarding the first statement, I like to look upon the engineer as, indeed, the oldest representative of the so-called profession—as one of those groups or guilds, the members of which play some specialized part in the advancement of our civilization.

In this sense there has come about, as we know, a specialized group whose particular function has been and is the utilization of the materials and the energies of nature in the service of man. Even if we go back to prehistoric times, we find that there have been, in this sense, engineers since the days of the palaeolithic age, when man first found a way to fashion flint chips into tips for his arrows and spears and learned how to utilize the potential energy of a distorted elastic system—a strung bow—through a rapid transformation from potential to kinetic energy, as the arrow sped toward its mark.

Again there have been bridge engineers since the time when some one found a way by fire or flint axe to fell a tree across a stream or to utilize a wild grapevine to carry his weight from one shore to the other—the far away prototype of the noble suspension bridges of modern times. There have been naval architects since the day when some one found how to hollow a log by fire and stone axe, and thus to utilize one of the basic laws of fluid mechanics. There have been metallurgical engineers since the days of Tubal Cain and long before.

If we go back to the great prototype of the engineer, we have Prometheus, who in Greek myth first learned how to bring down fire from heaven and subdue it into the service of man. A civilization without fire at its service is unthinkable, and down through the ages, both before and since the written record, its influence and significance can be plainly traced.

Enough on this phase. We surely have good ground, if we so choose, to consider ourselves as, perhaps, the oldest of those specialized groups into which the progress of civilization has forced the subdivision of those whose task it is to carry it forward. If, now, we turn to the second statement, that the world cannot get along without the engineer, the truth of this is perhaps obvious. At least, the world will not be willing to forego those things which the scientist and the engineer have jointly provided, in the advancement of what we call our civilization.

If, then, we as a group or guild must carry on, we come at once to the heart of our present inquiry. What is the engineers' part in the co-operative enterprise which we call civilization; what are its boundaries; are we occupying its full breadth and length, and what of the future? How may we better address ourselves to the task of a more adequate and perchance a better-balanced occupancy of this important domain of human activity?

The engineer has been defined as one who is concerned with the utilization of the materials and the energies of nature in the service of man. Amplifying, in terms which define his field of activity and which stand in the preamble to the constitution of American Engineering Council: "Engineering is the science of controlling the forces and of utilizing the materials of nature for the benefit of man, and the art of organizing and directing human activities in connection therewith."

Note may be taken of certain implied specializations in this definition: Thus, obviously, we are specially concerned with the constructive materials of nature and again with the inorganic energies. These may be accepted as limiting, in suitable fashion, the field of our work. On the other hand, this modern concept contains an amplification of the very highest order of importance. We are concerned with the "organizing and directing of human activities." That is, not only are we concerned with inanimate materials and inorganic energies, but also with the human agencies through which our ends are to be attained.

For our purposes in this brief statement, I should like to make some reference to 3 phases or aspects under which this broad subject might be considered.

*An address delivered by Doctor Durand at the Third World Power Conference held in Washington, D. C., during the first week in September 1936; published on recommendation of AIEE committee on education. At this meeting, Doctor Durand was elected president of the World Power Conference.

1. The terminal products of the activity of the engineer.
2. The raw materials from which such products are formed.
3. The social and broad humanistic problems which have resulted, at least in large measure, from the work of the engineer.

Regarding the first of these, I shall say only a word. We are all familiar with the terminal products of our own work. If we consider a period of only 200 years, say from the time of Watt, we have a world made over, at least as to the material content of our civilization. If we take a shorter period of say 50 to 60 years, we have a world almost made over as compared with the material content of life in the 70's and 80's of the last century.

My only point in recalling this phase of the work of the engineer is to emphasize its magnitude and the extent to which it has changed the conditions of life: social, economic, and politic. These conditions are known to us all; they are a part of our daily life, and in what I have further to say, they may simply be taken for granted.

Regarding the second of these, having to do with the raw materials comprising primarily the various constructive materials and energy, by some combination of which this transformation in the conditions of life has been brought about, some more extended word may be appropriate.

So far as the constructive materials of engineering are concerned, I shall, in the present admittedly brief and incomplete survey of the subject, refer only to the inorganic constituents of the earth's crust and outer envelope—in particular, to the mineral and metallic constituents of the earth's crust. So far as sources of energy are concerned, I shall refer only to heat as drawn from carbon and hydrocarbon sources and to falling water.

We must view the constructive materials as a bank deposit, not one drawing interest, but one out of which we are gradually and surely exhausting the principal. Only in part can our more important structural materials, *e. g.*, the ferrous compounds, be used over and over. Even with such multiple use, there is loss. We cannot completely capture the products of disintegration and reconstitute them into useful products. Neither is nature, so far as we can determine, now engaged in the enlargement of her initial deposits. The result is a gradual but continuous loss of our principal; and to that process, there is but one end—ultimate exhaustion.

The same is true with our carbon and hydrocarbon deposits used as a source of heat and transformed by our thermodynamic processes into useful work. We are gradually but surely exhausting our coal deposits and our provision of petroleum and natural gas.

Falling water—that is water caught up by the sun's heat into the upper air, carried over the high places of the earth, precipitated and caught and allowed to flow through our power-producing mechanisms—is the only source which partakes of the character of an annual dividend. Presumably as long as the sun radiates heat as at present, so long may we count on this seasonal or annual dividend, representing, as it does, a small bit of the energy which the sun is constantly radiating off into space and which, indeed in the end, will (in the absence of some cosmic interference) reduce that star to cold dead state. The picture is not a cheerful or and I shall not dwell on it. In a sense, all o materials of construction and all our sources power come from bank deposits which seem to subject to inevitable and continuous depletio Only in the case of falling water, among the sourc of present practical importance, do we seem to rea out to sources which lie outside the earth itself.

All of this, of course, is well-known; but whi well enough known, the question may at least asked whether we, as engineers, have given the fac the weight which their significance deserves. W in a sense, have appointed ourselves as custodians these deposits. They constitute a trust. We ca not evade the responsibility for their wise and effe tive use. Have we in the past and are we no living up to the full measure of this responsibilit We can hardly, I think, answer in the affirmative.

However, two things may be said in attempte attenuation of our fault. First, for some of the deposits, the quantity is so large and anything a proaching exhaustion is so remote that there is n occasion for worry; and in any event, new or su stitute sources may be found long before such condition begins to make itself felt.

While the supply in some cases may be large, for example in coal and in iron ore, it is far from th same in other cases, as for example petroleum an natural gas. But, again, here come newly devise techniques for the transformation of coal into pro ucts similar to those which we now derive by prefe ence from petroleum or natural gas. All of thi however, only serves to put forward the evil da and these or like excuses form, in the last analysi no valid justification for needless waste. The u of heat in the development of useful work is a on way street. In order to transform a part of th heat into useful work, we must let down anoth and larger part from a high level to a low; and on at the low level there is no way of restoration to th higher level whence it came, except indeed, by th letting down of a still larger quantity to the low level. Here the second law of thermodynamics i poses its rule—the ever-increasing entropy of thermodynamic system.

Much the same is true with the useful structur materials. Their utilization involves, at least some extent, progress along a one-way street, mark by the inevitable loss of some part of that which w wish to preserve and use. As to new sources energy, perchance the use of organic wastes or organic material grown or prepared for the purpos or again the direct heat of the sun or the intern heat of the earth, or again the internal energy of th atom, I make no present count.

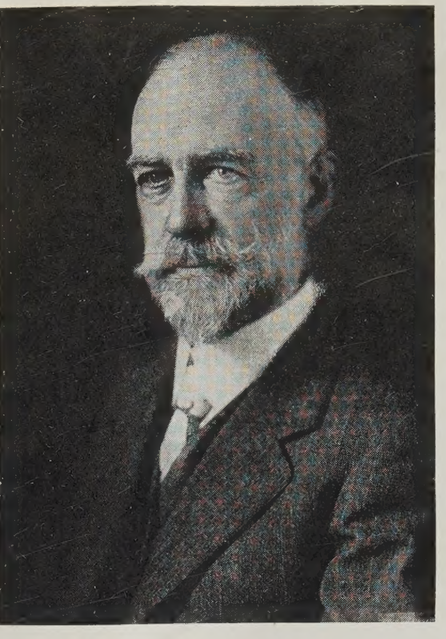
The practical, efficient, and economic use of an of the first-mentioned sources, in substitution f those which we are now employing, seems difficul and the possibility of our ever finding the key f unlocking the stores of atomic energy and of devisi methods for its safe control seems remote. In an event, in none of these possible substitutes do w find excuse for waste in our present sources of suppl

Again, as an excuse, it may be urged that t

economic and wise use of the materials and agencies of nature is only in part the responsibility of the engineer. Such matters are more widely the concern of the public at large; of the law makers; of the moulders of public opinion. This is perhaps true, but we, as engineers, can scarcely find here an adequate alibi for failure to take our own share in all efforts directed toward the development of a wise and forward-looking policy governing the use of these gifts which nature has placed at our disposal.

And who is there outside of our own guild likely to understand the significance of waste in the use of the gifts of nature, and the end toward which such waste inevitably tends. We cannot escape the fact that, in a direct sense, we are responsible to society at large and to future generations for the wise and economic use of these gifts of Nature in the development and utilization of which we are now engaged, as our share in the work of the progress of civilization. If society needs to be awakened, if new laws are needed, we must remember that we are more than engineers; we are members of society and members of the body politic. We must take our part in arousing society and our due share in the work of framing, enacting, and enforcing salutary laws and regulations looking to the ends which I have indicated. If we do not, it can hardly be expected that others will. In this way lies plainly the open path of duty for the engineer.

May we now turn to a consideration of the social, economic, and political problems which have been, shall we say, a by-product or at least an outgrowth of the work of the engineer?



Doctor Durand

We have already seen that through the co-operative work of the scientist and the engineer, the world has, in a material sense, been made over. The enumeration of details is not necessary. Compare the material content of our civilization of the time just preceding James Watt, 200 odd years ago, with the present; or again, that of the period just following the Civil War between our states—within the memory of many of us—with that of the present time. In a large sense, the external world is new; but have we made commensurate and parallel progress in the adaptation of ourselves to these new external conditions? A changed world externally must call for adaptations especially in our nervous and emotional systems, to all these new appeals to interest and stimulus. There is here needed a growth in wisdom directed to the most beneficial use of these new conditions of life. We may indeed ask if we have grown in wisdom in the use of these new products of science and engineering, commensurate with the conditions themselves, which these new products have brought about.

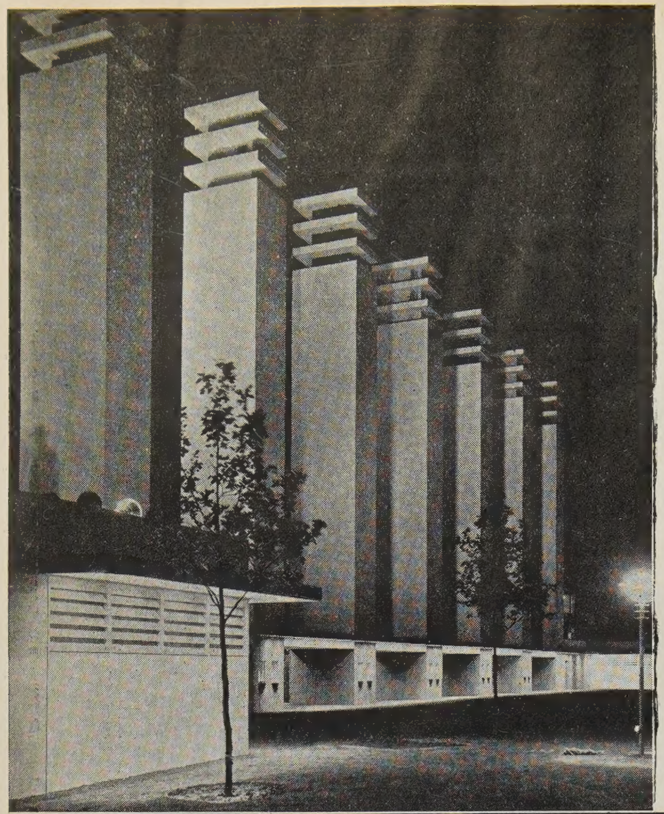
It would be a brave man, I believe, who would be prepared to assert and defend the affirmative. If we mean by wisdom, a sense of values, an appreciation of the significant and abiding as contrasted with the insignificant and transient, a capacity for effective judgment based on accurate analysis of the elements into which our problems resolved themselves—then we can hardly say that we are wiser than our fathers or our grandfathers, or even wiser than those of centuries long gone by. We have enormously more information, but that is a different thing.

My question in brief is whether the world of today is indeed as well-adjusted to the material content of our present civilization as was that of, for example, 50 or 75 years ago, to the content of that day and age. This new environment, these new conditions of life which have been brought about by the material advances of recent times have brought with them new and pressing problems, social, economic, political, and international. The time available admits of no discussion in detail. The displacement of human operatives by mechanical agencies, the tendency toward the concentration of populations in large centers, the problems of capital and labor, the new conditions and agencies of warfare—these are only examples.

Now what is our responsibility as engineers with regard to these problems? It is clear that the responsibility is not ours alone; but it is equally certain a share is ours because here again no class or stratum of society stands nearer to the source of these problems than do we who have shared in the creation and production of their causes. We cannot evade the responsibility which rests upon us to take our due share, even the lead, in the study of the problems which our own activities have, in a large measure, developed.

What I am urging is a quickened sense on the part of the engineer, of his responsibilities, not alone in a purely professional sense, but as a citizen of his community, of his state, of his country, of the world; a responsibility in the fulfillment of which he will take such part as he may in the earnest study of social, economic, and political problems, and in particular of the special conditions which his own activities have brought about, to the end that we may attain some better condition of balance as between the material content of our present-day civilization and the uses which we are making of it.

Some Lighting Features

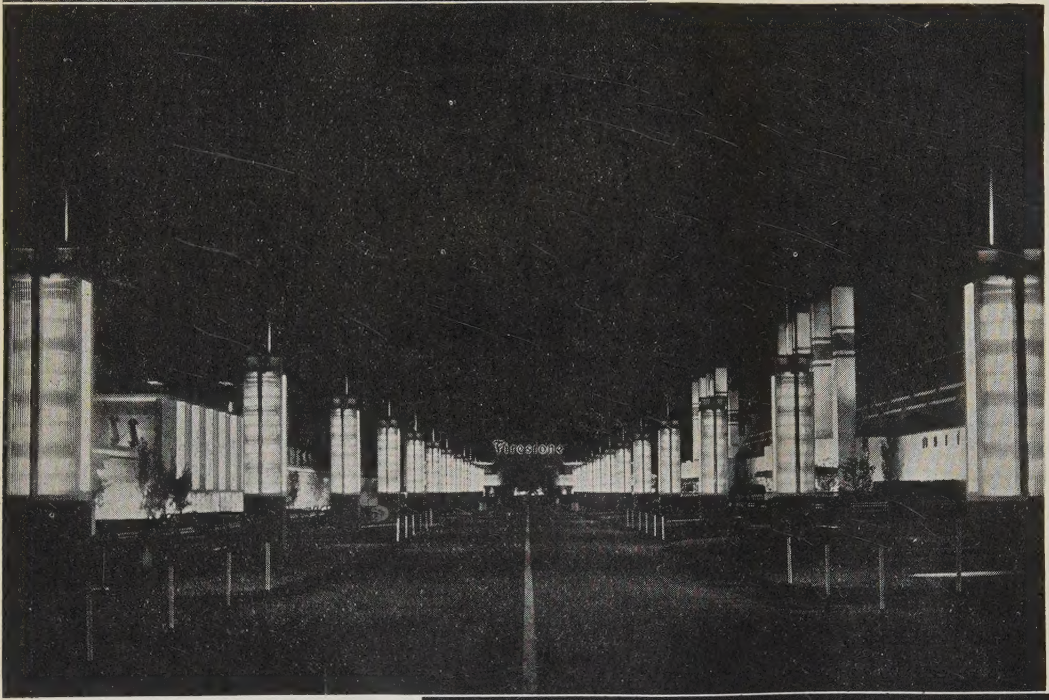


(Left) Main entrance to exposition grounds

AS THE 1936 Great Lakes Exposition in Cleve Ohio, closes, it will be remembered primarily f entrancing nighttime aspect. Although but 5 mont the making, the close co-operation of architects and il nating engineers resulted in the application of lig many new and interesting ways.

Outstanding of the many eye-catching spectacles the main vista with the Hall of Progress on one side Automotive Building on the other, and the always l Firestone display at the end. Contributing to this co scene were the 24 architectural pylons illuminating a as decorating this main axis. The pylons consist polished corrugated metal curved to form a backgr for the lamps which were concealed in the vertical s Clear 40-watt tubular lamps were employed and the from them was reflected by the corrugated metal series of wide bands. Natural-colored green 40 lamps were used to produce sparkling spikes of lig contrast with the white. The principal illuminatio this area was furn by the luminous pan the sides of each p Each pylon was equ with 12 75-watt in frosted lamps and deep cavities were with sandblasted r glass.

At each entrance t Hall of Progress, 2 ern pylons challenge tention, especially night when their beauty was evident. pylons were illumi by lamps concealed bands, which thems appeared in silhou Each pylon housed 40-watt inside-fr lamps on 8-inch ce The fins at the top each indirectly light 3 500-watt General tric exposition type lights.



(Above) Noteworthy among the applications of new lighting effects were the large pylons of polished corrugated metal flanking the main east and west axis

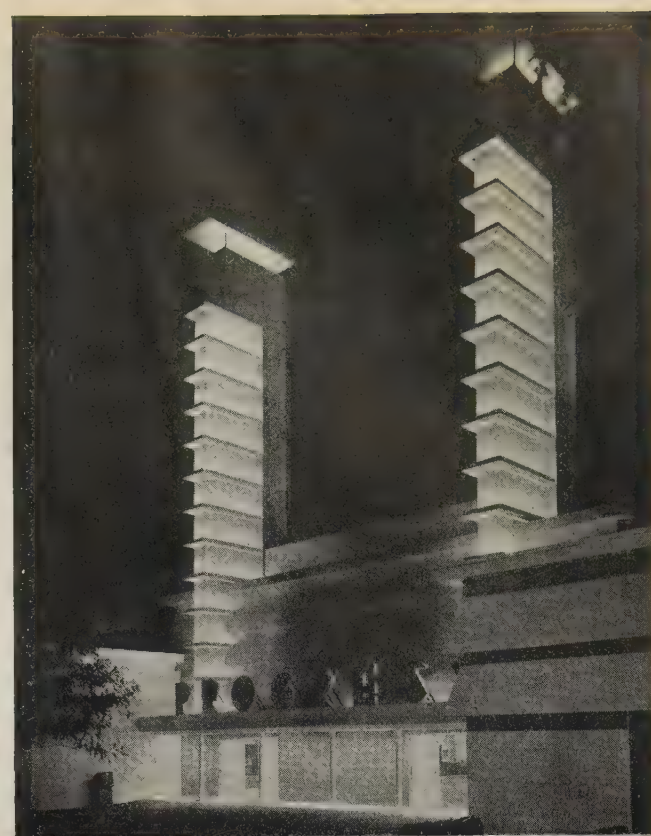
(Right) This display featured the lighting attractions that other expositions have made so well known—singing color fountains and large double-silhouette sign

e Great Lakes Exposition

ne Hall of Progress was distinguished by 2 command- entrances, each of which was composed of 7 large ster treatments of painted corrugated metal. Each ster was lighted by a vertical trough carrying 25 att inside-frosted lamps on 26-inch centers. The ghs provided an interesting silhouette effect. The sters are decorated in yellow and white which con- ted favorably with other illumination in this area.

erminating this uniquely lighted avenue was the stone Building. This building had lighting features other expositions have made so well-known—large ble-silhouette signs and singing color fountain. The was built with 3 planes with lamps behind the first 2 es. The front plane face was lighted in amber con- ally changing in brightness, while green and blue nging in intensity was used for the plane. A sequence circuit control vided the change. From any view- point the letters appeared to shrink swell with the changes in bright- . The 6 General Electric fountains each lighted by 2 red 250-watt, 2 n 200-watt, 2 amber 150-watt, and lue 400-watt Mazda lamps. By ns of a unique electric control de- , the brightness of the lamps in the erwater projectors react to the n, tempo, and volume of the music dcast from the loud-speaker sys- .

utstanding among the concession dings was that of the Highbee pany, modern in design and lighted in mobile color. The octagonal tower was lighted with 8 troughs, each carrying 3 color circuits. Four of the troughs concealed the letters Higbee which appeared in silhouette. Other indirect lighting completed the picture.



(Right) Pylons on the entrances to the Hall of Progress were modern in design and appealing in character

(Above) The main entrances to the Hall of Progress were distinguished by painted corrugated metal pilasters lighted by troughs which appeared in silhouette

(Left) Silhouette strips and letters lighted in mobile color made this building one of the most outstanding of the exposition

Contributed by AIEE committee on production and application of light, A. L. Powell, *chairman*.

Lightning Investigation on a 220-Kv System—II

Further investigation of the lightning performance of the circuit formerly known as the Wallenpaupack-Siegfried line is reported in this paper, which presents measurements of structure currents as made by surge-crest ammeters and additional data on the effect of counterpoises and ground wires.

By
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LIGHTNING RESEARCH has been conducted for the past 10 years by the Pennsylvania Power and Light Company and the General Electric Company, chiefly on that portion of the Pennsylvania-New Jersey 220-kv interconnection circuits between Wallenpaupack and Siegfried, Pa. This circuit, formerly known as the Wallenpaupack-Siegfried line,[1,2] is of single-circuit flat-top steel-tower construction. No changes in line construction were made during 1934 or 1935. About 37 miles of line is unprotected by overhead ground wires, 24 miles of line has 2 conventional aluminum-conductor steel-reinforced overhead ground wires, and 3.8 miles has special lightning-stroke diverting cables.[3] A 2.6-mile section with overhead ground wire over High Knob was provided during 1929 with a continuous buried counterpoise, and during 1930 and 1931 towers in the remaining 21.4 miles were provided with 4 buried crowfoot or radial tower-footing grounding cables 50 feet long. The purpose of the counterpoise and crowfoot ground cables was to reduce tower-footing resistances and improve line performance.

This paper will be devoted chiefly to measurements of lightning current in structures by means of surge-crest ammeters,[4] and to line performance, and deals only with the section of line between Wallenpaupack and Siegfried.

Records of insulator flashovers, line tripouts, magnetic oscillograms, and surge indicators[2] have been kept regularly. Lightning stroke recorders were used until 1936, but the results being similar to those previously reported[2] are not included here, nor are records from lightning severity meters.[2] Reference will be made only to the method of locating li[ne] tripouts[5] by means of ratios of the dynamic curren[t] flowing at times of faults.

By surge-crest ammeter is meant a method [of] measuring the maximum or crest value of lightnin[g] currents. Technique includes use of small lin[ks] capable of being magnetized by electromagnet[ic] fields, and retaining a definite proportion of the[ir] magnetization for long periods after the field h[as] disappeared. In practice, links are supported [in] nonmagnetic brackets near to tower legs or oth[er] conductors which may carry lightning current.

After storms, links can be tested in the field [or] replaced by unmagnetized links and measured at a[ny] convenient time and place. The intensity of ma[g]netization and direction of polarity can be interprete[d] in terms of the crest magnitude and direction of flo[w] of the transient lightning current.

During 1933 surge crest ammeter links were su[p]ported at distances of 2 and 8 inches from tower leg[s]. During 1934 the sensitivity was increased by movin[g] the links one inch closer to the tower. In additio[n] about half of the 311 towers were provided with [2] brackets instead of but one; also special bracke[ts] were applied to the High Knob counterpoise. Lin[ks] were supported at distances of $1^1/_4$ and 4 inches fro[m] the center of the number 2/0 conductor. Bracke[ts] were installed at distances of 3, 100, 200, and 3[00] feet from the nearest tower leg, and on both sides [of] a tower. Seven towers were equipped. Durin[g] 1935 the number of counterpoise installations wa[s] increased from 56 to 104, and the entire 13 spa[ns] were equipped. Also, one crowfoot cable only [at] 50 towers (a distance of about 10 miles) was pr[o]vided with a similar bracket located at the mid-poi[nt] of the 50-foot cable.

Lightning Current Measurements

During the period 1933–35 a total of 282 registr[a]tions of tower-structure currents was obtained at 1[?] towers, and 135 registrations from buried counte[r]poise or crowfoot ground cables. Of the 124 recor[ds] of tower current, all 32 registrations during 1933 we[re] of necessity from single-leg installations. Durin[g] 1934 and 1935, 20 and 72 records were from 1-le[g] and 4-leg installations, respectively. These latter 7[2] records were composed of from 1 to 4 registratio[ns] per tower.

In magnitude, structure-current records range[d] from a trace (about 1,000 amperes) to about 55,0[00] amperes and the maximum probable stroke curre[nt] was about 110,000 amperes. All structure curren[ts] save a very small one have been of negative polarit[y]; that is, from negative cloud to positive eart[h]. (During 1936 a few very small positive records ha[ve] also been obtained.) Of the structure curre[nt] records, 66 per cent have been less than 10,0[00] amperes in magnitude, and only 11 per cent e[x]ceeded 30,000 amperes. For this line the ve[ry] severe currents occasionally registered on oth[er] systems have not been experienced.

It is impracticable to present a table showing [all] 124 structure current records as well as counterpoi[se] currents; accordingly, only those 44 records

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1. For all numbered references see list at end of paper.

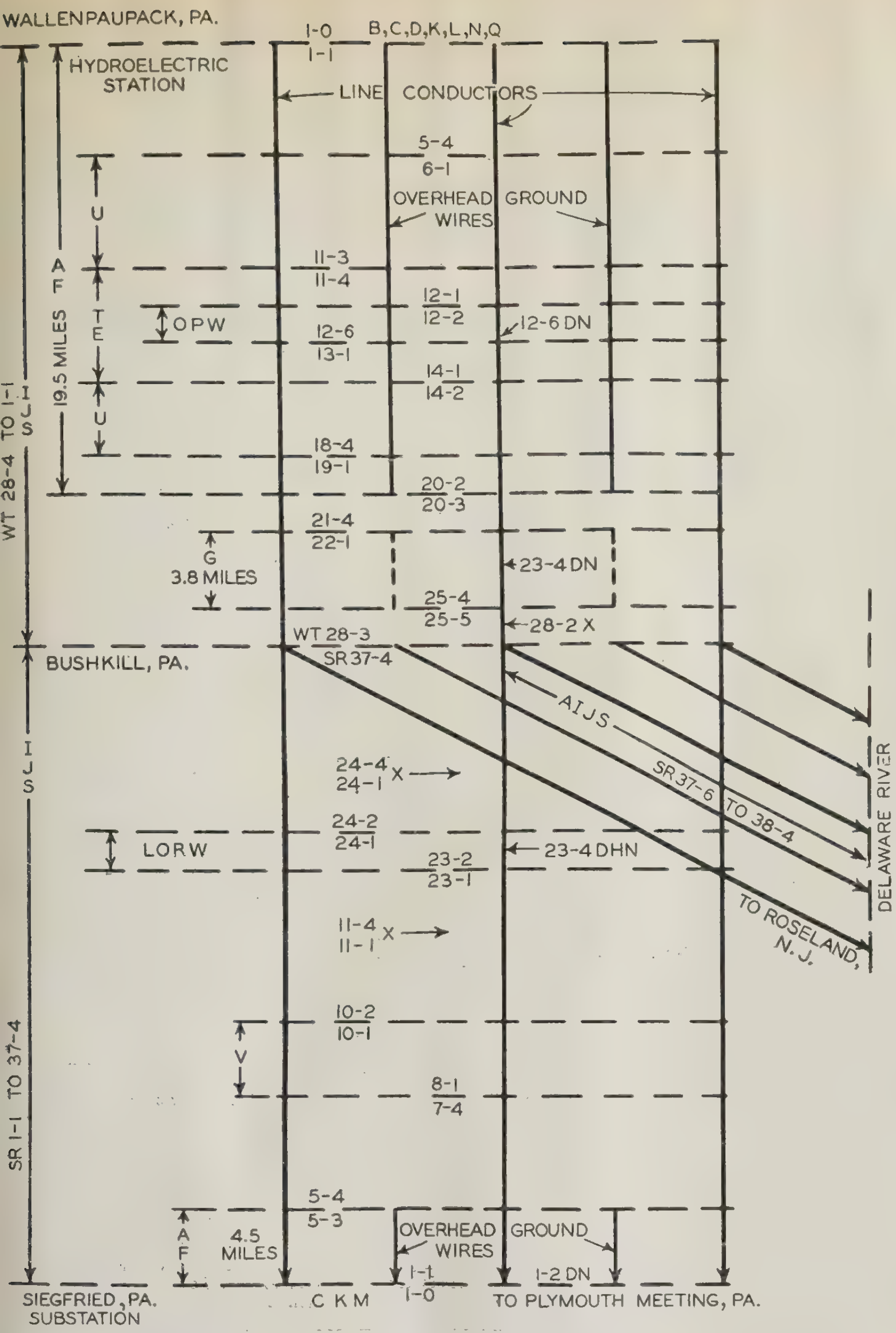

Fig. 1. Schematic diagram of Wallenpaupack-Siegfried line and Bushkill-Delaware River addition

A—Conventional overhead ground wires installed during summer of 1927 from Wallenpaupack to *WT*20-2, and from Siegfried to *SR*5-3, and in 1932 from Bushkill to Delaware River

B—Lightning laboratory installed at Wallenpaupack during spring of 1928; expanded during 1929 and closed that fall

C—High speed recording ammeters installed at Wallenpaupack and Siegfried in 1929, to measure line residual amperes during line faults

D—Six voltage-measuring antennas installed at Wallenpaupack in 1929 and 5 antennas at 4 other locations during 1930, each equipped with a surge voltage recorder. Not used after 1930

E—Continuous buried counterpoise installed in spring of 1929 over High Knob (rocky high-resistance hill)

F—Crowfoot counterpoises (4 50-foot tower-footing grounding cables) installed on 19.5-mile section of line with overhead ground wire in summer of 1930, and on 4.5-mile section with overhead ground wire in spring of 1931

G—Lightning stroke diverting cables (specially designed overhead ground wires insulated from top of structures, guyed to earth, and bonded back to tower footings through extra-long crowfoot counterpoises) installed in summer of 1930

H—Lightning laboratory installed at tower *SR*23-4 in Cherry Valley in spring of 1930, and closed that fall

I—Lightning stroke recorders installed on every structure of line between Wallenpaupack and Siegfried during 1930. A few installations made during 1929. During 1932 installations were made on Bushkill-Delaware River section

J—Surge (flashover) indicators installed on each insulator assembly from Wallenpaupack to Siegfried during 1930 and from Bushkill to Delaware River during 1932

K—Thyrite lightning arresters of 20 units installed on *W* phase at Wallenpaupack and Siegfried line dead ends, 1930. Removed during winter of 1931–32, redesigned, cut down to 16 units, and installed at Wallenpaupack on *W* and *Y* phases, spring of 1932

L—Hull gaps (experimental lightning arresters) installed on *W* and *X* phases at Wallenpaupack dead ends, and on *W* phase at towers from *SR*23-2 to 24-1, 1930. Removed, fall 1931

M—Two magnetic oscillographs installed at Siegfried in 1930 to record phase and line residual currents, voltages, etc.

N—Lightning severity meter installed near tower *SR*23-4 during 1930 and at antenna sites during 1931, as well as at locations distant from this line

O—Pipe lightning rods installed at 5 towers from *WT*12-2 to 12-6, and at 5 towers from *SR*23-2 to 24-1, 1931. Each rod was insulated from structure top and grounded through a resistor, across which was a lightning stroke recorder to measure ohmic voltage drop

P—Overhead ground wires at towers from *WT*12-2 to 12-6 insulated from tower tops, cross bonded, and grounded through resistors. At each tower the resistor was shunted by a lightning stroke recorder to measure ohmic voltage drop

Q—High-speed recorder installed at Wallenpaupack in 1931 to record phase and line residual amperes

R—Experimental expulsion protective gaps installed on *W* phase at 5 towers from *SR*23-2 to 24-1 replacing Hull gaps during spring of 1932. Removed in spring of 1934

S—Links for surge-crest ammeter installed at a few towers during 1932 and at every line structure during spring of 1933. During 1934 about half of the total structures had brackets installed on all 4 legs instead of but one

T—During 1934 surge-crest-ammeter brackets installed on continuous counterpoise between *WT*12-1 and 13-1. During 1935 installation was completed from *WT*11-4 to 14-1

U—During 1935 surge-crest-ammeter brackets installed on one crowfoot cable from *WT*6-1 to 11-3, inclusive and from *WT*14-2 to 18-4, inclusive. During 1936 24 structures, mostly within these sections, had additional grounding cable added, and 3 additional surge-crest-ammeter brackets on new and existing grounding cable

V—Expulsion protective gaps installed on *W* phase at 10 towers from *SR*8-1 to 10-1, inclusive, in fall of 1935

W—Surge-crest ammeters installed on lightning rods at towers from *SR*23-2 to 24-1, and from *WT*12-2 to 12-6, also on overhead ground wires at latter towers, 1936. All lightning stroke recorder-resistor installations removed

X—Transpositions at these points. Phases change as follows (from Wallenpaupack to Siegfried) *YXW* to *YWX* to *XYW* to *WXY*

Y—Surge-voltage recorders installed at varying locations and in varying numbers from 1926 to 1932. Locations not shown

Z—Wallenpaupack-Siegfried line placed in [illegible] in spring of 1926. Bushkill-Roseland line added in spring of 1932

magnitude 10,000 amperes or greater are shown in detail in table I. The remaining 80 records are summarized in magnitude in table II.

Strokes to overhead ground wires and strokes which directly contact grounded structures seldom cause insulator flashover unless the currents and tower footing resistances are large enough to result in a dangerously high potential of the tower footing. Conversely, very small strokes of only a few thousand amperes, and which occasionally fail to leave even a trace of record, are capable of causing insulator flashovers on unprotected sections of line. A few typical examples are shown in table III.

When only one bracket is installed on a tower, th registration is multiplied by 4 to obtain the probab total structure current. This procedure is a logic one, but must occasionally result in errors. Whe the total current is small, registrations may not occu on all 4 legs and when the current is large the di tribution between legs may also be unequal. Tab IV illustrates this, and shows why the accuracy o one-leg measurements is less than those from all legs.

When a continuous counterpoise is in use, such a the one from *WT*11–4 to 14–1 connecting 14 towe over High Knob, the legs connected to the counte

Table Ia—Surge-Crest Ammeter Records of 10,000 Amperes or Greater—To Unprotected Line or to Structures on Whic Overhead Ground Wires Terminate

Item Number	Year	Line Name#	Tower Number	Total Structure Current, Amperes	Footing Ohms	Product RI × 10⁻³	Surge Indicator Operated	Insulators Flashed	Number of Tower Leg Brackets	Remarks
1*	35	SR	9–4	−54,400	28	1520	Yes	Yes	4	Two-phase flashover involving towers 9–2, 9–3, 9–5 and 10–1
2*	33	SR	31–3	−51,000	34	1740	Yes	Yes	1	Three-phase flashover at single tower
3*	33	SR	8–4	−49,000	20	980	No	No	1	Stroke to single tower
4*	33	SR	21–2	−42,000	20	840	Yes	No	1	Stroke to single tower
5*	33	SR	19–3	−40,000	45	1800	Yes	Yes	1	Three-phase flashover at single tower
6*	33	WT	27–6	−36,000	58	2100	Yes	Yes	1	Stroke to this tower also caused flashover at next tower
7*	35	SR	33–2	−30,790	18	555	No	No	4	Stroke to single tower
8*	34	SR	18–1	−30,000	28	840	Yes	Yes	1	Two-phase flashover at single tower
9*	35	SR	9–3	−28,850	28	810	Yes	Yes	4	Two-phase flashover—see item 1
10*	35	SR	13–4	−25,600	38	975	Yes	Yes	1	Two-phase flashover involving towers 12–6, 13–1, 13–2, 13–3, 13-
11*	35	SR	13–5	−24,600	42	1030	Yes	No	1	See item 10
12	35	SR	23–5	−22,180	39	865	No	No	4	See item 15
13	35	WT	28–4	−19,490	1		No	No	4	Associated with stroke to overhead ground wires at Bushkill
14	34	SR	16–3	−18,600	30	560	Yes	Yes	4	Single-phase flashover at single tower
15	35	SR	23–4	−18,150	23	420	Yes	Yes	4	Stroke divided between 23–4 and 23–5, but 23–5 insulators no flashed
16	35	SR	5–5	−14,870	45	670	Yes	?	4	Stroke divided between 5–4 and 5–5; see item 25
17	35	SR	7–1	−13,280	17	225	No	Yes	4	Single-phase flashover at 7–1 and 7–2; see item 24
18	33	WT	28–4	−12,000	1	12	Yes	Yes	1	Associated with stroke to open line between 28–3 and 28–4
19	33	SR	7–2	−12,000	12	140	Yes	Yes	1	
20	33	SR	8–1	−12,000	34	410	Yes	Yes	1	Single-phase flashover at 8–1 and 8–2
21	33	SR	9–2	−12,000	15	180	No	?	1	Single-phase flashover at 9–2 and 9–3; see item 28
22	33	SR	24–2	−12,000	20	240	Yes	Yes	1	Single-phase flashover at 24–1 and 24–2
23	33	SR	32–2	−12,000	13	160	No	No	1	Stroke to single tower
24	35	SR	7–2	−10,970	12	130	No	Yes	4	See item 17
25	35	SR	5–4	−10,830	15	165	Yes	Yes	4	See item 16
26	35	SR	9–2	−10,250	15	155	Yes	No	4	See items 1 and 9
27	35	SR	36–4	−10,080	50	505	Yes	Yes	4	Single-phase flashover at 36–4 and 36–5
28	33	SR	9–3	−10,000	28	280	Yes	Yes	1	See item 21

SR—Siegfried-Roseland; WT—Wallenpaupack tap.
* In these cases both inner and outer links registered greater than traces. In all other cases some or all of the outer links registered traces or zero.

Table Ib—Surge-Crest Ammeter Records of 10,000 Amperes or Greater—To Overhead Ground Wire Structures

Item Number	Year	Line Name#	Tower Number	Total Structure Current, Amperes	Footing Resistance, Ohms	Footing RI × 10⁻³	Surge Indicator Operated	Insulators Flashed	Number of Tower Leg Brackets	Former Footing Resistance, Ohms
1*	33	WT	8–5	−40,000	7	280	No	No	1	
2*	34	WT	12–3	−38,700	1.4	55	Yes	No	4	87
3*	33	SR	2–1	−38,000	15	570	Yes	No	1	
4*	33	SR	2–4	−36,000	10	360	No	No	1	
5*	33	SR	4–4	−35,000	19	670	No	No	1	
6*	33	WT	8–4	−35,000	8	280	No	No	1	
7*	34	SR	2–2	−30,200	15	450	No	No	4	
8	34	WT	12–5	−22,100	1.7	40	No	No	4	80
9	35	WT	12–4	−18,800	1.3	25	Yes	No	4	95
10	34	WT	12–2	−17,900	1.4	25	Yes	No	4	50
11	34	WT	12–1	−15,700	1.4	20	No	No	4	85
12	35	WT	12–2	−14,450	1.4	20	No	No	4	50
13	34	WT	1–5	−12,000	17	200	Yes	No	1	
14	33	WT	4–5	−10,000	26	260	No	No	1	
15	33	WT	12–4	−10,000	1.3	13	No	No	1	95
16	33	SR	2–5	−10,000	5	50	No	No	1	

SR—Siegfried-Roseland; WT—Wallenpaupack tap.
* In these cases both inner and outer links registered greater than traces. In all other cases some or all of outer links registered traces or zero.

poise conduct the greater part of the total measured structure current as shown in table V. (The letters *WT* indicate a tower on the Wallenpaupack tap.)

Tripouts and Flashovers

Table VI shows that only 13 towers in overhead ground wire sections of line have been flashed over, as compared with 325 on unprotected line. Reducing these data to numbers per 100 circuit miles per year, the comparison is more striking, namely, 5.7 compared to 76.8 or 7 per cent as against 93 per cent. Of these towers 9, or perhaps 11, were flashed before tower-footing resistances were reduced by counterpoise or crowfoot cables. All 13 occurred prior to 1933 and no structure-current records were obtained but it is known that in at least some cases the stroke was severe.

Table II—Numbers of Surge-Crest Ammeter Records of Less Than 10,000 Amperes

Range of Current, Amperes	Number of Records
1,000–1,990	16
2,000–2,990	11
3,000–3,990	7
4,000–4,990	20
5,000–5,990	5
6,000–6,990	7
7,000–7,990	6
8,000–8,990	6
9,000–9,990	2
	80

Table III—Six Representative Lightning-Stroke Records to Unprotected Line, to Isolated Structures, or to Overhead Ground Wires, Illustrating Effect of Tower-Footing Resistance

Case Number	Tower Number	Overhead Ground Wires	Tower Footing Resistance, Ohms	Structure Amperes	Product, Kv	Surge Indicated on Phase	Flashed Insulators on Phase	Tripout	Remarks
1	SR35–5	No	43	None measured		*X*	*X*	1934, No. 9	Single-phase fault; 8 surge-crest-ammeter links per structure, current less than 1,000 amperes per structure
	SR36–1	No	13			*X*	*X*		
2	SR27–1	No	34	− 3,000	100	*Y*	*Y*	1935, No. 11	Single-phase fault; small stroke to unprotected line
	SR27–2	No	28	− 2,000	60	*Y*	*Y*		
3	SR 9–2	No	15	−10,250	150	*W*		1935, No. 7	Three-phase fault; severe stroke to unprotected line
	SR 9–3	No	28	−28,850	810	*W, Y*	*W, Y*		
	SR 9–4	No	28	−54,400	1,520	*W, X*	*W, X*		
	SR 9–5	No	34	− 7,420	250	*W, Y*			
	SR10–1	No	15	− 8,230	130	*Y*			
4	SR 8–4	No	20	−49,000	980	None	None	None	Severe strokes to single structures showing effect of tower-footing resistance
5	SR31–3	No	34	−51,000	1,740	*Y, W, X*	*Y, W, X*	1933, No. 11?	
6	SR 2–1	Yes	15	4,000	60	None	None	None	Severe stroke to section of line with overhead ground wire
	SR 2–2	Yes	15	30,200	450	None	None		
	SR 2–3	Yes	15	4,000	60	None	None		

In table VII are shown line tripouts. Excluding 2 tripouts caused by faults near Roseland, N. J., and about which nothing is known except the tower flashed and phase involved, and 56 tripouts whose locations have not been satisfactorily determined, only 4 tripouts occurred on sections equipped with overhead ground wire as against 108 on unprotected line. Reduced to tripouts per 100 circuit miles per year, these numbers become 1.8 and 25.5, a ratio of 7 to 93. A rough estimate of the average lightning severity for the periods 1926–30 and 1931–35 can be obtained by comparing numbers of open line plus unknown location tripouts for these periods. The numbers are 95 and 69, yet during these periods there were 3 and 1 overhead ground wire tripouts, and the first 3 tripouts occurred before tower-footing grounding was improved, footing resistances being 98, 43, and 26 ohms. The numbers of phases involved were 2, 3, and 1 respectively, the 2 higher resistance towers suffering multiphase flashovers. The stroke causing the 1932 tripout must have been quite severe because surge indicators operated at 3 successive towers, yet only one phase was involved at a single (13 ohm) tower.

Table VIII shows that most open-line tripouts involve only one phase: there have been 16 single-phase faults (all on open line) where surge-crest ammeters have been in use. In most cases the recorded structure currents were small. Except in one case, the maximum tower-footing potential was 560 kv with an average value of about 300 kv, and the maximum structure current was 18,600 amperes. In all but 3 cases insulators were flashed on the same phase at 2 adjacent towers. For these 3 exceptions structure currents were measured at 2 adjacent towers and it seems likely that flashover occurred at both towers but failed to mark the insulators at one structure.

There was one case that may be an exception. A stroke occurred to or near *WT*27–6 causing a structure current of 36,000 amperes and a tower-footing potential of 2,100 kv. Flashover occurred apparently from structure to conductor, and at the next tower, *WT*28–1, flashover occurred from conductor to structure and a trace of current (about 4,000 amperes) was recorded. Just why this stroke did not cause a multiphase flashover at 27–6 is not clear. Perhaps the structure current estimate of 4 times

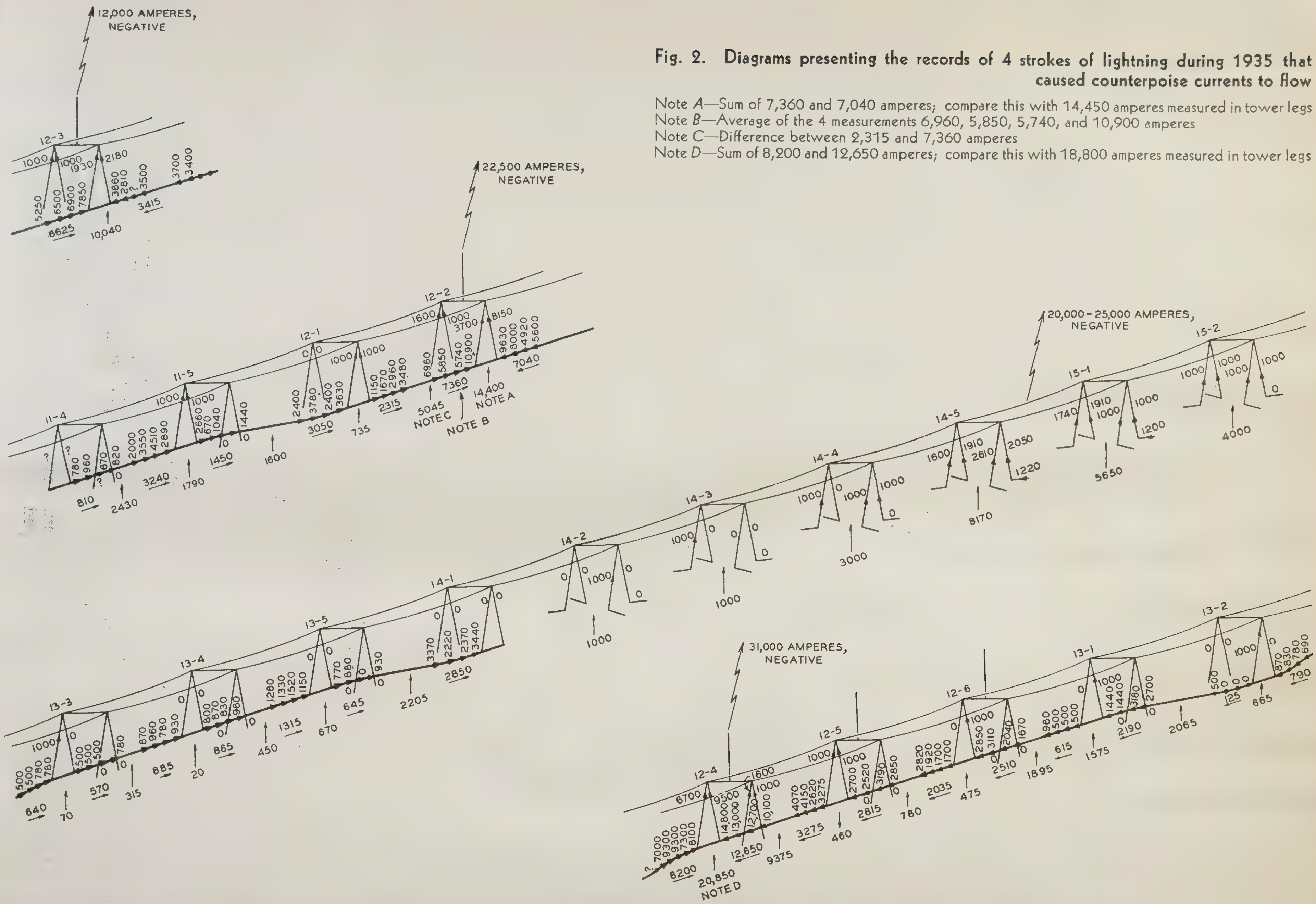

Fig. 2. Diagrams presenting the records of 4 strokes of lightning during 1935 that caused counterpoise currents to flow

Note *A*—Sum of 7,360 and 7,040 amperes; compare this with 14,450 amperes measured in tower legs
Note *B*—Average of the 4 measurements 6,960, 5,850, 5,740, and 10,900 amperes
Note *C*—Difference between 2,315 and 7,360 amperes
Note *D*—Sum of 8,200 and 12,650 amperes; compare this with 18,800 amperes measured in tower legs

Table IV—Structure-Current Distribution in Standard Structures Not Connected to Counterpoise and Having 4-Leg Installation

Item Number	Line Name#	Tower Number	Total Current, Amperes	Current in Leg 1	2	3	4	Per Cent Deviation From Equal Distribution			
(a) Records of 10,000 amperes or greater, and in which all registrations of surge-crest ammeter links, both inner and outer, were greater than traces											
1	SR	2–2	30,200	7,200	7,800	7,500	7,700	−4.6	+ 3.3	− 0.7	+ 2.0
2	SR	9–3	28,850	7,400	7,300	7,500	6,650	+2.6	+ 1.2	+ 4.0	− 7.8
3	SR	9–4	54,400	12,500	14,500	11,200	16,200	−8.1	+ 6.6	−17.6	+19.1
4	SR	33–2	30,790	7,060	6,130	10,600	7,000	−8.3	−20.4	+37.7	− 9.0
(b) Records of 10,000 amperes or greater, and in which some registrations of outer links of surge-crest ammeter were either traces or zeros											
1	SR	5–4	10,830	1,000‡	1,000†	1,680‡	7,150	−63.1	− 63.1	−38.0	+163.8
2	SR	5–5	14,870	2,860†	3,780†	1,680†	6,550	−23.0	+ 1.5	−54.7	+ 76.2
3	SR	7–1	13,280	1,680†	7,310	2,690†	1,600‡	−49.4	+120.2	−19.0	− 51.8
4	SR	7–2	10,970	3,110†	3,530†	2,100†	2,230‡	+13.4	+ 28.7	−23.4	− 18.7
5	SR	9–2	10,250	3,110†	3,280†	1,000†	2,860†	+21.4	+ 28.0	−61.0	+ 11.6
6	SR	16–3	18,600	6,800	6,300	2,800†	2,700†	+46.3	+ 35.5	−39.8	− 42.0
7	SR	23–4	18,150	1,600†	11,000	3,700†	1,850†	−64.7	+142.3	−18.4	− 59.2
8	SR	23–5	22,180	6,550	6,800	2,350†	6,480	+18.1	+ 22.6	−57.6	+ 16.9
9	SR	36–4	10,080	1,930†	2,860†	2,690†	2,600†	−23.4	+ 13.5	+ 6.7	+ 3.2
10	WT	28–4	19,490	8,400	1,000†	1,000‡	9,090	+72.4	− 79.5	−79.5	+ 86.6

(c) Records of 5,000 amperes or less

Number of Registrations Obtained	Inner Link Reading Greater Than Trace, Outer Link Reading Trace	Inner and Outer Link Readings Both Traces	Inner Link Reading Trace, Outer Link Reading Zero
1	1	2	5
2	1	3	9
3	0	1	6
4	0	0	5

SR—Siegfried-Roseland; WT—Wallenpaupack tap. † Outer link reading trace only. ‡ Outer link reading zero.

Table V—Current Distribution in Structures Connected to Counterpoise (Counterpoise Connected to Legs 1 and 2 Only)

Item Number	Line Name#	Tower Number	Total Current, Amperes	Current in Leg 1	2	3	4	Ratio of Currents Total 1 and 2	Total 3 and 4
(a) Records where all registrations of both inner and outer links of surge-crest ammeter were greater than traces									
1	WT	12–3	38,700	12,200	12,200	7,000	7,300	63	37
(b) Records where all registrations of surge-crest ammeter from counterpoise legs were greater than traces, but noncounterpoise leg registrations were either trace or zero									
1	WT	12–1	15,700	5,800	6,400	1,800†	1,700‡	78	22
2	WT	12–2	17,900	6,700	7,200	1,700‡	2,300†	78	22
3	WT	12–5	22,100	9,500	9,400	1,900†	1,300†	85	15
4	WT	12–4	18,800	9,500	6,700	1,000‡	1,600‡	86	14
(c) Records where no outer link registration exceeded a trace									
1	WT	12–4	6,400	1,800‡	3,600†	0	1,000‡	84	16
2	WT	12–3	6,110	2,180†	1,930†	1,000‡	1,000‡	67	33
3–11*	WT	11–5	2,200	2,200‡	0	0	0	100	0

WT—Wallenpaupack tap. * Eight other similar records omitted from this table. † Outer link reading trace only. ‡ Outer link reading zero.

9,000 amperes is too great, and possibly the stroke occurred to the conductor quite near the tower, permitting most of the stroke current to flow through that tower; if so this case is really similar to all other single-phase faults except that the current was larger than usual and mostly limited to a single tower.

Table IX presents data for multiphase flashovers where surge-crest ammeters were in use. In the first 4 cases flashover involved all 3 phases simultaneously and the apparent tower-footing potentials agree with accepted laboratory values of 14 and 16 unit insulation strength. Items 1 and 2 involved 5 towers each, and were caused by large strokes. Items 3 and 4 involved one tower only and were caused by somewhat smaller strokes.

Items 5 and 6 are very similar and involved 2 phases only. Item 5 appears to be a simultaneous flashover judging from magnetic oscillograms, while item 6 is known to be the result of 2 successive single-phase faults. Because items 5 and 6 are similar as regards magnitude of stroke current and tower footing potential, and one is known to be a combination of single-phase faults to open line, both will be considered to be single phase. It is significant that the apparent tower-footing potential was insufficient to

have caused a flashover from structure to conductor, as was the case in the 15 single-phase flashovers previously described.

Counterpoise and Tower Footing Grounding Cables

The High Knob counterpoise which was installed during the spring of 1929 has effectively prevented insulator flashovers from occurring on that section of line. Prior to its installation 13 towers suffered flashover during 3 years time, and at least 2 of these towers were flashed after overhead ground wires were in use. The largest measured stroke to this section of line was in excess of 75,000 amperes and the largest single structure current was 38,700 amperes.

Figure 2 shows the records of 4 strokes of lightning which caused counterpoise currents to flow. The largest of these strokes was about 31,000 amperes. A surprising feature of measurements like these (several more have been obtained during 1936) is the considerable distance traveled by the counterpoise currents. For example, the stroke to tower *WT*12–4 drew current from nearly a mile south, and the stroke to towers 14–5 and 15–1 drew currents from about a mile and a half away.

In this diagram each arrowhead or dot on a tower leg, or along the counterpoise, represents a surge-crest ammeter installation. The direction of current flow is shown by the direction of the arrowhead from positive to negative, and the crest current magnitude is shown by the adjacent figure. The measurements from each group of 4 adjacent counterpoise brackets have been averaged, and the sums or differences of these averages are shown as currents flowing up through towers, or from the earth into the counterpoise. At towers 14–5 and 15–1 measurements were obtained as shown in the tower-footing grounding cables of leg number 2.

This diagram presents the records left by 4 separate strokes during 1935. Records from the stroke to tower 12–2 further south than shown were obliterated by the strokes to towers 12–3 and 12–4. Similarly, records from the stroke to tower 12–4 further north than shown were obliterated by the strokes to towers 12–3 and 12–2. Purely to avoid confusion the 4 records have been shown on separate sections of the diagram.

Consider the records associated with tower 12–2. Beginning at 11–4 an average current of 810 amperes was measured. This current increased to 3,240 amperes before it reached 11–5, and the additional 2,430 amperes is represented as leaking from the earth on to the counterpoise. At tower 11–5 only 1,450 amperes leaves the tower, and the difference of 1,790 amperes must flow up to the overhead ground wires; as a matter of fact about 2,000 amperes was recorded by the tower-leg installations.

In a similar manner the counterpoise current is increased in every span except one by leakage from the earth, and decreased at every tower by current flowing upward to the overhead ground wires. Although in this instance all 4 strokes were negative, records obtained during 1936 show both negative and positive strokes. The latter have all been very small and might have escaped detection had only tower-leg brackets been in use.

This diagram shows how the counterpoise draws current from considerable distances and avoids concentration of current in the earth at the tower footings and consequent high tower-to-earth potentials.

There have been no flashovers to towers equipped with overhead ground wires and tower-footing grounding cables during the past 3 years. The

Table VI—Flashed Towers on the Wallenpaupack-Siegfried Line

Construction	Year 1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	Total	Per Cent	Number per 100 Circuit Miles per Year
Overhead ground wires*	0	3	4	4	0	2	0**	0	0**	13	3.5	5.7	
Unprotected line***	18	4	74	71	45	36	15	25	14	23	325	86.7	76.8
Unknown	0	11	22	0	0	0	0	0	0	0	33	8.8	
Miscellaneous	0	0	1†	0	1‡	1‡	0	1‡	0	0	4	1.0	

* Average mileage, 8.5 years, 26.7.
** One flashover on overhead ground wire in New Jersey.
*** Average mileage, 10 years, 42.3.
† Reduced insulation near Wallenpaupack station.
‡ End towers of overhead ground wire sections.

Table VII—Tripouts of the Wallenpaupack-Siegfried Line*

Construction	Year 1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	Total	Average per 100 Circuit Miles per Year
Overhead ground wires			1	1	1	0	1	0*	0	0*	4	1.8
Unprotected line	15	4	2	10	14	20	8	13**	11	11	108	25.5
Unknown		3	11	27	9	0	1	2	1	2	56	

* One tripout on section with overhead ground wire occurred due to fault in New Jersey during the years 1933 and 1935 but not included in original Wallenpaupack-Siegfried line.
** One tripout caused by flashover at *WT*28–4 (end tower of section with overhead ground wire).

maximum structure current measured was 40,000 amperes. The maximum tower-footing potential was 670 kv, which occurred at a tower of 19 ohms resistance and with a current of 35,000 amperes.

Summarized Results for the Wallenpaupack-Siegfried Line, 1926–35

1. Overhead ground wires intercept direct lightning strokes and conduct them safely to earth. Excluding 3 flashovers at end towers and 1 at a reduced insulation tower near Wallenpaupack, only 13 towers with overhead ground wire have suffered flashover, compared with 325 on unprotected line. All 13 flashovers occurred prior to 1933 and no surge-crest ammeter records are available. Tower footing resistances ranged from 20 to 116 ohms, with an average value of 48 ohms. All evidence obtained during the last 3 years verifies the assumption that flashovers on protected lines are occasioned only by high tower-footing potential exceeding the line insulation strength.

2. The continuous counterpoise installed over High Knob during 1929 reduced tower-footing resistances to very low and uniform values, and has effectively prevented any insulator flashovers from occurring in that section. During 1934 and 1935 structure currents were measured which would have caused at least 2 and possibly 4 towers to flash over had the counterpoise not been in use (items 2, 8, 9, and 11 of table I*b*).

3. The counterpoise serves to drain charges from a considerable length of earth and prevents concentration of earth currents at tower footings. In this manner tower-footing potentials are kept to low values and even large lightning strokes are unable to cause flashover from tower to line conductor.

4. Tower-footing grounding cables, or crowfeet, installed during 1930 and 1931 reduced tower-footing resistances by about half. Since then 2 or possibly 4 isolated towers have suffered flashover, all involving one phase only. At 2 towers the flashover involved the middle phase only, which has only 14 units of insulation (2 less than the outside) but the time of occurrence is unknown. In the first certain case a severe stroke occurred causing surge indicators to operate on 16 unit assemblies at 3 successive towers but insulators were found burned at only one. This flashover caused a line tripout. In the other certain case all 3 assemblies have uniform 14 unit insulation, and this flashover has not been correlated with a line tripout.

5. The lightning stroke diverting cables installed during 1930, and which are specially constructed overhead ground wires with an extensive buried tower-footing network, have effectively protected the 18 towers which they cover.

6. All lightning strokes except a very minor one have been of negative polarity, from negative clouds to grounded positive structures. The maximum stroke current (sum of measured currents in adjacent structures) was about 110,000 amperes, but most have been much less, and some have been so small that the surge crest ammeter failed to give registrations. These latter are known only by the insulator flashovers they have caused.

Table VIII—Phases Affected in Tripouts of the Wallenpaupack-Siegfried Line, 1928–35

Number of Faulted Phases	Overhead Ground Wire Section: Number	Overhead Ground Wire Section: Per Cent	Unprotected Line Section: Number	Unprotected Line Section: Per Cent	Total: Number	Total: Per Cent
1	2*	50	71	80	73	78.5
2	1**	25	10	11	11	11.8
3	1***	25	8	9	9	9.7
Total	4	100	89	100	93	100

* Tower footing resistances 26 and 13 ohms. ** Tower footing resistance 98 ohms. *** Tower footing resistance 43 ohms.

7. Surge indicators have been of great value in locating insulator flashovers and in indicating towers which have been struck by lightning.

8. Lightning stroke recorders have also been of use, but their records have never been interpreted in a fully satisfactory manner, particularly in regard to current magnitude.

Table IX—Tower-Footing Potential Associated With Multiphase Insulator Flashovers

Item Number	Number of Phases Faulted	Total Stroke Current, Amperes	Maximum Tower Footing Potential, Kilovolts	Corresponding Footing Ohms	Corresponding Structure Amperes	Tripout	Number of Insulator Units in Flashed Assemblies	Number of Towers Affected
1	3	109,150	1,520	28	54,400	1935, No. 7	14	5
2	3	61,240	1,310	329	4,000	1935, No. 9	14	5
3*	3	51,000	1,730	34	51,000	1933, No. 11	14 and 16	1
4*	3	40,000	1,800	45	40,000	1933, No. 11	14	1
5	2	30,000	840	28	30,000	1934, No. 10	14	1
6**	2	20,000	900	101	8,950	1935, No. 8	14 and 16	2

* For these items a 3-phase flashover occurred at a single tower. Only one flashover caused a line tripout.
** This fault started single phase at 2 adjacent towers, and involved a second phase on one tower after 5 cycles.
Tripouts 1 and 4 in 1933 not reported because no surge crest ammeters were then installed.
Tripout 12, 1933, and tripout 4, 1934, omitted because flashovers occurred at mast structures where surge-crest ammeters do not operate satisfactorily or yield more than traces of current, if any.
All of these records were from open sections of line. The flashover strengths of 14 and 16 unit assemblies for positive surges are respectively 1,565 kv and 1,775 kv for 1.0x5 microsecond surge, 1,265 kv and 1,425 kv for 1.5x40 microsecond surge.

9. During September 1935 10 single-phase installations of expulsion protective gaps were made on 10 successive open-line towers. No results were obtained during the remainder of that season.

10. The method of locating faults by means of the ratios of dynamic fault currents has been of great assistance in permitting the various data pertaining to a lightning fault to be correlated.

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Electronic Transient Visualizers

Cathode-ray oscillographs, a few years ago found only in research laboratories, have become extremely useful tools in the hands of operating engineers, research technicians, and teachers alike; however, the usefulness of cathode-ray oscillographs of the ordinary portable type is limited almost entirely to the observation of sustained periodic phenomena, and is of little use in the study of transients. Methods of extending the field of application of this instrument to include visual observation of circuit and line transients are presented in this paper.

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THE IMPORTANCE of a complete understanding of the transient behavior of electrical systems results in the need, both in the commercial laboratory and in the class and lecture room, of simple oscillographic equipment for the study of transient phenomena in circuits and lines. The ordinary portable cathode-ray oscillograph employing a relaxation oscillator to provide a linear time scale is entirely satisfactory in the observation of sustained periodic phenomena, but is of little help in the study of transients. Extension of the field of application of this instrument to include visual observation of circuit and line transients greatly increases its value. This can be done by the simple expedient of repeating the transients in synchronism with the sweep voltage. A "transient visualizer" that accomplishes this result by means of a relay opened and closed by the sweep oscillator has been described by the author.[1] Although this device functioned well at low frequency, it had the limitations imposed by relay inertia and contact chatter. Experiments then in progress on the replacement of the relay by a vacuum-tube circuit had not advanced sufficiently to justify a detailed description at that time. The present paper describes several types of electronic transient visualizers that have been evolved from the original circuit.

Before proceeding to a discussion of particular circuits it is important to note that the function of the transient visualizer is merely to initiate a transient at the beginning of each sweep of the luminous spot across the fluorescent screen, either by closing or opening the transient circuit. In order to be of most general applicability it is essential that the device should have the characteristics of a switch at least during the initiation and duration of the transient. It should not in itself possess inductance, capacitance, or resistance, and should not affect the behavior of the transient circuit for the duration of the transient. This requirement immediately prohibits the use of devices in which the charge or discharge of a capacitor or inductor is inherently associated with the action of that part of the switching device through which the transient current flows.

The 2-tube "parallel" switching circuit,[2,3] shown in simplest form in figure 1, fulfills the requirements if it is used properly. The circuit employs 2 grid-controlled arc rectifiers. To understand the operation of this circuit, it is necessary to recall that the grid of an arc rectifier is effective only in preventing the tube from firing, and, except at low values of anode current, has no effect upon the value of the anode current. Once the tube has fired, anode current can be stopped only by removing the anode voltage for a sufficient time to allow deionization to take place. The voltage drop through the tube is practically constant at a value somewhat greater than the ionization potential of the gas or vapor throughout the operating range of anode current, and the value of the anode current is determined solely by the external impedance of the anode circuit and by the applied voltage minus the constant tube voltage drop. Such a tube therefore has the characteristics of a zero-resistance switch that passes current in one direction only, in series with a counter electromotive force equal to the tube voltage drop. It may be so represented in an equivalent circuit.

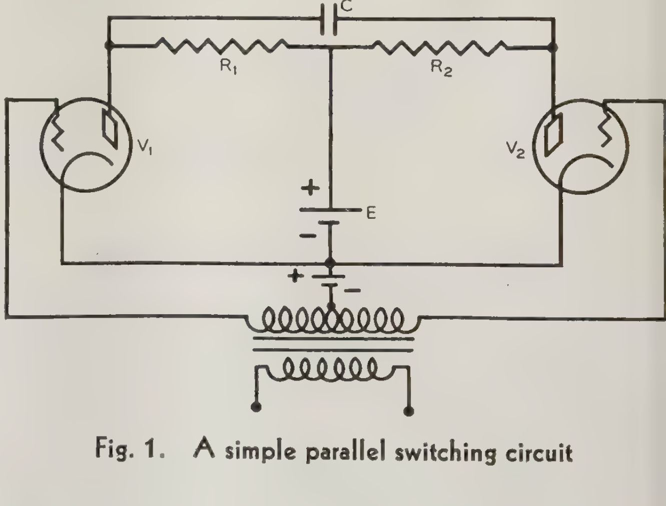

Fig. 1. A simple parallel switching circuit

The operation of the circuit of figure 1 is as follows. Suppose that the application of a pulse of positive grid voltage to tube 1 has caused it to fire. The flow of current through R_1 results in an IR drop

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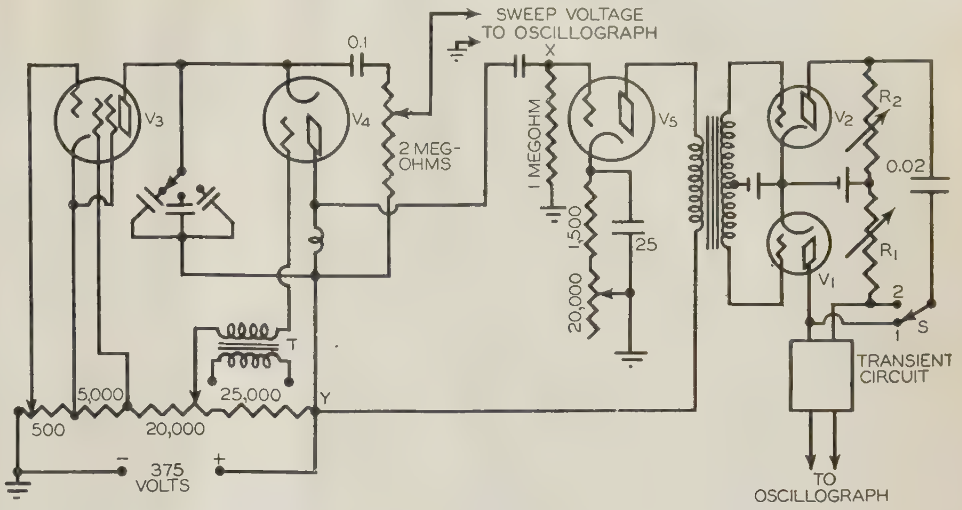
1. For all numbered references, see list at end of paper.

equal to the applied voltage E less the voltage drop V_{ad} through the tube. This charges the capacitor C in such a direction as to make the end adjacent to tube 1 negative. The application of another pulse of grid voltage of such polarity as to make the grid of tube 1 negative and that of tube 2 positive causes tube 2 to fire and lowers the voltage of its anode from E to V_{ad}. Since C cannot discharge instantaneously, the anode voltage of tube 1 must also be lowered by the amount $E - V_{ad}$ to the value $-(E - 2V_{ad})$. Because the tube passes current in only one direction, this negative anode voltage stops

Fig. 4. Transient visualizer circuit based upon the parallel switching circuit of figure 1

Numbers beside symbols indicate values of circuit constants in ohms and microfarads, except where specifically designated otherwise

V_1, V_2—Type *FG*-67 thyratrons or type 885 gas-discharge tubes
V_3—Type 58 pentode
V_4—Type 885 gas-discharge tube
V_5—Type 56 triode



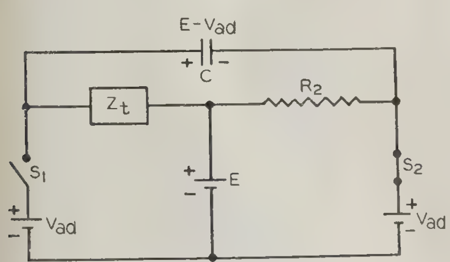


Fig. 2. An equivalent of the parallel switching circuit of figure 1, for observing starting transients

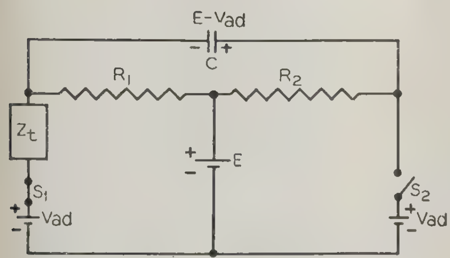


Fig. 3. An equivalent of the parallel switching circuit of figure 1, for observing stopping transients

the anode current of tube 1. The capacitor immediately starts to discharge and subsequently to charge in the reverse direction to a voltage equal to the IR drop in R_2, and eventually the anode of tube 1 again becomes positive. If the time taken for the anode of tube 1 to become positive exceeds the deionization time of the tube, and its grid is sufficiently negative, the discharge will remain extinguished. Current can be caused to transfer from one tube to the other merely by applying grid-voltage pulses of such polarity as to make the grid of the nonconducting tube positive (or insufficiently negative to prevent firing) and the other more negative.

If the resistance R_1 of figure 1 is replaced by the circuit of which the transient response is to be studied, then the firing of tube 1 will initiate a transient in this circuit. That the form of the transient will not be affected by the switching circuit can be seen by an examination of the equivalent circuit of figure 2, in which the tubes have been replaced by switches of zero resistance in series with counter electromotive forces equal to the tube voltage drops. Closing S_1 is equivalent to firing tube 1. Because there is no resistance in the tube and battery branches of the network, the current that flows through the transient circuit Z_t is unaffected by the other circuit parameters. This is not true when tube 2 fires (S_2 closes after S_1) because tube 1 is extinguished as soon as tube 2 fires, and the capacitor discharges through Z_t. Thus, the circuit of figure 2 will give unmodified starting transients, but distorted stopping transients.

Unmodified stopping transients in many types of circuits can be produced by placing the transient circuit in series with the anode of one tube, as in the equivalent circuit of figure 3. The firing of tube 2 (indicated by the closing of S_2) will extinguish tube 1 (indicated by the opening of S_1) if the transient voltage induced by the opening of S_1 remains less than the capacitor voltage for a length of time sufficient to allow the tube to deionize. This is possible with transient circuits whose impulsive response is not inductive. If tube 1 is extinguished, the transient circuit is connected to the remainder of the network only through the small interelectrode capacitance of the tube, and its transient behavior is practically independent of all but its own parameters.

To use these switching circuits in the oscillographic study of transients the grid excitation voltage of V_1 and V_2 is obtained from the sweep oscillator that provides the linear time axis. One transient is produced for each sweep of the luminous spot, and a stationary image is obtained. The sweep frequency may be made sufficiently low to allow each transient to die out so that it does not appreciably affect the form of the next.

Several circuits based on the parallel switching circuit were developed by John A. Bennett at Cornell University[4] and by the author at the University of Illinois. One of these is shown in figure 4. In this

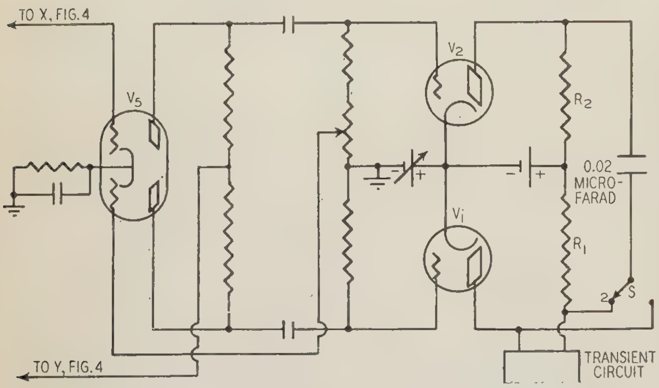
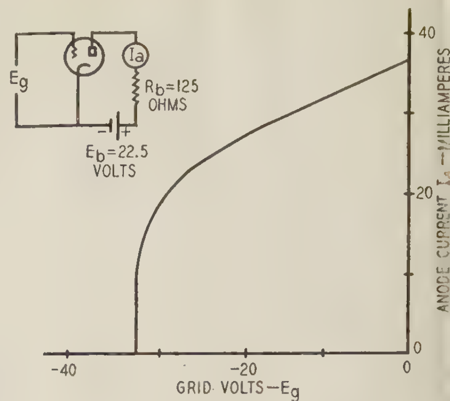


Fig. 5. A variation of the transient-visualizer circuit of figure 4

V_1, V_2—Type *FG*-67 thyratrons
V_5—Type 53 double triode

circuit the grid excitation is in the form of a short voltage pulse of one polarity, followed immediately by a short pulse of opposite polarity. By the use of a center-tapped coupling transformer the voltage is applied to the grids of the switching tubes V_1 and V_2 in such a direction that the grid of V_1 is made positive and that of V_2 negative during the first pulse, causing V_2 to fire shortly after V_1. By the introduction of suitable delay networks, by the adjustment of grid-bias voltages of V_1 and V_2, and by the reversal of primary or secondary transformer connections, the conduction time of V_1 can be varied.

Fig. 6. Curve of anode current versus negative grid voltage for a type 885 gas-discharge tube



When the switch S is in position 1 and R_1 is zero, the circuit is that of figure 2; when the switch is in position 2 the circuit is that of figure 3. The required difference of polarity of the pulses applied to the grids of V_1 and V_2 can also be obtained by the use of the "phase inverter" circuit of figure 5, instead of the center-tapped transformer of figure 4. In this modification, variation of the relative conducting times of V_1 and V_2 is accomplished by adjusting the grid-biasing voltage of these tubes.

Although the operation of these parallel switching circuits was found to be excellent at high currents, the action was observed to be somewhat erratic when an attempt was made to work with small currents. Further investigation revealed that the difficulty resulted from the fact that at small currents the anode current of grid-controlled arc-discharge tubes is not independent of grid bias, but may be decreased and even cut off by the application of

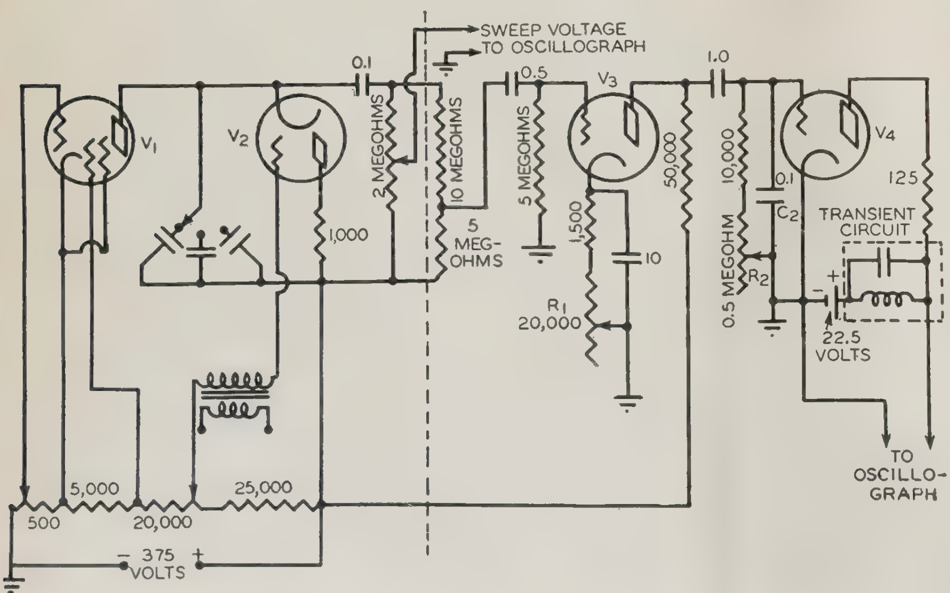


Fig. 7. Combined sweep-oscillator and single-tube transient-visualizer circuit

Numbers beside symbols indicate values of circuit constants in ohms and microfarads, except where specifically designated otherwise

V_1—Type 58 pentode
V_2, V_4—Type 885 gas-discharge tubes
V_3—Type 56 triode

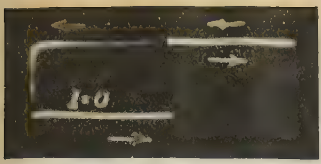


Fig. 8. Oscillogram of starting and stopping of current through a resistor

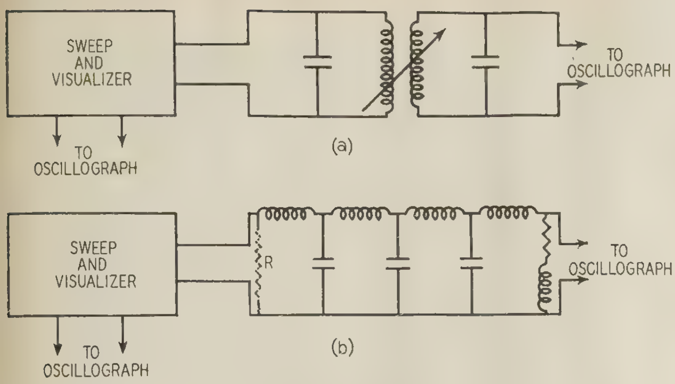


Fig. 9. Methods of connecting visualizer for the study of transients

(a)—For coupled circuits *(b)*—For lines

negative grid voltage. A typical curve of anode current versus negative grid voltage for a type 885 "gas discharge" tube is shown in figure 6.

Fortunately, this dependence of anode current upon grid voltage, although undesirable in the 2-tube circuit, makes possible the design of a simplified

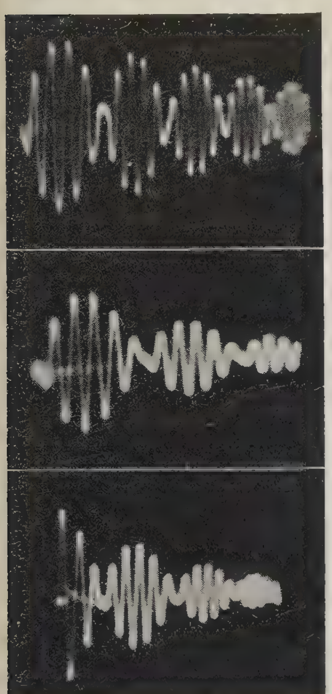


Fig. 10. Oscillograms of transient oscillations in coupled circuits

single-tube circuit. If the saw-tooth voltage of the sweep oscillator is applied to the grid of a type 885 tube in such a direction that the abrupt voltage change at the end of the sweep makes the grid highly negative, then, because of the rapidity of this change in voltage, the anode current of the type 885 tube will fall to zero in a small fraction of a second. The tube fires instantaneously when the negative grid voltage has fallen to a critical value, the magnitude of which depends upon the applied anode voltage. The tube has, therefore, the required switching characteristics both for starting and interrupting current flow, and may be used in initiating repeated transients in synchronism with the sweep voltage. The combined sweep oscillator and single-tube transient visualizer is shown in figure 7. The portion of the circuit to the left of the dotted line is a standard sweep oscillator; the portion to the right represents the addition that must be made to the circuit for transient visualization. In many transient studies the 22.5-volt battery in the visualizer circuit may be replaced by a tap on the main anode-voltage divider. The 125-ohm resistance in the anode circuit of V_4 may be reduced or eliminated if the resistance of the transient circuit is sufficient to limit the current to a value so small that the grid can

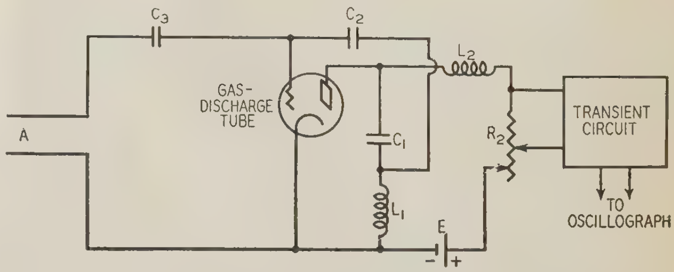


Fig. 11. Transient-visualizer circuit based upon the relaxation-oscillator circuit

extinguish the tube (50 milliamperes or less). The function of C_2 is to delay the firing of V_4 until after the luminous spot has returned to the beginning of its timing sweep. The conduction time of V_4 is adjusted by varying R_1 and R_2.

Because of the dependence of anode current upon grid voltage of the gas-discharge tube, the transient initiated by this circuit would be distorted if there were appreciable variation of grid voltage during the life of the transient. Actually this grid voltage variation is small, because the tube fires when the grid is less than 4 volts negative, and flow of grid current tends to prevent the grid from becoming positive.

Figure 8 shows an oscillogram of current through a resistance, started and interrupted by the transient visualizer. The direction in which the oscillogram was traced is indicated by the arrows. The abruptness with which the current rises and falls affords proof of the suitability of the device for transient visualization. The portion of the oscillogram that shows the rise of current was traced during the forward sweep. The sweep frequency was about 100 cycles; therefore, this portion was traced during approximately 0.01 second. The part of the oscillogram showing current interruption was traced during the return sweep of the luminous spot. The estimated time of return is 0.00002 second. The

lack of abruptness of current cut-off may be attributed to distributed circuit capacitance and inductance. Figure 9 shows the manner in which the device may be used in the study of transient oscillations in coupled circuits and in lines. The function of the resistance R in figure 9*b* is to discharge the line between successive chargings, and need be used only when the line has low leakage and is open at the far end. Typical transients obtained by the use of this visualizer are shown in figure 10. The oscillograms were obtained by photographing the screen of an oscillograph tube with an ordinary uncorrected double-convex lens. The stability of the image is indicated by the sharpness of the photographs despite exposures as long as 15 seconds.

Figure 11 shows another grid-controlled rectifier tube circuit[5] which, although it does not meet the switching requirements set up at the beginning of this paper, is frequently useful. Assuming that C_1 has been charged by the battery, and that the tube grid is negative, the action is as follows: The voltage from the sweep oscillator, properly amplified, is applied at A in such a direction that the sudden voltage change at the end of the sweep makes the grid positive. This causes the tube to fire and allows C_1 to discharge through the tube. Because of the action of L_1, anode current continues to flow until C_1 is charged with opposite polarity to a voltage somewhat lower than the applied voltage. The voltage induced in L_1 causes a flow of grid current during this discharge and therefore charges the capacitors C_2 and C_3 in such a direction as to leave the grid negative. Because of the rectifying action of the grid, this charge cannot leak off through the grid, and so the tube is prevented from firing until another positive impulse is applied at A. The capacitor C_1 starts recharging through R_2 and L_2 in a manner determined by the voltage E, the capacitance C_1, the resistance R_2, and the inductances L_1 and L_2, and, to some extent, the constants of the transient circuit that shunts R_2. The resulting voltage pulse produced in R_2 sets up transient oscillations in the transient circuit. In certain studies the transient circuit may be connected in series with, or in place of R_2, instead of in shunt with R_2. The impulse applied to the transient circuit may be obtained from a resistance in series with L_1 as shown in figure 12. This gives impulses of shorter duration. The impulse may also be obtained from the capacitor C_1, or the inductances L_1 and L_2. Interaction of the impulse generator circuit with the transient circuit can be prevented by the use of a single-stage buffer amplifier between the generator and the transient circuit. A wide choice of impulse shape may be obtained by varying the circuit parameters of the impulse generator and the point from which the impulse is obtained. The shapes of typical impulses are shown in figure 13. The portions of these curves lying between points *a* and *b* correspond to the time during which the tube conducts. Increase of L_1 increases the length of this portion of the impulse; increase of R_2 lengthens the remainder of the impulse; and increase of C_1 lengthens both parts of the impulse. In the circuit of figure 12 the grid bias necessary to keep the tube from firing is obtained from a "C battery," instead of by the rectifying action of the grid.

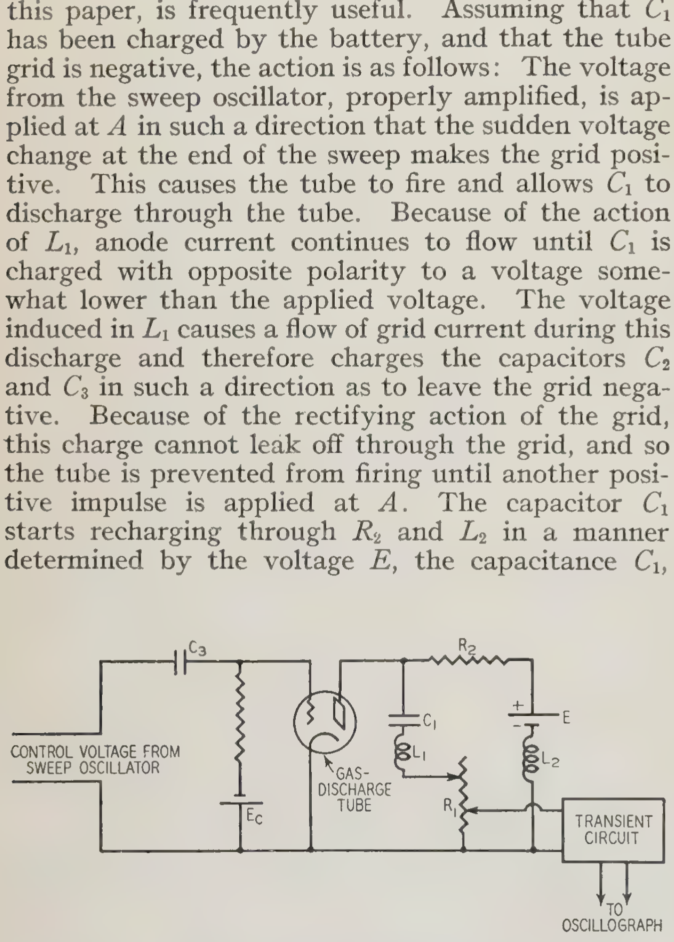


Fig. 12. A variation of the transient-visualizer circuit of figure 11

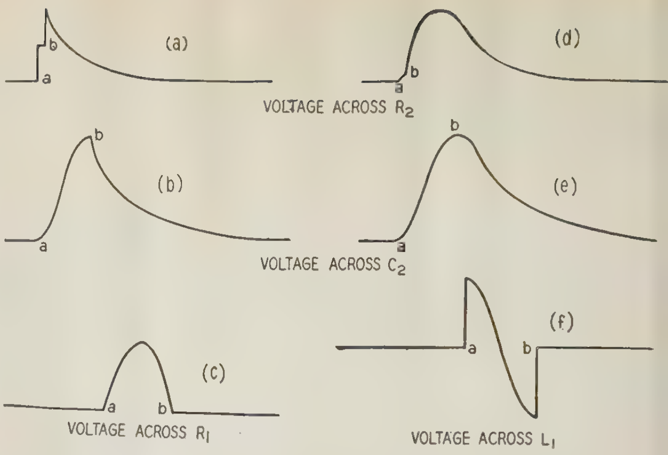


Fig. 13. Typical voltage impulses produced by the circuits of figures 11 and 12

(*a*) and (*b*)—$L_2 = 0$

(*d*), and (*e*)—$(R_1 + R_2)^2 = \dfrac{4(L_2 + L_1)}{C_1}$

(*c*) and (*f*)—$L_1 \ll L_2$ and $R_1 \ll R_2$

The circuits described have been found to be of great value in the lecture demonstration and laboratory study of circuit and line transients. They may prove to be of value in commercial laboratories. The possibility of observing directly the effects of variation of circuit parameters is an outstanding advantage of this method. An important feature of these circuits is that they can be added with little complication to a standard sweep oscillator.

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Calculation of Resistances to Ground

Formulas for practical use in the calculation of the resistances from grounding conductors of various forms to the earth are given in this paper and their use illustrated by examples. The accuracy of the formulas varies considerably, as discussed in the paper, but is sufficiently good that the methods should be helpful to those whose work involves problems of grounding.

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AN ELECTRICAL CONNECTION to the ground requires consideration of the engineering problem of obtaining the lowest number of ohms to ground for a given cost. This problem involves the need of formulas for comparing different arrangements of ground conductors. A collection of such formulas for d-c resistance is given in this paper, with some discussion as to their relative accuracy. In order to compare 2 arrangements of conductors, it is usual to assume that they are both placed in earth of the same uniform conductivity. It is well-known that there is usually considerable variation of earth conductivity in the vicinity of ground conductors, but that effect is a separate problem.

Cylindrical Conductor

A very common type of ground connector is that of a vertical ground rod. Such a rod is an isolated cylinder and the flow of electricity from it into and through the ground is calculated by the same expressions as is the flow of dielectric flux from an isolated charged cylinder. That is, the problem of the resistance to ground of the ground rod is essentially the same as that of the capacitance of an isolated cylinder whose length is very great compared to its radius. The following formula for the latter case has been given by E. Hallén:[1]

$$\frac{C}{L} = \frac{1}{\lambda} + \frac{1.22741}{4\lambda^2} + \frac{2.17353}{8\lambda^3} + \frac{11.0360}{16\lambda^4} \cdots$$

where

$\lambda = \log_\epsilon (2L/a)$
$2L$ = length of the isolated cylinder in centimeters
a = radius of the cylinder in centimeters
C = capacitance in absolute electrostatic units, or statfarads
$\log_\epsilon$ denotes natural logarithm

By taking unity divided by this series, an expression is obtained which is more convenient for the present purpose, and which is more rapidly convergent, to a slight extent, as follows:

$$\frac{1}{C} = \frac{1}{L}\left(\lambda - 0.306852 - \frac{0.17753}{\lambda} - \frac{0.5519}{\lambda^2} \cdots\right) \tag{1}$$

This formula has been checked, and close agreement found, by a successive approximation calculation[2] in which uniform distribution of charge is first assumed and then other distributions of charge are successively added, so as to keep the potential of the cylinder the same throughout. The formula has been checked also by Dr. F. W. Grover[3] by a successive approximation method using mechanical integration.

An approximate method of calculation which is used for a great many shapes of conductors is the average potential method of Dr. G. W. O. Howe.[4] This consists in assuming uniform charge density over the surface of the conductor and calculating the average potential. Then the approximate capacitance is taken as equal to the total charge divided by the average potential. This method is correct within 2 or 3 parts in 1,000 for a long straight antenna wire, and within less than 1 per cent for a cylinder of the proportions of a ground rod. However, an estimate of its accuracy should be made for each shape or combination of conductors with which it is used, because in some cases it gives an error of several per cent. For example, it gives a value of $1/C$ for a thin round plate which is 8 per cent too high (see the paragraph following equation 35).

Assuming a uniform charge density q per centimeter along the cylinder, on its curved surface, the potential at P (figure 1) because of the ring dy is, by equation 6, page 153, reference 10,

$$\frac{q\,dy}{a}\left[\frac{a}{r} - \frac{1}{2^2}\frac{a^3}{r^3}\left(\frac{3y^2}{r^2} - 1\right) + \cdots\right] \tag{2}$$

where $r^2 = a^2 + y^2$ and y is the distance from P to the ring.

Integrate equation 2 from $y = 0$ to $L - x$ and also from 0 to $L + x$, obtaining the potential at P resulting from the parts of the cylinder to the right and left of P. Then multiply by dx/L and integrate from $x = 0$ to L, giving the average potential of the cylinder resulting from uniform charge density on its curved surface, as follows:

$$\frac{V_{av}}{2q} = \log_\epsilon\frac{4L}{a} - 1 + \frac{a}{L}\left(\frac{1}{2} + \frac{1}{8} + \frac{1}{128} + \cdots\right) - \frac{a^2}{L^2}\left(\frac{3}{16} - \frac{1}{32}\cdots\right) + \frac{a^4}{L^4}\left(\frac{1}{64} - \frac{1}{1024}\cdots\right) \tag{3}$$

$$\frac{1}{C} = \frac{V_{av}}{2qL} = \frac{1}{L}\left(\log_\epsilon\frac{4L}{a} - 1 + 0.63\frac{a}{L} - 0.16\frac{a^2}{L^2} + 0.015\frac{a^4}{L^4}\cdots\right) \tag{4}$$

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1. For all numbered references see list at end of paper.

If, as is often done, only the first term of equation 2 is used, the result is

$$\frac{1}{C} = \frac{1}{L}\left(\log_e \frac{2L + \sqrt{a^2 + 4L^2}}{a} + \frac{a}{2L} - \sqrt{1 + \frac{a^2}{4L^2}}\right) \tag{5}$$

which, when expanded, becomes

$$\frac{1}{C} = \frac{1}{L}\left(\log_e \frac{4L}{a} - 1 + 0.5\frac{a}{L} - 0.06\frac{a^2}{L^2} + 0.002\frac{a^4}{L^4}\cdots\right) \tag{6}$$

It may be seen that equation 6 does not have the right coefficients for terms in a/L and so equations 6 and 5 should not be used. For long cylinders of the proportions of wires or vertical ground rods, the terms in a/L and its powers are of very small effect and may be omitted, thus giving the practical formula

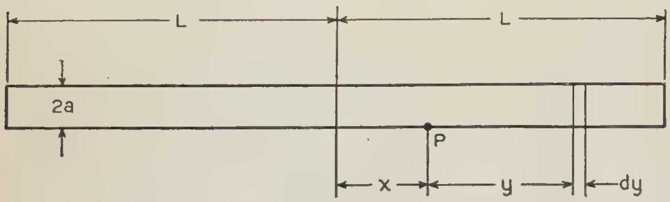

Fig. 1 (above). Cylindrical conductor

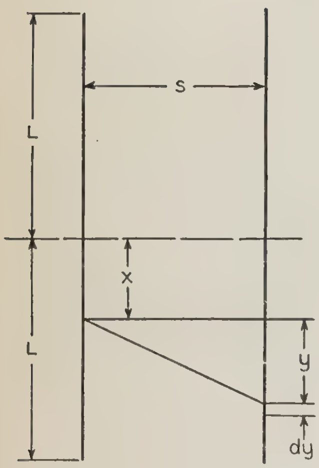

Fig. 2 (left). Two rods connected in parallel

$$\frac{1}{C} = \frac{1}{L}\left(\log_e \frac{4L}{a} - 1\right) \tag{7}$$

This gives an error of less than one per cent in practical cases of resistance to ground. The error is due to the approximation inherent in the average potential method and if a more accurate formula is desired, equation 1 should be used. The small error in equation 7 cannot be avoided by including terms in a/L or by using more complicated expressions in logarithms or inverse hyperbolic functions such as equation 5 or its equivalent forms.

The formula

$$\frac{1}{C} = \frac{1}{L}\log_e \frac{2L}{a} \tag{8}$$

which is based on the capacitance of an ellipsoid of revolution of the same diameter and length as the cylinder, has a larger error than equation 7, and should not be used. Logarithms to base 10 may be used, by noting that $\log_\epsilon p - 2.303 \log_{10} p$.

The method of changing a formula for capacitance into one for resistance to ground, or through the ground, may be found by considering the simple case of 2 parallel plates whose distance apart is small and the effect of whose edges may be neglected.

If each of the 2 plates has an area of B square centimeters and if the charge density on one is q per square centimeter and that on the other $-q$ per square centimeter, the number of lines of dielectric flux through air from one plate to the other is $4\pi qB$. The volts per centimeter in the space between the plates is equal to $4\pi q$, the density of the lines, and the potential difference V between the plates is $4\pi qs$, where the separation of the plates is s centimeters. Then

$$\frac{1}{C} = \frac{V}{qB} = \frac{4\pi s}{B} \tag{9}$$

For the flow of electricity between the same plates when they are embedded in earth of resistivity ρ abohms per centimeter cube, the resistance between the plates in abohms is

$$R = \frac{\rho s}{B} \tag{10}$$

Thus in this case,

$$R = \frac{\rho}{4\pi C} \tag{11}$$

where C is in statfarads. If ρ is in ohms per centimeter cube, R will be in ohms. Equation 11 shows merely the relation between the units and has nothing to do with the geometry of the flow of dielectric flux and current which in equations 9 and 10 is represented by the letters s/B. Equation 11 applies to any conductor or combination of conductors (see reference 11, appendix III, page 219).

The resistance of a buried straight wire of length $2L$ centimeters, no part of which is near the surface of the ground, is given by equations 7 and 11 and is

$$R = \frac{\rho}{4\pi L}\left(\log_\epsilon \frac{4L}{a} - 1\right) \tag{12}$$

A vertical ground rod which penetrates to a depth of L centimeters must be considered along with its image in the ground surface. The voltage and the shape of current flow are the same as for a completely buried cylinder of length $2L$ centimeters, but the total current is half as much, making the resistance twice as great. Therefore, the resistance to ground of a vertical ground rod of depth L centimeters is

$$R = \frac{\rho}{2\pi L}\left(\log_\epsilon \frac{4L}{a} - 1\right) \tag{13}$$

In general, if C includes the capacitance of the images of a conductor or conductors in the earth, the resistance to ground is

$$R = \frac{\rho}{2\pi C} \tag{14}$$

Vertical ground rods are widely used, sometimes in large groups, because they can be driven in place with little or no excavation and because they are likely to reach permanent moisture and earth of good conductivity.

Two Ground Rods in Parallel

Let 2 ground rods of radius a centimeters be as shown in figure 2, and let them be electrically in parallel. Find the air capacitance C of the 2 rods and their images, that is, of 2 cylinders each of length $2L$. Let the cylinders have a uniform charge of q per centimeter. The potential at a point on one cylinder at a distance x from its center, caused by $q\,dy$ on the other cylinder, is

$$\frac{q\,dy}{\sqrt{s^2 + y^2}}$$

The potential at x caused by the other cylinder is obtained by integrating this expression from $y = 0$ to $L - x$ and from $y = 0$ to $L + x$. The average potential on one cylinder caused by uniform charge on the other is then obtained by multiplying by dx/L and integrating from $x = 0$ to L, and is

$$q\left(2 \log_e \frac{2L + \sqrt{s^2 + 4L^2}}{s} + \frac{s}{L} - \frac{\sqrt{s^2 + 4L^2}}{L}\right) \tag{15}$$

For large values of s/L, this becomes

$$\frac{2qL}{s}\left(1 - \frac{L^2}{3s^2} + \frac{2}{5}\frac{L^4}{s^4} \dots\right) \tag{16}$$

For small values of s/L, it is

$$2q\left(\log_e \frac{4L}{s} - 1 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4} \dots\right) \tag{17}$$

Add the average potential of the cylinder caused by its own charge,

$$\frac{2qL}{C} = 2q\left(\log_e \frac{4L}{a} - 1\right) \tag{18}$$

Divide by $4qL$, the total charge on the 2 cylinders, thus obtaining the value of $1/C$ for the pair of ground rods and their images. Then, by equation 14, the resistance to ground of the pair of rods is

$$R = \frac{\rho}{4\pi L}\left(\log_e \frac{4L}{a} - 1 + \log_e \frac{2L + \sqrt{s^2 + 4L^2}}{s} + \frac{s}{2L} - \frac{\sqrt{s^2 + 4L^2}}{2L}\right) \tag{19}$$

or, for large values of s/L,

$$R = \frac{\rho}{4\pi L}\left(\log_e \frac{4L}{a} - 1\right) + \frac{\rho}{4\pi s}\left(1 - \frac{L^2}{3s^2} + \frac{2}{5}\frac{L^4}{s^4} \dots\right) \tag{20}$$

or, for small values of s/L,

$$R = \frac{\rho}{4\pi L}\left(\log_e \frac{4L}{a} + \log_e \frac{4L}{s} - 2 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4} \dots\right) \tag{21}$$

A short alternative calculation for equation 20, which is found by trial to be good for spacings of 20 feet or more, is to replace each ground rod by a half-buried sphere which is equivalent in resistance to one isolated ground rod. This procedure can be illustrated by a numerical example. The resistance to earth of a ground rod of $^3/_4$-inch diameter and 10-foot depth, by equation 13, is

$$R = \frac{\rho}{2\pi L} \times 6.155$$

The hemisphere which is buried and its image above the ground surface make a complete sphere whose air capacitance as an isolated conductor is, by a well-known proposition, equal to its radius A in centimeters. By equation 14, the resistance to ground of the hemisphere is

$$R = \frac{\rho}{2\pi A}$$

Then the radius of the hemisphere which is equivalent to the isolated ground rod is, by equation 13,

$$A = \frac{L}{\log_e \frac{4L}{a} - 1} = \frac{L}{6.155}$$

Here, A and L may be both in centimeters or both in feet and so $A = 10/6.155 = 1.625$ feet.

The capacitance of 2 equal spheres at a distance s, center to center, connected in parallel, is readily calculated when s is not small and the charges can be assumed uniformly distributed around the spheres. By symmetry, the spheres will carry equal charges. Let each charge be q. The potential of the surface of each sphere is

$$q\left(\frac{1}{A} + \frac{1}{s}\right)$$

and

$$\frac{1}{C} = \frac{1}{2}\left(\frac{1}{A} + \frac{1}{s}\right)$$

where the dimensions are in centimeters. Then

$$R = \frac{\rho}{4\pi}\left(\frac{1}{A} + \frac{1}{s}\right) \tag{20a}$$

This is the same as using the first term of the second part of equation 20.

From numerical examples, it is found that the results of equations 19 and 20a differ by about 0.5 per cent for 20-foot spacing between 2 ground rods, and by a few per cent for 10-foot spacing.

Groups of Ground Rods

The potential of a rod caused by its own charge and the charges of several other rods can be found by using equation 16 or 17 several times. Similarly, the potential of the surface of a sphere caused by its own charge and the charges of a number of other spheres is

$$\frac{q_1}{A} + \frac{q_2}{s_2} + \frac{q_3}{s_3} + \dots$$

where for an approximate calculation q_1, q_2, q_3, etc., may be assumed equal.

For somewhat better accuracy, the values of q for the rods near the center of the group may be taken a little lower than the values for the rods near the outer parts of the group, by an amount sufficient to make the potential of each rod the same. Thus the values for the outer parts may be taken equal to q, and others to $0.95q$, $0.9q$, etc., according to judgment and to the test of equal potentials.

In order to design groups of ground rods for transmission line towers, stations and substation grounds, etc., and to decide on the best number and spacing of rods, it is desirable to be able to compare various groups of rods, assuming uniform conductivity of the soil. Accordingly, sets of curves are given in figures 3 to 6.[2] From them it is possible to estimate how many rods and how much area will be required for a certain number of ohms to ground in a given locality, from measurements on a few temporary test ground rods.

The basis of comparison is the resistance of one isolated ground rod of $^3/_4$-inch diameter and 10-foot depth. By means of figure 3, the resistance of single rods of various depths can be found and, if desired, the resistivity of the soil. In figure 4 the conductivity of 2, 3, and 4 rods is given in terms of the conductivity of isolated rods, and in figure 5 results are shown for larger numbers of rods.

In figure 6 the information is presented in different form, and the lowest resistance that can be obtained from a given area is shown. This information is often of value as it may be the means of preventing the wasteful attempt of putting additional ground rods in an area which could not give the desired low resistance even if an infinite number of rods were used. For instance, if 36 ground rods are distributed as in figure 7 over a square area of 10,000 square feet, that is, 100 by 100 feet, the spacing will be 20 feet. Figure 6 shows that the resistance is 1.3 times the resistance of an infinite number of rods in the same area. Therefore, no matter what the resistance in ohms may be or what the uniform conductivity of the ground may be, if it is desired to have less than 75 per cent of the resistance with 36 rods there is no use in putting more rods in the area of 10,000 square feet but it will be necessary to use a larger area.

Figure 6 shows also that if there are more than 10 rods 10 feet deep, there is no use in having closer than 10-foot spacing.

It is generally desirable not only to utilize the ground area effectively, but to make effective use of the rods, inasmuch as they represent a considerable cost. If one wishes the rods to be at least 60 per cent as effective as they would be if isolated, that is, to have at least 60 per cent of the conductivity of isolated rods, it may be seen from both figures 5 and 6 that it is necessary to use over 20-foot spacing for 16 rods in a square area, about 30-foot spacing for 25 rods, and over 40-foot spacing for 49 rods.

The curves of figures 5 and 6 are based on uniform distribution of ground rods in square areas, as shown in figure 7, the boundary of the area running through the outer rods. Although the curves are computed for square areas, they give an estimate for rectangular areas. The curves cannot be used for a part of a group of rods nor for a group which is near other groups, but a reading from the curves is to apply to

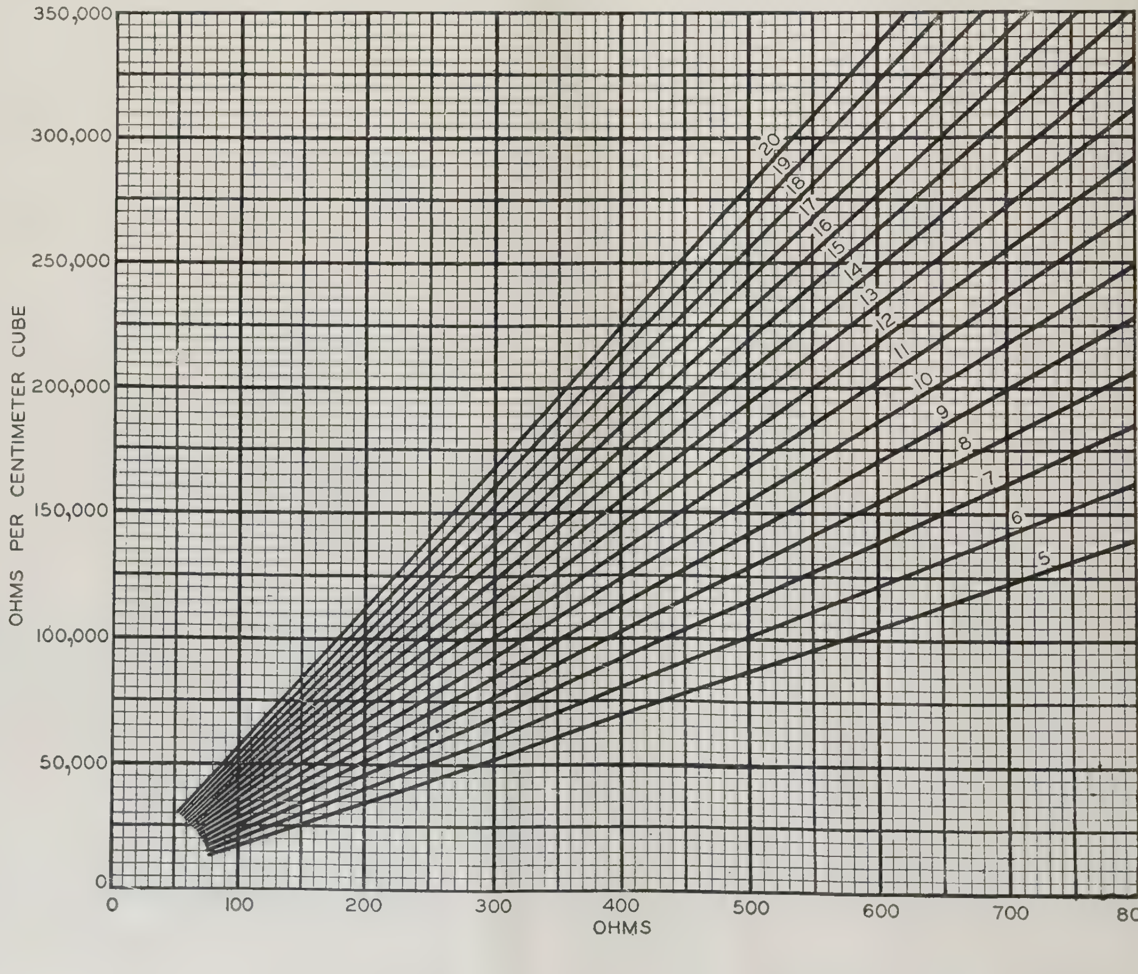

Fig. 3. Resistance of one ground rod of $^3/_4$-inch diameter

For low resistances divide both scales by 10; for high resistances multiply both scales by 10

Numbers on curves are depths in feet

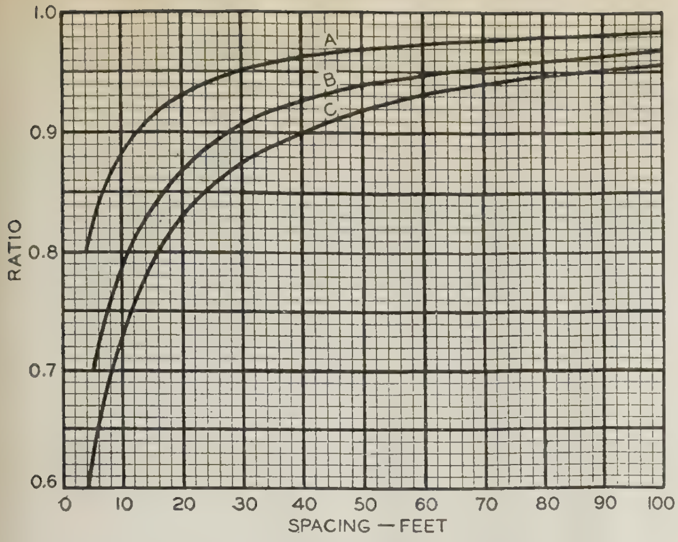

Fig. 4. Ratio of conductivity of ground rods in parallel to that of isolated rods

Ground rods are $^3/_4$ inch in diameter, 10 feet deep; 3 rods on equilateral triangle, 4 rods on square
A—Two rods. B—Three rods C—Four rods

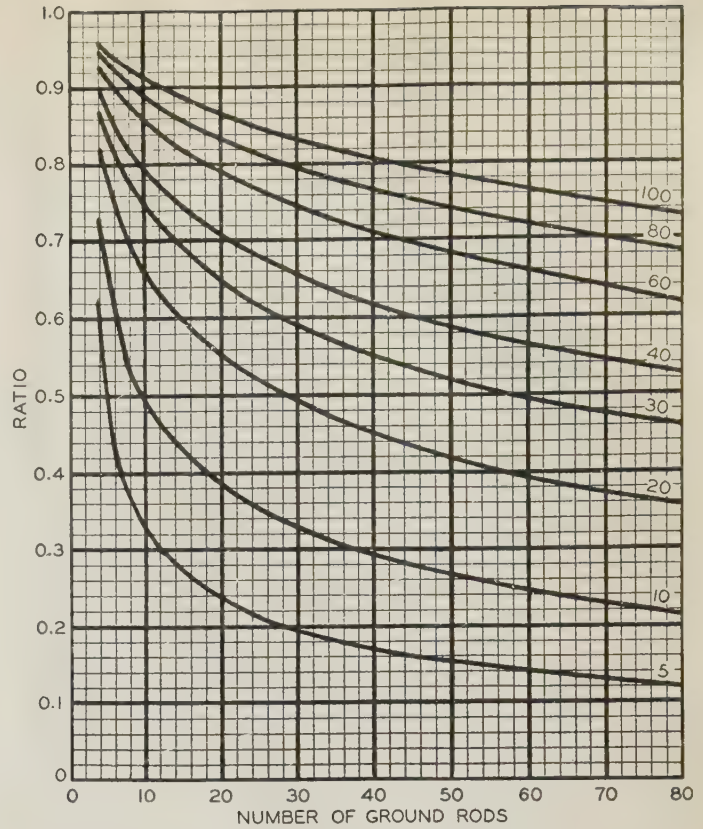

Fig. 5. Ratio of conductivity of ground rods in parallel on an area to that of isolated rods

Numbers on curves are spacings in feet

an area which contains all the ground rods in the vicinity.

In the calculation of cases involving more than 4 rods or the spheres which were substantially equivalent to them, equal charges were assumed for the spheres. The potential of a corner sphere was calculated and that of an innermost sphere and the average taken, thus approximating the Howe average potential method. For a small number of spheres this was checked and good agreement obtained by computing the actual charges by simultaneous equations. For a large number of ground rods, the results shown in figures 5 and 6 are approximate.

The effect of the buried wires used to connect the ground rods together was not included in the computations for the figures which have been described. If the conductivity of each ground rod be assumed to be increased by the same percentage by the connecting wires, the latter will have little effect on the comparison of different groups of rods made by means of the curves.

Buried Horizontal Wire

In some cases, connection to the earth is made by means of a buried horizontal wire. The image of this wire in the ground surface requires the use of equation 19 or 21 where, in this case, the length of the buried wire is $2L$ centimeters and the distance from the wire to its image is s centimeters, which is twice the distance from the wire to the ground surface. If the image were not taken into account, a serious change in the result would often ensue.

Example. Length of No. 4/0 wire, 200 feet; depth, 10 feet; ρ, 200,000 ohms per centimeter cube. $R = 57.6$ ohms

Two Parallel Buried Wires

The resistance to ground of 2 parallel buried wires, including the effect of their images in the ground surface, is calculated by the same method as for 4 ground rods at the corners of a rectangle. The average potential of one of the wires caused by its own charge, equation 18, is to be added to the average potential caused by each of the other wires. Thus, one item is to be computed by equation 18 and 3 items by equation 15 or 17, using 3 values of s. The sum, divided by the total charge of the 4 wires, $8qL$, is $1/C$ and then equation 14 can be applied.

Example. To find: the resistance to ground of 2 wires, 100 feet long, 0.46 inch in diameter, 7 feet apart, and 10 feet below the surface of the ground, the resistivity of the ground being 200,000 ohms per centimeter cube. The wires are connected in parallel, electrically.

Potential of a wire caused by its own charge, by equation 18	$16.51\,q$
Potential by equation 15 or 17 for $s = 7$	$4.84\,q$
Potential by equation 15 or 17 for $s_1 = 20$	$2.99\,q$
Potential by equation 15 or 17 for $s_2 = 21.19$	$2.89\,q$
Potential	$27.23\,q$

Total charge $= 4 \times 1200 \times 2.540 \times q = 12{,}190\,q$

$$\frac{1}{C} = \frac{27.23}{12{,}190}$$

$$R = \frac{\rho}{2\pi C} = \frac{200{,}000 \times 27.23}{2\pi \times 12{,}190} = 71.1 \text{ ohms}$$

Right-Angle Turn of Wire

If the buried wire forms a right angle each arm of which is L centimeters in length the depth below

the ground surface being $s/2$, the resistance to ground including the effect of the image wires is

$$R = \frac{\rho}{4\pi L}\left(\log_\epsilon \frac{2L}{a} + \log_\epsilon \frac{2L}{s} - 0.2373 + 0.2146\frac{s}{L} + 0.1035\frac{s^2}{L^2} - 0.0424\frac{s^4}{L^4}\ldots\right) \quad (22)$$

where a is the radius of the wire.

Example. Length of each arm of No. 4/0 wire, 100 feet; depth, 10 feet; ρ, 200,000 ohms per centimeter cube. $R = 59.4$ ohms

Three-Point Star

If there are 3 buried wires of length L which meet each other at 120 degrees, the resistance to ground including the effect of images is

$$R = \frac{\rho}{6\pi L}\left(\log_\epsilon \frac{2L}{a} + \log_\epsilon \frac{2L}{s} + 1.071 - 0.209\frac{s}{L} + 0.238\frac{s^2}{L^2} - 0.054\frac{s^4}{L^4}\ldots\right) \quad (23)$$

Example. Length of each arm of No. 4/0 wire, 100 feet; depth, 10 feet; ρ, 200,000 ohms per centimeter cube. $R = 43.9$ ohms

Four-Point Star

$$R = \frac{\rho}{8\pi L}\left(\log_\epsilon \frac{2L}{a} + \log_\epsilon \frac{2L}{s} + 2.912 - 1.071\frac{s}{L} + 0.645\frac{s^2}{L^2} - 0.145\frac{s^4}{L^4}\ldots\right) \quad (24)$$

Example. Same values of dimensions as for 3-point star. $R = 37.3$ ohms

Six-Point Star

$$R = \frac{\rho}{12\pi L}\left(\log_\epsilon \frac{2L}{a} + \log_\epsilon \frac{2L}{s} + 6.851 - 3.128\frac{s}{L} + 1.758\frac{s^2}{L^2} - 0.490\frac{s^4}{L^4}\ldots\right) \quad (25)$$

Example. Same values of dimensions as for 3-point star. $R = 31.1$ ohms

Eight-Point Star

$$R = \frac{\rho}{16\pi L}\left(\log_\epsilon \frac{2L}{a} + \log_\epsilon \frac{2L}{s} + 10.98 - 5.51\frac{s}{L} + 3.26\frac{s^2}{L^2} - 1.17\frac{s^4}{L^4}\ldots\right) \quad (26)$$

Example. Same values of dimensions as for 3-point star. $R = 28.2$ ohms

Equations 22 to 26, being simple power series, save considerable time in computing numerical results.

If L is in centimeters and ρ in ohms per centimeter cube, R is in ohms. Also, if L is in inches and ρ in ohms per inch cube, R is in ohms. Inside the brackets, only ratios of dimensions occur. The numerator of each fraction must be in the same units as the denominator of that fraction. Note that s is the distance from the wire to the image, and is twice the distance from the wire to the ground surface.

In order to estimate relative accuracy, the potential of the wire at various distances from the center of a 4-point star was plotted and compared with the potential distribution of a round plate. The poten-

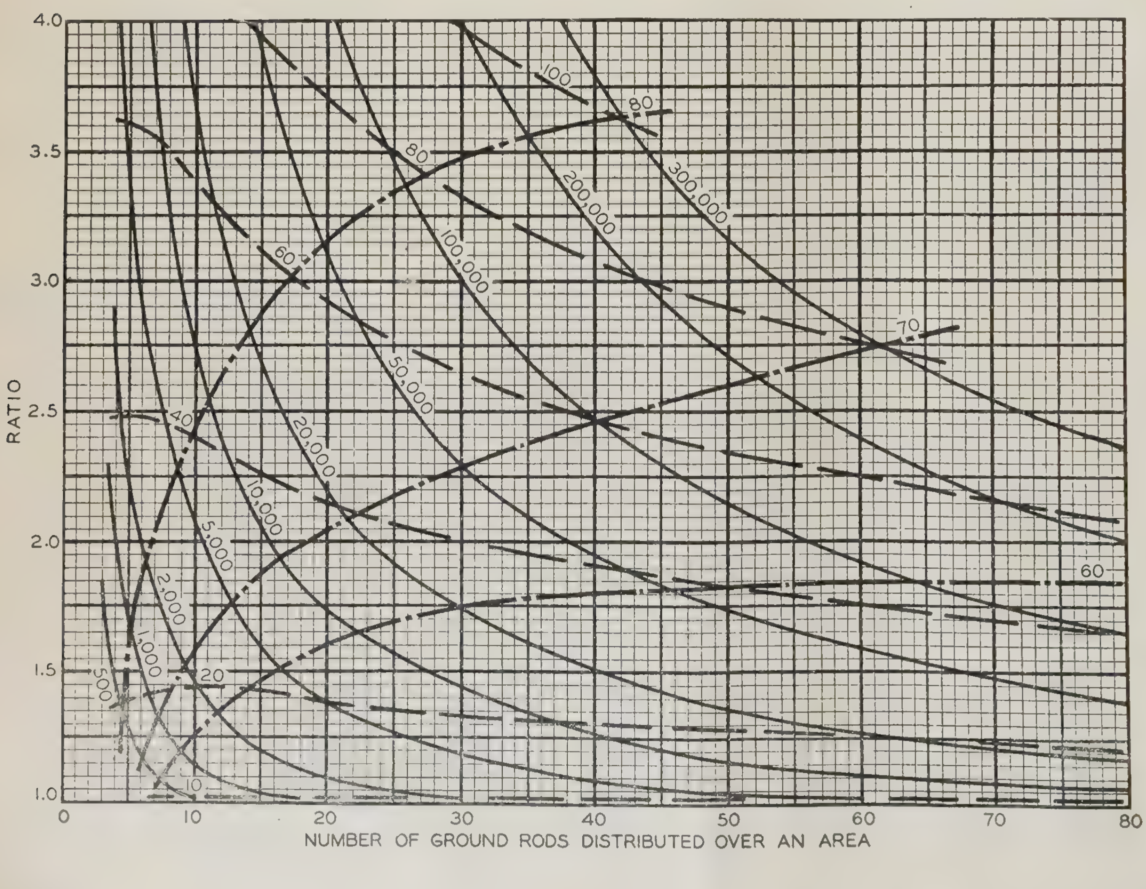

Fig. 6. Ratio of the resistance of a number of ground rods in parallel distributed over an area to the resistance with an infinite number of rods

Ground rods are 3/4 inch in diameter, 10 feet deep

——— Areas in square feet

– – – – – Spacings in feet

— — — – Effectiveness of rods in per cent

tial distribution in the 2 cases showed approximately the same amount of deviation from the average potential. Accordingly, it can be concluded that the resistance of stars of wire obtained by equations 23 to 26 is an approximation within several per cent.

In deriving the formulas for stars of wire, the average potential on one branch caused by its own charge is computed and added to that caused by each of the other branches and the images. The total potential, divided by the total charge of all the branches and their images, gives $1/C$ and then equation 14 is used.

For these computations, the formulas in appendix 2 of reference 3 are used. In this connection, the average potential of one wire caused by the charge on another equal wire which meets it at one end at angle θ is given by the simplified expression

$$V_{av} = 2q \log_\epsilon[1 + \operatorname{cosec}(\theta/2)] \tag{27}$$

If θ should be extremely small, the radius of the wire would need to be brought into the computation, and equation 27 would be inapplicable.

Table I—Approximate Formulas Including Effect of Images

One Ground Rod Length L, radius a	$R = \frac{\rho}{2\pi L}\left(\log_\epsilon \frac{4L}{a} - 1\right)$	(13)
Two Ground Rods $s > L$	$R = \frac{\rho}{4\pi L}\left(\log_\epsilon \frac{4L}{a} - 1\right) + \frac{\rho}{4\pi s}\left(1 - \frac{L^2}{3s^2} + \frac{2}{5}\frac{L^4}{s^4}\ldots\right)$	(20)
Two Ground Rods $s < L$	$R = \frac{\rho}{4\pi L}\left(\log_\epsilon \frac{4L}{a} + \log_\epsilon \frac{4L}{s} - 2 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4}\ldots\right)$	(21)
Buried Horizontal Wire Length $2L$, depth $s/2$	$R = \frac{\rho}{4\pi L}\left(\log_\epsilon \frac{4L}{a} + \log_\epsilon \frac{4L}{s} - 2 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4}\ldots\right)$	(21)
Right-Angle Turn of Wire Length of arm L, depth $s/2$	$R = \frac{\rho}{4\pi L}\left(\log_\epsilon \frac{2L}{a} + \log_\epsilon \frac{2L}{s} - 0.2373 + 0.2146\frac{s}{L} + 0.1035\frac{s^2}{L^2} - 0.0424\frac{s^4}{L^4}\ldots\right)$	(22)
Three-Point Star Length of arm L, depth $s/2$	$R = \frac{\rho}{6\pi L}\left(\log_\epsilon \frac{2L}{a} + \log_\epsilon \frac{2L}{s} + 1.071 - 0.209\frac{s}{L} + 0.238\frac{s^2}{L^2} - 0.054\frac{s^4}{L^4}\ldots\right)$	(23)
Four-Point Star Length of arm L, depth $s/2$	$R = \frac{\rho}{8\pi L}\left(\log_\epsilon \frac{2L}{a} + \log_\epsilon \frac{2L}{s} + 2.912 - 1.071\frac{s}{L} + 0.645\frac{s^2}{L^2} - 0.145\frac{s^4}{L^4}\ldots\right)$	(24)
Six-Point Star Length of arm L, depth $s/2$	$R = \frac{\rho}{12\pi L}\left(\log_\epsilon \frac{2L}{a} + \log_\epsilon \frac{2L}{s} + 6.851 - 3.128\frac{s}{L} + 1.758\frac{s^2}{L^2} - 0.490\frac{s^4}{L^4}\ldots\right)$	(25)
Eight-Point Star Length of arm L, depth $s/2$	$R = \frac{\rho}{16\pi L}\left(\log_\epsilon \frac{2L}{a} + \log_\epsilon \frac{2L}{s} + 10.98 - 5.51\frac{s}{L} + 3.26\frac{s^2}{L^3} - 1.17\frac{s^4}{L^4}\ldots\right)$	(26)
Ring of Wire Diameter of ring D, diameter of wire d, depth $s/2$	$R = \frac{\rho}{2\pi^2 D}\left(\log_\epsilon \frac{8D}{d} + \log_\epsilon \frac{4D}{s}\right)$	(29)
Buried Horizontal Strip Length $2L$, section a by b, depth $s/2$, $b < a/8$	$R = \frac{\rho}{4\pi L}\left(\log_\epsilon \frac{4L}{a} + \frac{a^2 - \pi ab}{2(a+b)^2} + \log_\epsilon \frac{4L}{s} - 1 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4}\ldots\right)$	(31)
Buried Horizontal Round Plate Radius a, depth $s/2$	$R = \frac{\rho}{8a} + \frac{\rho}{4\pi s}\left(1 - \frac{7}{12}\frac{a^2}{s^2} + \frac{33}{40}\frac{a^4}{s^4}\ldots\right)$	(32), (36)
Buried Vertical Round Plate Radius a, depth $s/2$	$R = \frac{\rho}{8a} + \frac{\rho}{4\pi s}\left(1 + \frac{7}{24}\frac{a^2}{s^2} + \frac{99}{320}\frac{a^4}{s^4}\ldots\right)$	(32), (38)

Buried Ring of Wire

The capacitance of an isolated ring of round wire is given by

$$\frac{1}{C} = \frac{2}{\pi D}\log_\epsilon \frac{8D}{d} \tag{28}$$

where the diameter of the ring, D centimeters, is

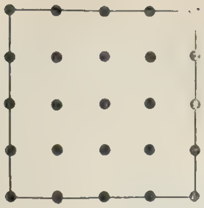

Fig. 7. Distribution of ground rods used for calculating the curves

much larger than the diameter of the wire, d centimeters. The resistance to ground of a deeply buried ring is then $R = \rho/4\pi C$ as in equation 11.

The capacitance of a ring and its image at distance s is given by

$$\frac{1}{C} = \frac{1}{\pi D}\left(\log_\epsilon \frac{8D}{d} + \log_\epsilon \frac{4D}{s}\right) \tag{29}$$

where s is considerably larger than d and also considerably smaller than D.[12] The resistance to ground of a buried horizontal ring, taking the effect of the ground surface into account, is then $R = \rho/2\pi C$ as in equation 14. Note that s is twice the distance from the ring to the ground surface.

Buried Strip Conductor

The capacitance of an isolated strip conductor, whose length $2L$ centimeters is large compared with its width a centimeters or its thickness b centimeters, is given by

$$\frac{1}{C} = \frac{1}{L}\left(\log_\epsilon \frac{4L}{a} + \frac{a^2 - \pi ab}{2(a+b)^2}\right) \tag{30}$$

For a description of the derivation, see equation 27 of reference 2. The thickness b should be less than about $1/8$ of the width a.

In most cases, the effect of the image should be included, as follows:

$$\frac{1}{C} = \frac{1}{2L}\left(\log_\epsilon \frac{4L}{a} + \frac{a^2 - \pi ab}{2(a+b)^2} + \log_\epsilon \frac{4L}{s} - 1 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4}\cdots\right) \tag{31}$$

where s is the distance in centimeters from the strip to the image, that is, twice the distance to the ground surface. The resistance to ground, when equation 31 is used, is $R = \rho/2\pi C$ as in equation 14.

Round Plate

The capacitance of a single isolated thin round plate is given by [13]

$$\frac{1}{C} = \frac{\pi}{2a} = \frac{1.571}{a} \tag{32}$$

where a is the radius of the plate in centimeters.

It is of interest to show that the average potential method, if used in this case, produces an error of 8 per cent, giving a value of $1/C$ which is 8 per cent too large. Expressions for the potential caused by a ring carrying uniform charge density are given in reference 10, pages 11 and 153. From these is obtained the average potential of a thin round plate of radius a centimeters caused by a uniform charge density q per square centimeter in the form of a series whose sum is

$$V_{av} = \frac{2\pi qa}{3}\left(2 + \frac{1}{4} + \frac{3^2}{4^2 \times 6} + \frac{3^2 \times 5^2}{4^2 \times 6^2 \times 8} + \ldots\right) \tag{33}$$

The sum of this slowly converging series can be found by comparing it with the series

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots \tag{34}$$

Multiply the terms within the brackets of equation 33 by 0.401723 to make the eleventh terms of equations 33 and 34 alike. It is found that $1/C$ or $V_{av}/\pi a^2 q$ is slightly greater than $1.69721/a$. Multiply the terms within the bracket of equation 33 by 0.329595 to make its tenth term equal to the eleventh term of equation 34 and all its succeeding terms distinctly less than the corresponding terms of equation

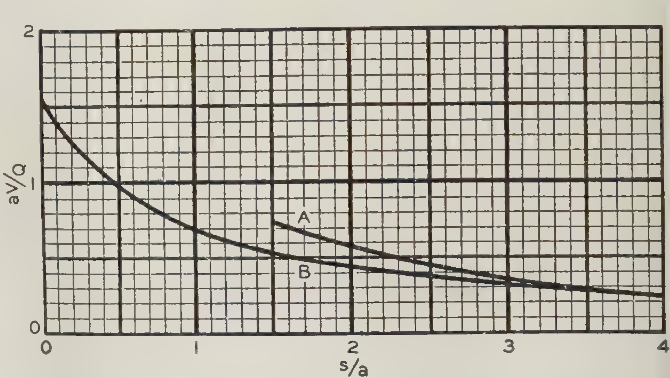

Fig. 8. Average potential on a round plate caused by an equal plate

A—Same plane *B*—Parallel planes

a — radius of plates in centimeters

34. It is found that $1/C$ is less than $1.70169a$. The difference between the 2 limiting values is 0.27 per cent and an inspection of the series shows that the value of $1/C$ by the average potential method is much nearer the smaller limit than the larger, and so is nearly equal to

$$\frac{1.6972}{a} \tag{35}$$

The true value of $1/C$ is, however, by equation 32,

$$\frac{\pi}{2a} = \frac{1.5708}{a}$$

The average potential method, therefore, gives a value of $1/C$ for a thin round plate which is 8 per cent too high.

This result has been confirmed by Dr. F. W. Grover by a method using mechanical integration, which gave

$$\frac{V_{av}}{\pi a^2 q} = \frac{1.6966}{a}$$

There is a connection between the error caused by the average potential method and the amount of the

variation in potential over the conductor caused by uniform charge distribution. The potential of the center of the plate resulting from uniform charge distribution is $2\pi aq$, using the formulas of reference 10. However, the average value is

$$\frac{1.697}{a} \times \pi a^2 q = 1.697\pi aq$$

Thus, the potential of the center is seen to be 18 per cent higher than the average potential. This comparatively large percentage may account in some degree for the 8 per cent error in the case of a round plate, resulting from the use of the average potential method. In the case of the cylinder of the length considered in this paper, the potential of the middle point was 4.7 per cent higher than the average potential (see reference 2) and in that case the average potential method was shown to give a very nearly correct value of $1/C$.

Two Round Plates in Parallel Planes

The potential caused by a charged round plate, at points not near the plate, is given by the last series on page 154 of reference 10. Integrating that series over the surface of a second disk having the same axis and the same length of radius as the first, the average potential on the second disk caused by the charge on the first is

$$V_{av} = \frac{Q}{s}\left(1 - \frac{7}{12}\frac{a^2}{s^2} + \frac{33}{40}\frac{a^4}{s^4} \cdots\right) \qquad (36)$$

where Q is the total charge in statfarads on the first disk, a is the radius in centimeters of both plates, and s is the distance in centimeters between the 2 plates.

This power series should not be used unless the last term is quite small and so the largest value of a/s for which it is useful is about $^1/_2$. Since equation 36 gives the average potential, it is not a precise calculation for capacitance or resistance. However, the order of its precision may be estimated by finding the potential at the center of the second disk, which is $(Q/s)\tan^{-1}(a/s)$ (see the series on page 154 of reference 10).

Where $s = 2a$, the potential at the center is 4 per cent greater than the average potential, and for larger values of the separation s the discrepancy is smaller. It may be remembered that for one isolated round plate the potential of the center was 18 per cent more than the average potential, and the error in the value of capacitance was 8 per cent. In the case of the calculation for one isolated ground rod of average proportions the potential at the middle of the cylinder is 4.7 per cent greater than the average potential, and the error in the capacitance is less than 1 per cent.[2] It may be concluded that the use of average potential gives the same order of accuracy in the case of equation 36 as in the case of a long cylinder.

Values of aV/Q calculated by equation 36 are plotted in figure 8. This curve has been extended to apply to small values of s by taking the average values obtained by using the 2 series on page 154, reference 10, to compute the potential at the rim and at the center of the second disk caused by the charge on the first disk. This process is shown to give good results by comparing it with equation 36 for s/a between 2 and 4. The potential at the rim is equal to the potential at the center when $s = 0$. More accurate values for the curve of figure 8 could be computed by dividing the circular area into

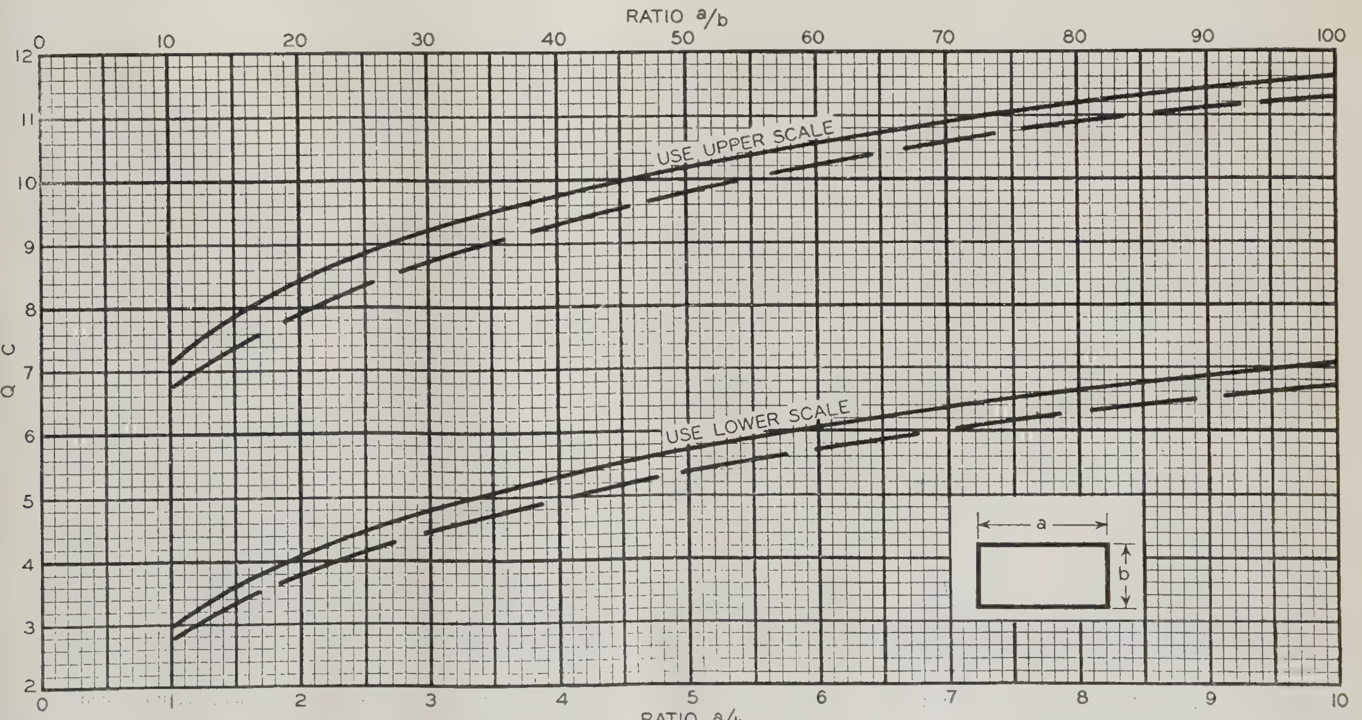

Fig. 9. Capacitance of a rectangular plate

Dimension a in centimeters; C in statfarads

bands, computing the potential for each band, and averaging, this being a process of mechanical integration. However, no matter how carefully this might be done, the result would be subject to the error inherent in the average potential method.

It may be seen from equation 36 that when the separation is large, the potential on one plate caused by the other is given closely by

$$V = \frac{Q}{s} \tag{37}$$

which is the same as assuming that the charge is on the surface of a sphere, or concentrated at a point. For example, when $s/a = 5$, the result of equation 37 is 2 per cent larger than that of equation 36 and the value of Q/s itself is only 13 per cent of the potential of an isolated plate to which it is to be added. Accordingly, equation 37 can usually be used for the images of the buried plates, which are at distances s_1 and s_2, center to center.

Where the 2 round plates are connected in parallel, then in the capacitance problem the plates and their images all carry charges equal to Q. By symmetry, the potential is the same for them all, and is made up of the following 4 items which are added together. First, the potential, $\pi Q/(2a)$, given by equation 32, caused by the plate's own charge; second, the potential given by equation 36, caused by the other coaxial buried plate which lies in a vertical plane parallel to that of the first plate; third, the potential Q/s_1, caused by the plate's image, where s_1 is the distance in centimeters from the center of the plate to the center of the image, that is, s_1 is twice the distance to the surface of the ground; fourth, the potential Q/s_2, caused by the other image.

The sum of these 4 items is equal to V and the capacitance of the 4 plates is given by $1/C = V/(4Q)$. Then the resistance to ground of the 2 buried plates connected in parallel is $R = \rho/(2\pi C)$ by equation 14.

Where the 2 round plates are connected in series, in the resistance problem current flows from one to the other through the ground and in the capacitance problem one plate carries a charge Q and the other $-Q$. The images carry charges Q and $-Q$, each being of the same sign as the charge directly beneath it. Equations 11 and 14 are still used to change from the capacitance problem to the resistance problem.

Two Round Plates in the Same Plane

Where the 2 plates of radius a in centimeters are in the same plane, the average potential on one resulting from the charge on the other is

$$V_{av} = \frac{Q}{s}\left(1 + \frac{7}{24}\frac{a^2}{s^2} + \frac{99}{320}\frac{a^4}{s^4} + \ldots\right) \tag{38}$$

In this case, there is more error in the use of average potential than in equation 36, for when $s/a = 2$, the potential at the center differs from the average potential by 19 per cent. As in other cases, if the term in a^2/s^2 is negligibly small compared to 1, the simple expression Q/s may be used and the error resulting from the use of average potential is negligible.

Rectangular Plates

The capacitance of an isolated thin rectangular plate, a centimeters by b centimeters, according to the average potential method, is given by[9]

$$\frac{1}{C} = 2\left(\frac{1}{a}\log_\epsilon\frac{a+\sqrt{a^2+b^2}}{b} + \frac{1}{b}\log_\epsilon\frac{b+\sqrt{a^2+b^2}}{a} + \frac{a}{3b^2} + \frac{b}{3a^2} - \frac{(a^2+b^2)\sqrt{a^2+b^2}}{3a^2b^2}\right) \tag{39}$$

The potential of the center of a square plate is 18 per cent greater than the average potential, and this difference is 15 per cent for a rectangle whose length is 5 times its width. Accordingly, the correction for the average potential method found for a circular plate will apply, and 8 per cent should be subtracted from the value of $1/C$ given by equation 39 for a square or nearly square plate, and almost as much should be subtracted for a rectangular plate of length about 5 times the width. The full lines of figure 9 show values calculated by equation 39 and the broken lines show estimated corrected values (see the paragraphs following equation 32 of reference 2).

A formula for the average potential of a rectangular plate caused by a uniformly distributed charge on a similar plate in the same plane can be given but it is not short and it is subject to the errors inherent in the average potential method. It seems better to replace the rectangular plates by circular plates of the same area and on the same centers, and to use equation 36, 37, or 38 for the effect of one plate on another.

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A-C Characteristics of Dielectrics—II

Predetermination of the a-c characteristics of dielectrics by d-c measurements was discussed in an earlier paper.[1] The present paper further shows that the method of 3 exponentials, although predicting accurately the in-phase component of current caused by reversible absorption, usually yields results of less than the true value in predicting the quadrature component. Charts are presented that reduce the computations required in the study of a great number of specimens.

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THE SCOPE of the theoretical developments and experimental results presented in an earlier paper[1] dealing with the analysis of power factor and dielectric loss concerns itself primarily with the analytical and experimental proof that the a-c behavior of a dielectric at 60 cycles may be accurately predicted from suitable d-c measurements. This is done by an empirical determination of the equation for the relaxation function of the dielectric at a given temperature, followed by the application of on Schweidler's method,[2] which is available at any frequency, provided that the continuous-potential charge and discharge currents may be measured over initial time intervals comparable with the alternating period.

The present paper considers further, in part I, the possibilities and limitations of the method of 3 exponentials, giving additional theoretical evidence of the accuracy of the method as applied to the in-phase component of current resulting from reversible absorption and showing that, in general, this method yields results which fall short of the true values in predicting the capacitance or quadrature component.

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* The work upon which this paper is based was performed by the author while research assistant at The Johns Hopkins University; he is now (1935–37) Guggenheim fellow in physics at the Massachusetts Institute of Technology, Cambridge, on leave of absence from the National University of Mexico and the School of Mechanical and Electrical Engineering at Mexico City.

For all numbered references see list at end of paper.

The second part of the paper gives the description, method of construction, and application of 2 charts that were designed for the purpose of reducing to a minimum the labor of numerical computation entailed by the study of a great number of specimens.

I. Increase in Capacitance Caused By Reversible Absorption

The method of 3 exponentials developed earlier[1] is based on the possibility of resolving the decaying relation between discharge current and time for a dielectric at a given voltage and temperature into the sum of 3 negative exponentials as shown in figure 1. The subsequent application of Von Schweidler's analysis[2] shows that the quadrature and in-phase components of current contributed under alternating stress by the anomalous property of reversible absorption are directly proportional to the charging current of the geometric capacitance and that the constants of proportionality are respectively the A and B integrals defined by equation 4 as given in reference 1. It is thus evident that the computed a-c behavior depends entirely, as far as reversible absorption is concerned, on the A and B integrals and their manner of computation. The purpose of the present discussion is to show from a theoretical point of view, by considering the variation with frequency of the A and B integrals, that the method of 3 exponentials as applied in the present studies will predict accurately the correct value of B, but generally will yield a computed value of A which is definitely smaller than the observed or experimental value. In other words, it is shown that the increase in capacitance caused by reversible absorption, as computed by the method outlined, generally will be lower than the actual increase as measured from the difference between the observed capacitance at 60 cycles and the geometric capacitance observed at a frequency sufficiently high to be considered infinite.

Residual Relaxation Function

The evaluation of the A and B integrals presupposes a knowledge of the relaxation function $\varphi(t)$, which characterizes the empirical relation between discharge current and time for the dielectric under continuous potential. The relaxation function is a decaying function of time which possesses a continuous derivative everywhere and which can always be represented by a series of negative exponentials of time. The number of such exponentials that will reproduce accurately the relaxation function depends entirely upon the nature of the problem and the range of time for which the empirical current-time relation is known. Thus, in the preceding case,[1] it was shown that 3 exponentials were necessary and sufficient in most cases for the analytical expression of the relaxation function corresponding to the range of time from 1 to 12 milliseconds, since the computed values of the quadrature and in-phase components of current caused by reversible absorption, and hence the computed values of the A and B integrals, turned out to be essentially independent of the par-

ticular way in which the relaxation function was resolved into the sum of 3 exponentials, even though this resolution was otherwise quite indeterminate.

On account of the nature of the method of 3 exponentials, it is apparent from figure 1 that the excess of the ordinates of the empirical current-time relation over the computed ordinates becomes increasingly greater for values of time larger than the value t_0, 12 milliseconds in this case, at which the first tangent is drawn. For values of time between zero and the time corresponding to the first measurable ordinate of the empirical current-time relation usually in the close neighborhood of one millisecond there is no direct information concerning the discrepancy between the empirical and computed ordinates. However, it is the contention of the author that in most cases as will be shown later the computed values of current again fall short of the real values. Hence it appears reasonable to conclude that, in general, the method of 3 exponentials constitutes an accurate analytical reproduction of a given portion of the relation between discharge current and time, and that, for values of time before and after the specified range the computed ordinates are smaller than the real ordinates.

Thus, if the complete empirical current-time relation were known for values of time from zero to infinity, it would be possible to plot

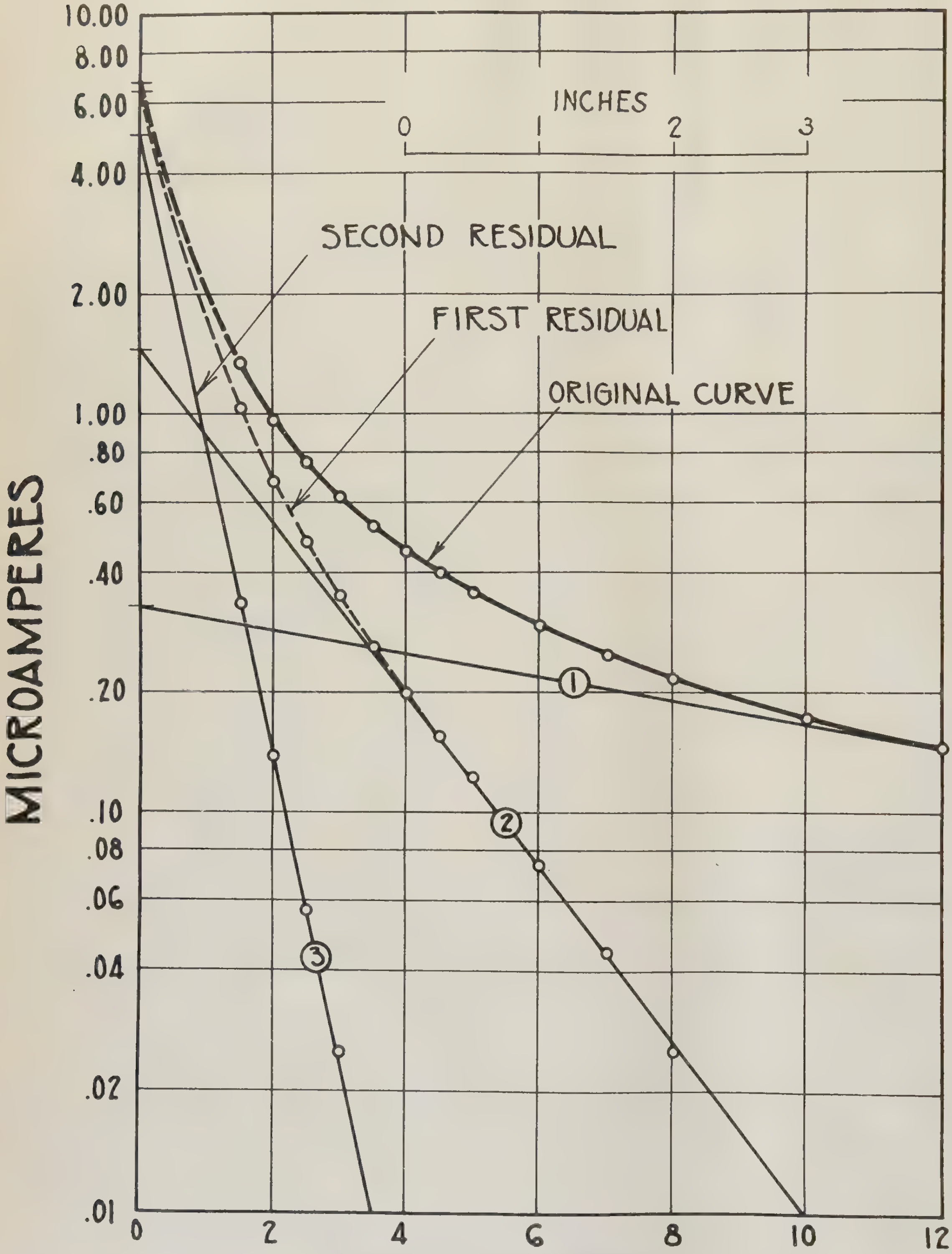

Fig. 1. Application of the method of 3 exponentials to a relation of charge and discharge current with time

Dry paper specimen *A7* at 60 degrees centigrade; 1,500 volts

as a function of time, the differences between the real ordinates and the corresponding ordinates computed by means of the method of 3 exponentials, and the resulting curve for the residual relaxation function would look, in general, very much like figure 2, where t_1 denotes the time corresponding to the first measurable ordinate and t_0 the time at which the first tangent is drawn. The range of

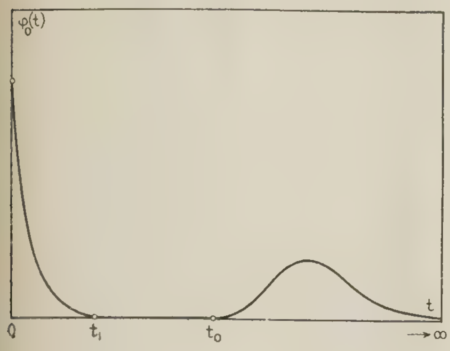

Fig. 2. The residual relaxation function

$\varphi_1(t)$—Empirical relaxation function

$\varphi_2(t) = \sum_1^3 \beta_k \epsilon^{-\alpha_k t}$—Sum of 3 exponentials

$\varphi_0(t) = \varphi_1(t) - \varphi_2(t)$—Residual relaxation function

time from t_1 to t_0 exhibits zero discrepancy throughout and corresponds to the range for which the method of 3 exponentials is accurately applicable. It is evident from figure 1 and from the nature of the relaxation function that, for t greater than t_0, the curve of discrepancy versus time (figure 2) rises to a maximum and then finally decays to zero, because both the empirical and the computed curves decay to zero for sufficiently large values of time. For $0 < t < t_1$, as shown in the following, there is at present only indirect evidence to draw the curve in the manner illustrated in figure 2.

Contribution of Reversible Absorption to the In-Phase Component of Current

Figure 5 of reference 1 gives the variation with frequency of the A and B integrals for a relaxation function expressible as a single exponential as characterized by the inverse time constant α and the constant multiplier β. This variation may be expressed analytically as follows:

$$\left.\begin{aligned} A(u) &= \frac{\beta}{2\alpha}\,\epsilon^{-u}\operatorname{sech} u \\ B(u) &= \frac{\beta}{2\alpha}\operatorname{sech} u \end{aligned}\right\} \quad (1)$$

where u is a new independent variable, a function of the frequency, defined by the relation $\epsilon^u = \omega/\alpha$.

When the relaxation function is expressed by the sum of several exponentials, there will be associated with each exponential, and hence with each set of α and β values, a pair of curves similar to those defined by equation 1 and giving the variation with frequency of the individual contributions of each exponential to the A and B integrals. In order to construct the over-all frequency variation of the A and B integrals, it becomes necessary to plot on the same set of axes, using log ω as the independent variable, each pair of curves, one pair to each exponential, drawing the point of inflection of A and the associated maximum of B at a value of ω equal to the corresponding value of α for each particular exponential. Thus, the variation of the B integral with log ω, for example, is a composite curve made up of the sum of simple hyperbolic secant curves, each one of maximum value $\beta_k/2\alpha_k$, occurring for values of frequency such that $\omega = \alpha_k$ where $k = 1,2$, etc., denotes the ordinal number of each exponential term appearing in the relaxation function. In applying the method of 3 exponentials, it has been shown that the maximum of a fourth exponential that might be introduced occurs at a frequency far removed from the frequency being studied, and that its contribution to the B integral or the total in-phase component is negligible. The maximum of each contribution occurs at a value of ω equal to the inverse time constant α of that exponential. Therefore, only the exponentials with values of α in the neighborhood of ω at the frequency at which correlation is intended contribute an appreciable amount to the loss component $B(\omega)$.

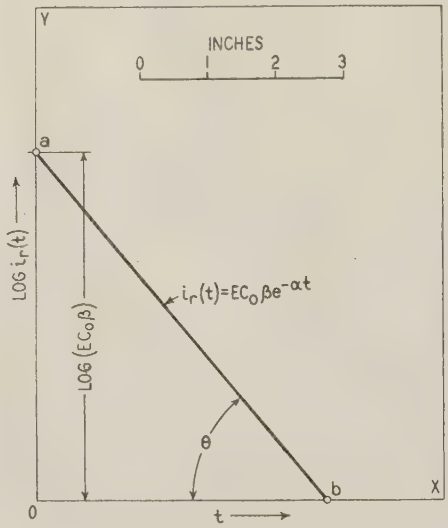

Fig. 3. Graphical representation in semilogarithmic co-ordinates of a hypothetical current-time relation defined by a single exponential term

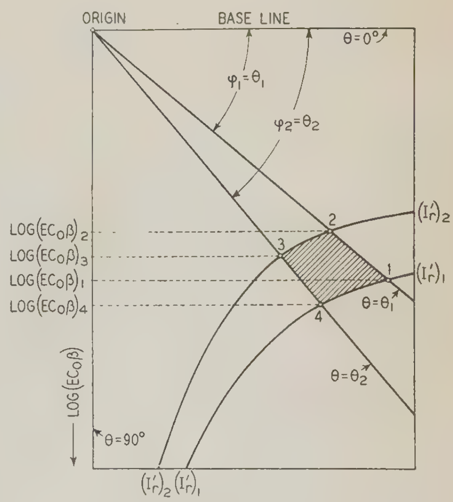

Fig. 4. General shape of the family of curves $I_r' =$ constant

The same reasoning, applied to examine the time range within which the d-c measurements will give

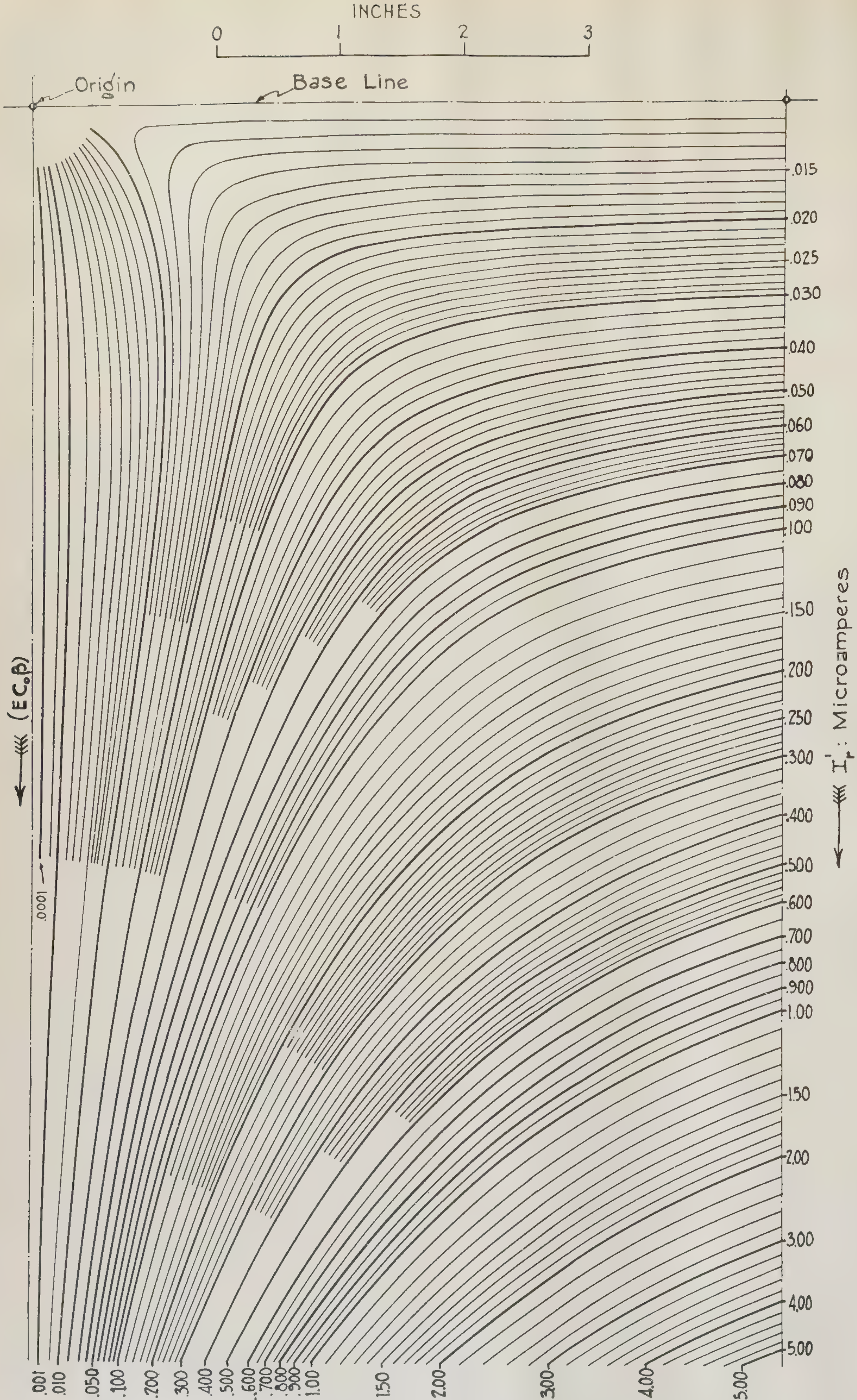

Fig. 5. Chart 2, for the computation of the in-phase component of current, under alternating stress, from a knowledge of the current-time relation under continuous potential, corresponding to a single exponential term

accurate predic
tion of the a-c be
havior, show
that the values o
α obtained for th
3 exponential
must range in th
neighborhood o
the value of
corresponding t
the frequency a
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from t_1 to t_0, mus
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portion of th
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sum of severa
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acterized b
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far removed fron
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in-phase compo
nent of current re
sulting from ab
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$t > t_0$, the curv
of residuals (fig
ure 2) is no longe
expressible as th
sum of simple ex
ponentials, bu
the direct nu
merical integra
tion of B show
again that th
contribution o
this portion of th
relaxation func
tion is negligible

Increase in Capacitance Because of Absorption

The composite curve showing the frequency variation of the A integral may be constructed in exactly the same manner as that of the B integral by using once again $\log \omega$ as the independent variable. Here the set of curves is defined by the first equation 1, one for each exponential, and each one symmetrical with respect to its own point of inflection which occurs at a value of frequency such that $\omega = \alpha_k$ where $k = 1,2$, etc., denotes the ordinal number of each exponential. The ordinates to the point of inflection in each case have the value $\beta_k/2\alpha_k$. For frequencies sufficiently low, each exponential contributes to the quadrature component $A(\omega)$ its maximum value β_k/α_k, whereas for frequencies sufficiently high, all the individual contributions vanish asymptotically.

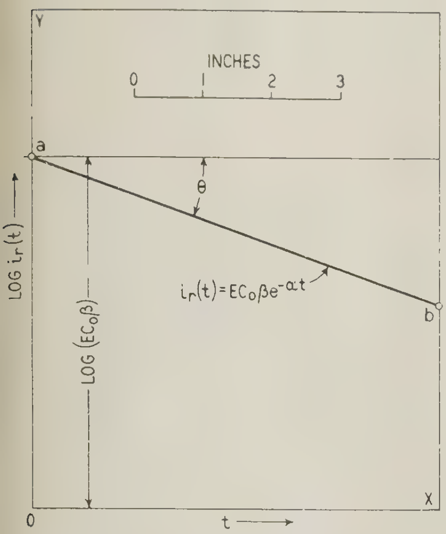

Fig. 6. Slowly decaying exponential term

For a given frequency, those exponentials with values of α considerably smaller than the value of ω for which agreement is being examined exhibit contributions to the quadrature component which have already practically vanished, since their points of inflection are to be found in the composite curve at values of ω considerably smaller than the value which is the object of discussion. Conversely, those exponentials characterized by values of α in the neighborhood of the value of ω at which correlation is intended and all the exponentials with greater values of α, respectively, will contribute to the composite curve at ω a fractional amount of the maximum and the full amount. This statement justifies the contention, mentioned elsewhere, that the method of 3 exponentials is inadequate for the accurate prediction of the increase in capacitance caused by absorption. Evidently, from figure 2, if the residual relaxation function corresponding to $0 < t < t_1$ is not negligible and, hence, susceptible of resolution into the sum of several exponentials, then each of these exponentials, although contributing nothing to the in-phase component $B(\omega)$, will contribute practically its maximum value β_k/α_k to the quadrature component $A(\omega)$. For $t > t_0$, the curve of residuals is no longer expressible as the sum of simple exponentials, but here again the direct numerical integration of A shows that this portion of the relaxation function possesses a negligible contribution to the quadrature component. It then becomes evident that by neglecting the rapidly decaying terms that might enter into the relaxation function and which have escaped detection by practically vanishing before the beginning of the d-c oscillographic records, the method of 3 exponentials will usually yield results which are lower than the true values when predicting the capacitance or quadrature component, although it will predict accurately the correct value of the in-phase component of current resulting from absorption.

Experimental Results

In order to test out experimentally the preceding conclusions, it is found convenient to use the expression

$$\Delta C_r = C' - C_0 = AC_0 \tag{2}$$

which shows the increase in capacitance caused by reversible absorption as proportional to the geometric capacitance C_0. When all the quantities entering into equation 2 are known from observation and computation, it is possible to predict the increase in capacitance caused by absorption (AC_0) and to compare this computed increase with the actual measured increase ($C' - C_0$) taken as the difference between the capacitance measured at the given frequency of observation and at a frequency sufficiently high to yield the correct value of geometric capacitance.

Such comparison has recently been carried out by Whitehead and Greenfield,[3] who performed an extensive study of the dielectric properties of cellulose paper. Tables I and IV of their paper are of particular interest in the present discussion. Table I (page 1392, reference 3) gives their analysis of power factor and capacitance for a paper specimen at the constant temperature of 100 degrees centigrade for various stages of drying as indicated by the successive lower pressures of evacuation. Table IV (page 1500, reference 3) gives the same analysis for a standard state of dryness and various temperatures from 20 to 100 degrees centigrade. Both tables give for each value of the variable parameter, drying pressure in the first and temperature in the second, the measured and computed increases in capacitance caused by reversible absorption at 60 cycles. Except for one single case in each table, the computed increase in capacitance is found to be smaller than the corresponding measured increase, the discrepancies ranging from 3 to 15 per cent. It is significant that although both tables show the predicted increase in capacitance as consistently lower than the measured increase, the predicted power factor, when compared with the measured power factor, exhibits much smaller discrepancies without any definite tendency in sign. This experimental evidence strengthens the hypothesis that the relaxation function of a dielectric, as shown in figure 2, may contain rapidly decaying exponential terms, which escape detection by the amplifier oscillograph and which lead, when the

method of 3 exponentials is applied, to computed values of the increase in capacitance caused by absorption that may be considerably lower than the observed values.

Summary

1. The method of 3 exponentials has been developed as a convenient and sufficient expression for the relaxation function of a dielectric at a given temperature.

2. The method developed furnishes an empirical equation for the relaxation function which constitutes an accurate analytical reproduction valid only within the range of time for which the method was applied, and, for values of time before and after the specified range, the computed ordinates are smaller than the real ordinates.

3. As far as reversible absorption is concerned, the criterion for suitable d-c measurements which are to be used for the accurate prediction of the a-c behavior demands that the oscillographic d-c measurements cover initial intervals of time of the same order of magnitude as the period of the alternating potential.

4. The method of 3 exponentials (by neglecting the rapidly decaying exponential terms that might enter into the relaxation function of a dielectric and that have practically vanished by the beginning of the d-c oscillographic records thus escaping detection) although predicting accurately the correct value of the in-phase component of current due to absorption, will usually yield results which are lower than the true values when predicting the capacitance or quadrature component.

II. Power Factor and Dielectric Loss Charts

The computation of the in-phase component of current caused by reversible absorption has been shown to be

$$I_r' = BI_0'' \qquad (3)$$

where I_0'', equalling $\omega C_0 E$, is the charging current of the geometric capacitance and B must be computed from a knowledge of the α and β constants of the 3 exponentials into which the empirical current-time relation has been resolved. The application of the method of 3 exponentials to a great number of cases made it necessary and convenient to simplify the labor of numerical computation. The power factor and dielectric loss charts described in this section were thus constructed in an attempt to reduce to a minimum the numerical computations involved in the direct application of equation 3.

Construction of the "Pseudotopographic" Chart

To simplify the discussion, for the time being a hypothetical relation between discharge current and time defined by a single exponential term may be considered:

$$i_r(t) = EC_0\beta\epsilon^{-\alpha t} \qquad (4)$$

and the in-phase component of current may be computed which, by introducing in equation 3 the explicit relation defining the charging current of the geometric capacitance and using equation 8 of reference 1, may be expressed thus:

$$I_r' = EC_0\beta\left(1 + \frac{\alpha^2}{\omega^2}\right)^{-1} \qquad (5)$$

where E is the effective value of the applied alterna[ting] potential. Plotting equation 4 in semilogarit[h]mic co-ordinates, that is, the logarithm of the cu[r]rent against the time in seconds, gives the straig[ht] line shown in figure 3. The intercept on the y ax[is] is a measure of the triple product $(EC_0\beta)$ and t[he] slope of the straight line is a measure of the inver[se] time constant α through the relation

$$\alpha = k \tan\theta \qquad (\ldots$$

where θ is the angle that the line actually mak[es] with the x axis (figure 3) and k, a constant for a giv[en] set of co-ordinate axes, is given by the relation

$$k = \frac{m_y}{m_x}\log_\epsilon 10 = 2.3026\frac{m_y}{m_x} \qquad (\ldots$$

as determined by m_y and m_x, the respective modu[li] for the co-ordinate axes. All the experimental wo[rk] previously discussed was performed at a frequen[cy] of 60 cycles per second and the current-time relatio[ns] were all plotted in semilogarithmic co-ordinates 6 b[y] 9 inches as shown in figure 1, which is here repro[duced] to a reduced scale. The range of curren[ts] measured at different temperatures and voltages an[d] for different specimens determined the choice [of] modulus for each co-ordinate axis as follows:

m_y = 0.333 per inch (that is: 1/3 logarithmic cycle per inch)
m_x = 0.002 seconds per inch

Substituting these numerical values in equation [...] yields k = 383.8 radians per second.

From equations 5 and 6 it is possible to exhibit th[e] in-phase component of current I_r' as a function of [...] independent variables, namely, the y intercep[t] $(EC_0\beta)$ and the inclination θ with respect to the [...] axis. Thus

$$I_r' = (EC_0\beta)\cdot f(\theta) \qquad (\ldots$$

and

$$f(\theta) = \left(1 + \frac{k^2}{\omega^2}\tan^2\theta\right)^{-1} \qquad (\ldots$$

where $0 < \theta < \pi/2$. In equation 9 the numerica[l] coefficient k^2/ω^2 is very nearly unity at 60 cycles ([ω] = 377 radians per second) with the result that th[e] graphical representation of equation 9 looks essen[tially] like a cosine-squared curve.

The form of equation 8 suggests the possibility [of] the graphical representation of the curves I_r' = con[stant] in whatever manner may prove most conve[nient]. In this case it was agreed to represent point[s] in the plane by means of 2 co-ordinates, (1) down[ward] ordinate log $(EC_0\beta)$, and (2) clockwise pola[r] angle θ. The curves I_r' = constant (figure 4) the[n] were drawn through a sufficient number of compute[d] points by first assigning several values of θ in suc[cession], the corresponding values of $f(\theta)$ being deter[mined] from equation 9, and finally, by substitutin[g] in equation 8, for a given constant value of I_r', th[e] corresponding (downward) ordinates, log $(EC_0\beta$[)] were computed for the various chosen values of [θ]. This is illustrated clearly in figure 4 which gives th[e] general shape of the family of curves I_r' = constan[t] in the adopted system of co-ordinates. It is to b[e]

observed that the quadrilateral 1234 (figure 4) is formed by the intersection of any 2 given members of the family of straight lines through the origin $\theta =$ constant corresponding to the values $0 < \theta_1 < \theta_2 < \pi/2$, with 2 other given members of the family of curves $I_r' =$ constant corresponding to any 2 given values $(I_r')_1$ and $(I_r')_2$.

The chart plotted for the co-ordinate paper already described and following the prescriptions given herein is shown in figure 5. It is called a

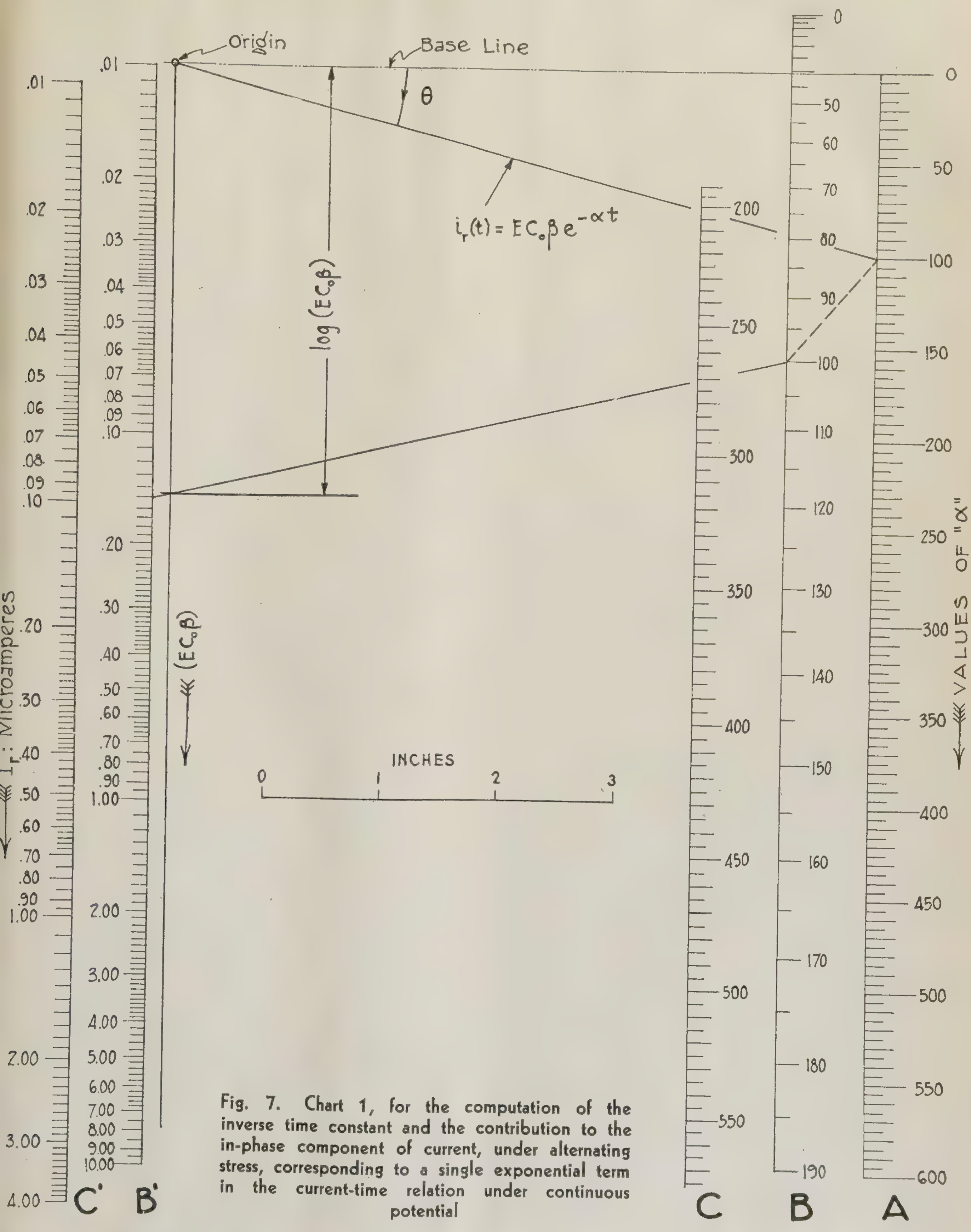

Fig. 7. Chart 1, for the computation of the inverse time constant and the contribution to the in-phase component of current, under alternating stress, corresponding to a single exponential term in the current-time relation under continuous potential

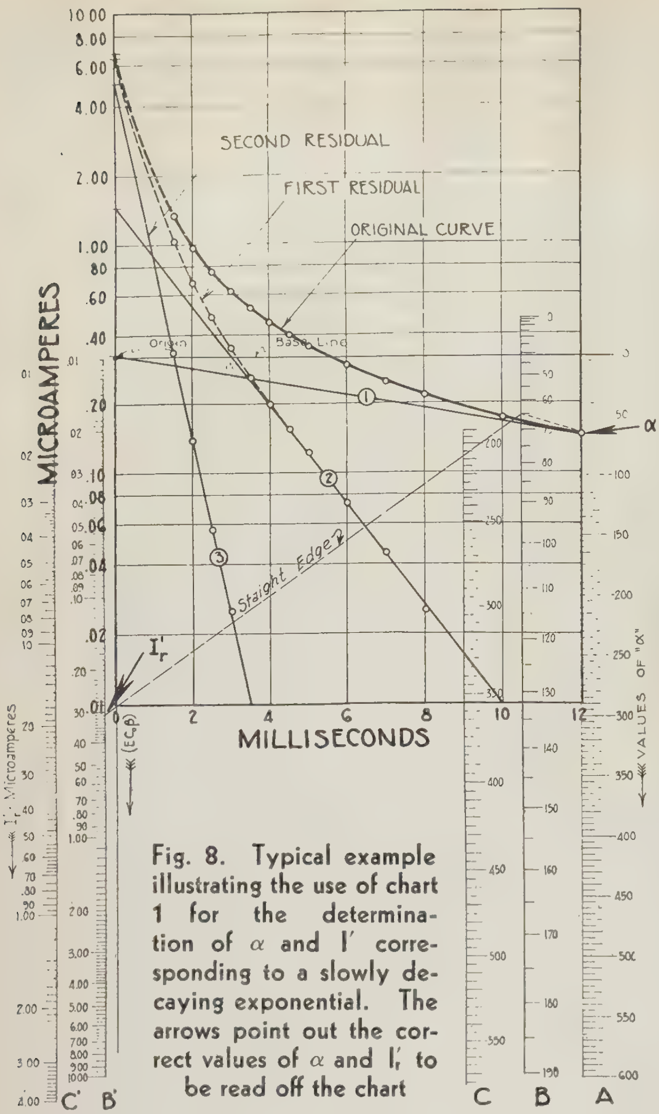

Fig. 8. Typical example illustrating the use of chart 1 for the determination of α and I' corresponding to a slowly decaying exponential. The arrows point out the correct values of α and I_r' to be read off the chart

"pseudotopographic" chart, because it constitutes essentially the mapping out of the co-ordinate plane into lines of equal values of the in-phase component of current caused by reversible absorption. Every point in this chart belongs to a member of the family of curves I_r' = constant and is furthermore characterized by 2 co-ordinates: its downward logarithmic ordinate, a direct measure of the triple product $EC_0\beta$; and its clockwise polar angle θ, a measure through equation 6 of the inverse time constant α. Conversely, to a given set of values of $EC_0\beta$ and α there corresponds a point of the co-ordinate plane through which passes a curve I_r' = constant, the value of which is given by equation 5. Therefore, this chart may now be used to determine graphically the in-phase component of current corresponding to the single exponential term of, say, figure 3, which is typical of the kind of single exponential terms amenable to the use of the "pseudotopographic" chart. By superposing the chart on the graph of figure 3, making the origin of the chart coincide with the point a, and by lining up the 2 respective vertical axes, the correct value of the in-phase component of current I_r' is read directly off the chart at the point b, the intersection of the straight line with the horizontal time axis. This is obviously true because th chart was drawn to give the actual value of I_r' fo such points as b in terms of the y intercept and th actual slope of the line ab. It may be noticed tha this chart, when used in the manner described a illustrated by the typical example given in figure and discussed in a later section, automatically deter mines the values of $EC_0\beta$ and θ and performs th

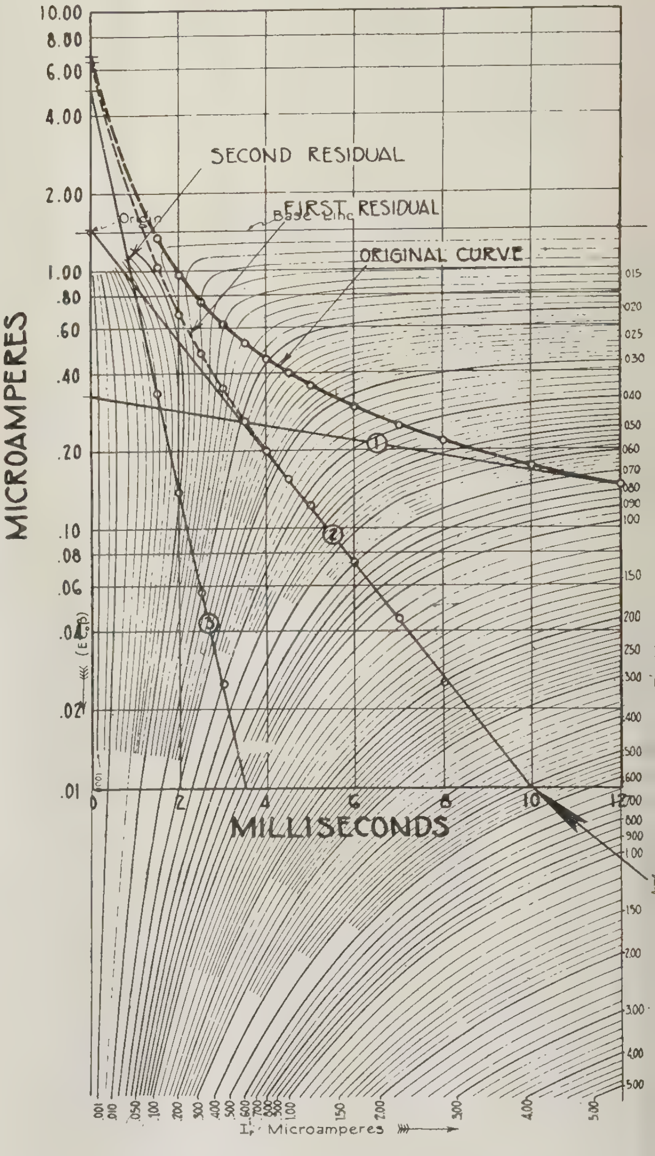

Fig. 9. Typical example illustrating the use of chart 2 for the determination of I_r' corresponding to a single exponential term. The arrow points out the correct value to be read off the chart

numerical operations embodied in equations 8 an 9 to give the required value of the in-phase compo nent of current I_r'. Thus, the direct application c this chart not only eliminates the labor of numerica computation, but actually dispenses with the evalua tion of the y intercept and the slope of the straigh line which characterizes, in semilogarithmic co ordinates, the decaying current-time relation define by a single exponential.

Construction of the Alignment Chart

When the straight line giving the graphical representation of equation 4 in semilogarithmic co-ordinates intersects the horizontal time axis outside of the co-ordinate paper, as shown in figure 6, the "pseudotopographic" chart described hereinbefore is no longer applicable. To treat such slowly decaying exponential terms it became necessary to resort to the nomographic or alignment chart shown in figure 7. This chart was designed to perform, for a given known value of α, the operations called for by equation 5. The procedure to be followed in constructing an alignment chart to perform a given set of numerical operations will be omitted here, since it is fully described in various manuals on the subject of graphical and mechanical computation such as the excellent book by J. Lipka.[4]

This alignment chart is used in a similar manner by superposing the chart on the graph of figure 6, making the origin of the chart coincide with the point a, and then lining up the respective vertical axes as with the "pseudotopographic" chart. The value of α is first read off scale A at the point of intersection of the straight line ab and the A scale. This point, as well as the subsequent application of the alignment chart, is illustrated by figure 8, which refers to a slowly decaying exponential somewhat different from the one given in figure 6, but to which the same principles apply; in fact, it is sufficient to recognize the point a as the intercept on the axis of ordinates and the point b as the intercept on a line parallel to the axis of ordinates and drawn 6 inches (on the full-size graphs) to the right of the origin. Then, with a straight edge from scale B (or C) the value of α already determined is lined up with the value of $EC_0\beta$, as set off on the proper vertical scale (figure 8) by the horizontal time axis of figure 6, and the value of the in-phase component of current I_r' is read directly from scale B' (or C') at the intersection of the straight edge with the corresponding scale. This point is illustrated by a typical example in figure 8. The purpose of the double range, scales B and C, is to furnish greater accuracy to the region more commonly used in practice.

Application of the Charts

To illustrate with a numerical application the power factor and dielectric loss charts described hereinbefore, the case illustrated in figure 1, which furnishes a typical relation between discharge current and time resolved into the sum of 3 exponentials, may be considered. Table I gives the numerical constants of the 3 exponentials, namely, the inverse time constant α and the triple product $i(0) = EC_0\beta$. The normal charging current I'' of this specimen at 1,500 volts and 60 cycles, computed from the apparent capacitance as measured by the power factor bridge,[5] was found to be 431×10^{-6} ampere.

The knowledge of the numerical constants of each exponential permits, by the use of equation 5, the computation of the individual contributions of each exponential to the in-phase component of current I_r'. The results of these computations are given in column 4 of table I. Similarly, the graphical resolution of the current-time relation as given in figure 1 allows the direct application of the charts in the manner indicated in the preceding. In figure 1 the 3 exponentials are represented by the straight lines marked 1, 2, and 3, respectively. Thus, to the *first exponential* apply the alignment chart to determine first its inverse time constant α and then, carrying out the instructions given before and illustrated in figure 8, determine its contribution to the in-phase component of current I_r'. To the *second and third exponentials* apply the "pseudotopographic" chart to determine directly their respective contributions as shown in figure 9 which here refers to the second exponential. These values and their sum are given in column 5 of table I. It may be seen by comparison of columns 4 and 5 that direct computation and the use of the charts yield numerical results which are in very close agreement.

From a knowledge of the total in-phase component of current I_r', either from direct computation or from the use of the charts, it is possible to compute at a given voltage, the contribution of reversible absorption to the phase defect angle ($\tan \psi_r$) and dielectric loss (W_r) of the specimen by means of the equations:

$$\tan \psi_r = I_r'/I'' \qquad W_r = EI_r' \tag{10}$$

where E is the effective value of the applied alternating potential and I'' is the charging current of the capacitor at the frequency of observation. The value of I'' may be computed from a knowledge of the apparent capacitance of the specimen, as determined by means of the Schering bridge, or may be measured directly with a suitable milliammeter; both methods were employed in this investigation.

Table I—Numerical Example Illustrating Use of Charts

Charge and Discharge Current-Time Relation at 1,500 Volts. Dry Paper Specimen *A*-7 at 60 Degrees Centigrade

(1) Exponential	(2) $\bar{\alpha}$, Radians per Second	(3) $i(0) = EC_0\beta$, Ampere $\times 10^{-8}$	(4) I_r', Ampere $\times 10^{-8}$ Computed	(5) I_r', Ampere $\times 10^{-8}$ From Charts
First	69	0.33	0.319	0.318
Second	500	1.45	0.525	0.522
Third	1,787	5.00	0.213	0.210
		Summation	1.057	1.050

References

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Impregnated Paper Insulation: The Inherent Electrical Properties (a book), J. B. Whitehead. John Wiley and Sons, New York, N. Y.; see appendix.

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4. Graphical and Mechanical Computation (a book), J. Lipka. John Wiley and Sons, New York, N. Y., 1918.

5. A High Sensitivity Power Factor Bridge, W. B. Kouwenhoven and Alfredo Baños, Jr. AIEE Transactions, v. 51, March 1932, p. 202–10.

Currents and Potentials Along Leaky Ground-Return Conductors

Current propagation and earth potentials for earth-return conductors with application to electric railway systems are discussed in this paper. General formulas are derived from which solutions may be obtained for problems such as propagation of track currents and voltages, and earth potentials imposed on neighboring conductors.

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THE OBJECT of this paper is to derive general formulas for currents and potentials of a long, straight, uniform wire lying on the earth's surface but separated therefrom by a thin conducting shell and subject to an arbitrary impressed electromagnetic field varying with the time as $\exp(i\omega t)$; a general formula is also given for earth potential for these conditions. These results generalize the ordinary treatment of transmission effects in earth return conductors. By specializing the field, solutions are obtained for conditions basic to the application of the results to problems associated with electric railway systems, including propagation of track currents and voltages, earth potentials imposed on neighboring communication lines, currents and potentials of neighboring cables and other conductors, and electrolysis problems. By introducing approximations, the relation of the propagation constant ordinarily employed to the basic circuit and earth constants is disclosed.

I—General Formulas

Physical Assumptions

A straight conductor of infinite length, radius a, and unit length internal impedance z, extends along the x axis at the surface of the earth, which is assumed to be homogeneous or horizontally stratified. To account for imperfect contact betwee conductor and earth, the conductor is assumed sur rounded by a shell of negligible thickness and unit length leakage conductance g, which in general is complex quantity dependent on the conductivit and dielectric constant of the shell. It is assume that the potential is constant over a given conducto cross section and that the axial current density ha circular symmetry about the conductor axis. Th conductor is located in an impressed electromagneti field varying with time as $\epsilon^{i\omega t}$ and having an axia electric intensity, $E^0(x)$, which is assumed constan over the conductor cross section. The impresse as well as the resultant electric force in the vicinit of the conductor may be decomposed into 2 com ponents. One of these components has a vanishin curl and is therefore the gradient of a scalar poten tial, which is here referred to as the earth potentia $V^0_e(x,y)$ being the impressed and $V_e(x,y)$ the re sultant earth potential.

General Solution

With the foregoing assumptions the conducto current, $I(x)$, the conductor potential $V(x)$, and th potential at the surface of the earth $V_e(x,y)$ are give by:

$$\left.\begin{aligned}
I(x) &= \int_{-\infty}^{\infty} \frac{e(u)}{\Delta(u)}\,\epsilon^{ixu}du \\
V_e^0(x,a) - V(x) &= \int_{-\infty}^{\infty} \frac{iue(u)}{\Delta(u)}\,[g^{-1} + q(u,a)]\,\epsilon^{ixu}du \\
V_e^0(x,y) - V_e(x,y) &= \int_{-\infty}^{\infty} \frac{iue(u)}{\Delta(u)}\,q(u,y)\,\epsilon^{ixu}du
\end{aligned}\right\} \tag{1}$$

where

$$\Delta(u) = u^2[g^{-1} + q(u,a)] + z + p(u,a)$$

$$\left.\begin{aligned}
p(u,y) &= i\omega \int_{-\infty}^{\infty} P(r)\,\epsilon^{iuv}dv \\
q(u,y) &= \int_{-\infty}^{\infty} Q(r)\,\epsilon^{iuv}dv
\end{aligned}\right\} r = \sqrt{v^2 + y^2}$$

$$e(u) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E^0(v)\,\epsilon^{-iuv}dv$$

Aside from r the functions $P(r)$ and $Q(r)$ depen on the frequency, the earth resistivity and structur the depth of the conductor below the surface of th earth, and at very high frequencies also on the di

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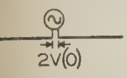
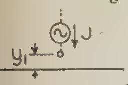
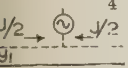
1. For all numbered references see list at end of paper.

The author is indebted to Miss Marion C. Gray and to R. M. Foster for a number of improvements in derivation, notation, and arrangement. John Riordan has verified much of the work, contributing several valuable suggestions. The author is also appreciative of the interest and advice of K. L. Maurer and H. M. Trueblood under whose supervision the paper was prepared; the present form of the derivation of the fundamental equations is a result of Doctor Trueblood's critical review.

lacement currents in the earth and in the air. They are known for a number of earth structures uch as homogeneous earth,[1] 2-layer earth,[2] and xponential variation in the earth resistivity with epth,[3] for wires on the surface of the earth and ith displacement currents neglected. The case of ires below the surface of a homogeneous earth is iven in the second of the papers referred to, which

Table I—Solutions of 4 Special Cases

$$\left\{\begin{matrix} I\ (x) \\ V\ (x) \\ V_e\ (x,y) \end{matrix}\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\begin{matrix} F\ (u) \\ F_1\ (u) \\ F_2\ (u)\end{matrix}\right\}\frac{\epsilon^{ixu}}{\Delta(u)}du$$

Case	F(u)	F₁(u)	F₂(u)
1 	$2V(0)$	$-2V(0)iu[g^{-1}+q(u,a)]$	$-2V(0)iuq(u,y)$
2 	$-Jiu[g^{-1}+q(u,a)]$	$J[g^{-1}+q(u,a)][z+p(u,a)]$	$Jq(u,y)[z+p(u,a)]$
3 	$-Jiuq(u,y_1)$	$J\cdot q(u,y_1)[z+p(u,a)]$	$J\{\Delta(u)q[u(y_1\pm y)] - u^2q(u,y_1)\cdot q(u,y)\}$
4 	$-J\frac{i}{u}p(u,y_1)$	$-Jp(u,y_1)[g^{-1}+q(u,a)]$	$-Jp(u,y_1)q(u,y)$

$\Delta(u) = u^2[g^{-1}+q(u,a)]+z+p(u,a)$ $\qquad p(u,y) = i\omega\int_{-\infty}^{\infty}P(r)\epsilon^{iuv}dv$

g = leakage conductance, mhos per meter

z = internal impedance, ohms per meter $\qquad q(u,y) = \int_{-\infty}^{\infty}Q(r)\epsilon^{iuv}dv$

a = conductor radius, in meters $\qquad r = (v^2+y^2)^{1/2}$

y_1 = separation from leaky conductor of energized electrode or isolated wire, in meters

y = separation from leaky conductor of the point at which the earth potential is taken, in meters

onductor at Surface of Homogeneous Earth

$P(r) = \frac{\nu\epsilon^{-\gamma r}}{2\pi r}\cdot\frac{\epsilon^{\gamma r}-1-\gamma r}{(\gamma r)^2}$ $\qquad Q(r) = \frac{\rho}{2\pi r}$

$p(u,y) = \frac{i\omega\nu}{\pi y\gamma^2}\left[|u|K_1(|uy|)-\sqrt{\gamma^2+u^2}\,K_1(y\sqrt{\gamma^2+u^2})\right]$ $\qquad q(u,y) = \frac{\rho}{\pi}K_0(|uy|)$

$\gamma = (i\omega/\rho)^{1/2}$ $\qquad \nu = 4\pi\cdot 10^{-7}$ henries per meter

$\omega = 2\pi f$ $\qquad \rho$ = earth resistivity in meter-ohms

K_0 and K_1 are Bessel functions of the second kind for imaginary arguments, of espectively zero and first order, as defined by Watson.

lso, although indirectly, contains expressions for $P(r)$ and $Q(r)$ which include displacement currents. The above functions and the functions $p(u,y)$ and $q(u,y)$ for homogeneous earth are given in table I.

The general solution may also be written in the orm:

$$\left.\begin{matrix} I(x) \\ V_e^0(x,a) - V(x) \\ V_e^0(x,y) - V_e(x,y)\end{matrix}\right\} = \int_{-\infty}^{\infty}E^0(v)\left\{\begin{matrix} h\ (x-v) \\ h_1\ (x-v) \\ h_2\ (x-v)\end{matrix}\right\}dv \qquad (2)$$

here $h(x)$, $h_1(x)$, and $h_2(x)$ are obtained from the ntegrals in equation 1 by taking $e(u) = {}^1/_{2\pi}$ and re respectively the conductor current, the conductor otential, and the earth potential obtained at x hen a unit voltage is applied across a break in the onductor at the origin.

Equation 1 (or 2) for the current is the solution of the following integro-differential equation:

$$\frac{d}{dx}\left[g^{-1}I'(x)+\int_{-\infty}^{\infty}I'(\tau)Q(s)d\tau\right] - zI(x) - i\omega\int_{-\infty}^{\infty}I(\tau)P(s)d\tau = -E^0(x) \qquad (3)$$

where the primes indicate differentiation and $s = [(x-\tau)^2+a^2]^{1/2}$.

The function $i\omega P(s)$ gives the axial electric force at the surface of the conductor at x for unit total axial current at τ, and the function $Q(s)$ gives the potential of the earth adjacent to the conductor at x for unit total radial current leaving the conductor at τ.

The usual differential equation of propagation is obtained from equation 3 by assuming that $I(\tau)$ and $I'(\tau)$ are practically constant and equal to $I(x)$ and $I'(x)$ in the range in which $P(s)$ and $Q(s)$ have any appreciable values. Equation 3 then reduces to the form:

$$\left[G^{-1}\frac{d^2}{dx^2} - Z\right]I(x) = -E^0(x) \qquad (4)$$

where

$$G^{-1} = g^{-1}+\int_{-\infty}^{\infty}Q(s)d\tau;\quad Z = z+i\omega\int_{-\infty}^{\infty}P(s)d\tau$$

The solution of equation 4 differs from the solution of equation 3 in that $\Delta(u)$ and $h(x)$ are changed into $\Delta_0(u) = u^2G^{-1}+Z$ and $h_0(x) = G\epsilon^{-\Gamma_0|x|}/2\Gamma_0$, where $\Gamma_0 = (GZ)^{1/2}$.

Since in the important part of the integration range $p(u,a)$ and $q(u,a)$ vary nearly logarithmically with u, large variations in u produce relatively small changes in these functions. For this reason equation 1 or 2 will give rise to approximately exponential modes of propagation, with propagation constant

$$\Gamma = [G(\beta)Z(\beta)]^{1/2}$$

where

$$G(\beta) = [g^{-1}+q(\beta,a)]^{-1},\quad Z(\beta) = z+p(\beta,a)$$

β being a constant which is so chosen that it gives a good approximation to $p(u,a)$ and $q(u,a)$ in the important part of the integration range. For $\beta = 0$, $G(0) = G$, and $Z(0) = Z$, the earth return impedance of the conductor.

With sufficient accuracy for most practical purposes β may be taken equal to Γ, in which case a transcendental equation for Γ is obtained. From this equation Γ may be obtained in terms of the basic constants of the problem: the earth resistivity, conductor radius, internal impedance, and leakage to ground, g. Such a relation is not obtained in the usual formulation, in which the leakage conductance G and the impedance Z are postulated quantities of a semiempirical nature.

With the above approximations the function $h(x)$ of equation 2 is changed into $G(\Gamma)\epsilon^{-\Gamma|x|}/2\Gamma$.

Solution by Successive Approximations

Equation 3 may be solved by successive approximations in the following way: The current is first assumed to be given by:

$$I(x) = I_1(x) + I_2(x) \tag{5}$$

where $I_1(x)$ satisfies the equation:

$$\left[G^{-1}(\Gamma)\frac{d^2}{dx^2} - Z(\Gamma) \right] I_1(x) = -E^0(x) \tag{6}$$

where $G(\Gamma)$, $Z(\Gamma)$, and Γ are determined as stated hereinbefore.

If equation 5 is inserted in equation 3 the resulting equation may be separated in 2 parts, one of which is identical with equation 6 and a second part which has the same form as equation 3 but a different electric force, $E_2^0(x)$ on the right-hand side.

The second term $I_2(x)$ may now be assumed of the form:

$$I_2(x) = I_{21}(x) + I_{22}(x) \tag{7}$$

where the first term satisfies the equation

$$\left[G^{-1}(\Gamma)\frac{d^2}{dx^2} - Z(\Gamma) \right] I_{21}(x) = -E_2^0(x) \tag{8}$$

The procedure may now be repeated to obtain an equation for $I_{22}(x) = I_{221}(x) + I_{222}(x)$, etc.

The current $I_2(x)$ is given by the following summation:

$$I_2(x) = \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \frac{e_n(u)\,\epsilon^{ixu}du}{u^2G^{-1}(\Gamma) + Z(\Gamma)} \tag{9}$$

where

$$e_n(u) = e(u)\left\{ \frac{u^2[q(\Gamma,a)-q(u,a)] + p(\Gamma,a) - p(u,a)}{u^2G^{-1}(\Gamma) + Z(\Gamma)} \right\}^n$$

Successive terms in this summation correspond respectively to $I_{21}(x)$, $I_{221}(x)$, etc.

II—Special Cases

In the following are considered 4 special cases of impressed electric force, which are shown together with their solutions in table I. Approximate formulas for these cases are given in table II for the special condition of homogeneous earth. These special cases were taken because of their practical application in problems connected with electrified railway systems.

Case 1

Discontinuity in conductor at origin; voltage applied across break. The electric force $E^0(x)$ is required to vanish at all points except near $x = 0$. The expression:

$$E^0(x) = \frac{E(0)l}{\pi}\int_{-\infty}^{\infty} \frac{\sin ul}{ul}\,\epsilon^{ixu}du$$

gives an electric force $E(0)$ in the interval $-l \leqq x$ ≦
and zero outside this interval. The expression f
$e(u)$ is then

$$e(u) = \frac{E(0)l}{\pi}\,\frac{\sin ul}{ul}$$

Inserting $e(u)$ in equation 1 and taking $V_e^0(x,a) =$
the limiting values $I(x)$, $V(x)$, and $V_e(x,y)$ are t
required quantities when $V(0)$ designates the lin
approached by $lE(0)$ as $E(0)$ becomes infinite at t
same time as l becomes zero.

Approximate Formulas. The approximate fo
mulas are obtained by approximating the functio
$p(u,y)$ and $q(u,y)$ in the manner described in part
The value of β has been taken equal to Γ, and t
formulas for $G(\Gamma)$ and $Z(\Gamma)$ given in table II a
obtained by taking the first terms in the expansi
of $p(\Gamma,a)$ and $q(\Gamma,a)$ for conductors at the surface
a homogeneous earth.* The following transce
dental equation for the propagation constant of t
current is then obtained:

$$\Gamma = [G(\Gamma)Z(\Gamma)]^{1/2}$$

where, with the notation of table II,

$$G(\Gamma) = \left[g^{-1} + \frac{\rho}{\pi}\log_\epsilon \frac{1.12\ldots}{a\Gamma} \right]^{-1}$$

$$Z(\Gamma) = z + \frac{i\omega v}{2\pi}\log_\epsilon \frac{1.85\ldots}{a\alpha}$$

The above equation may be solved by the meth
of successive approximations. A convenient fi
approximation is $\Gamma = \Gamma_1 = (gZ)^{1/2}$ since $Z = Z($
is the usual first approximation to $Z(\Gamma)$ and g
usually of the same order of magnitude as $G(\Gamma)$.

The earth potential is obtained as follows:

$$V_e(x,y) \cong -\frac{V(0)G(\Gamma)}{\pi}\int_{-\infty}^{\infty} \frac{iuq(u,y)}{u^2+\Gamma^2}\,\epsilon^{ixu}du$$

$$= -\frac{I(0)\Gamma}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q(r)\,\frac{iu\,\epsilon^{iu(x+v)}}{u^2+\Gamma^2}\,du\,dv$$

$$= I(0)\Gamma\left[\epsilon^{-\Gamma x}\int_{-x}^{\infty} \epsilon^{-\Gamma v}\,Q(r)dv - \epsilon^{\Gamma x}\int_{x}^{\infty}\epsilon^{-\Gamma v}Q(r)dv \right]$$

With $Q(r) = \rho/2\pi r$, $r = (v^2 + y^2)^{1/2}$, the formu
given in table II is obtained.

Case 2

*Conductor energized to a remote point over a perpe
dicular insulated wire.* This case may be obtain
from case 1 by application of the following propo
tion which follows from the reciprocal theorem:

$$\frac{I(x)}{2I(0)} = \frac{V(x)}{2V(0)}$$

where

$I(x)$ = current in case 2

* The functions $p(u,a)$ and $q(u,a)$ given in table I for homogeneous earth
obtained from pairs 917.8 and 918 of "Fourier Integrals for Practical Appli
tions" by G. A. Campbell and R. M. Foster, Bell Telephone System Techn
Publication, Monograph B-584.

$V(x)$ = conductor potential in case 1
$2I(0)$ = current entering conductor in case 2
$2V(0)$ = voltage applied in case 1

From this relation it follows that in case 2:

$$e(u) = -\frac{J}{\pi} iu[g^{-1} + q(u,a)]$$

CASE 3

Electrode near conductor energized to a remote point over a perpendicular insulated wire. In this case the impressed earth potential along the conductor is:

$$V_e{}^0(x) = JQ(R_1), \quad R_1 = (x^2 + y_1{}^2)^{1/2}$$

where y_1 is the separation of the energized electrode from the leaky conductor and J is the current entering the earth at the electrode.

From relations given in part I it follows by Fourier transformation (after changing u to $-u$) that:

$$Q(R_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty} q(u,y_1)\epsilon^{ixu}du$$

The impressed force along the conductor is:

$$E^0(x) = -\frac{d\,V_e{}^0(x)}{dx} = -\frac{J}{2\pi}\int_{-\infty}^{\infty} iuq(u,y_1)\epsilon^{ixu}du$$

Hence

$$e(u) = -\frac{J}{2\pi} iuq(u,y_1)$$

CASE 4

Conductor paralleled by an insulated wire which is energized to a remote point over a perpendicular insulated wire. In this case the impressed electric force along the conductor is:

$$E^0(x) = \frac{J}{2} i\omega \int_{-\infty}^{0} P(s_1)d\tau - \frac{J}{2} i\omega \int_{0}^{\infty} P(s_1)d\tau$$

where $s_1 = [(x - \tau)^2 + y_1{}^2]^{1/2}$, y_1 = separation of conductors, and $J/2$ and $-J/2$ are the currents in the insulated wire for x less than and greater than zero, respectively.

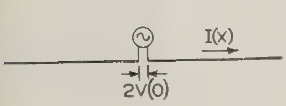
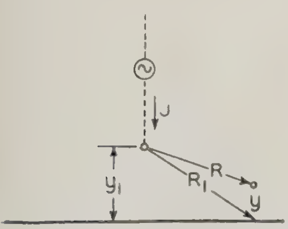
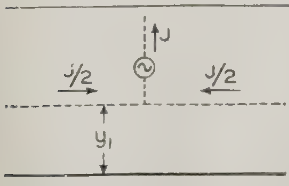
By change of variable of integration:

$$E^0(x) = -\frac{J}{2} i\omega\left[\int_{x}^{-\infty} P(r_1)dv - \int_{\infty}^{x} P(r_1)dv\right]; \quad r_1 = (v^2 + y_1{}^2)^{1/2}$$

From relations given in part I it follows by Fourier transformation (after changing u to $-u$) that:

$$i\omega P(r_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty} p(u,y_1)\epsilon^{iuv}du$$

Table II—Approximate Formulas

Case	Approximate Formulas	Notation
1 	$I(x) = V(0)\frac{G(\Gamma)}{\Gamma}\epsilon^{-\Gamma\lvert x\rvert} = I(0)\epsilon^{-\Gamma\lvert x\rvert}$ $V(x) = \pm I(0)\frac{\Gamma}{G(\Gamma)}\epsilon^{-\Gamma\lvert x\rvert}, \quad \pm x > 0$ $V_e(x,y) = I(0)\frac{\rho\Gamma}{2\pi}[\epsilon^{-\Gamma x}\phi(\Gamma x,\Gamma y) - \epsilon^{\Gamma x}\phi(-\Gamma x,\Gamma y)]$	$\phi(u,v) = \int_{-u}^{\infty}\frac{\epsilon^{-\tau}d\tau}{\sqrt{\tau^2+v^2}}$ $= J_0(v)\log_\epsilon\left(\frac{v}{w-u}\right) - \frac{\pi}{2}Y_0(v) + \sum_{n=1}^{\infty}\frac{1}{n!}A_n$
2 	$I(x) = \pm\frac{J}{2}\epsilon^{-\Gamma\lvert x\rvert}; \quad \pm x > 0$ $V(x) = \frac{J\Gamma}{2G(\Gamma)}\epsilon^{-\Gamma\lvert x\rvert}$ $V_e(x,y) = \frac{J}{2}\frac{\rho\Gamma}{2\pi}[\epsilon^{-\Gamma x}\phi(\Gamma x,\Gamma y) + \epsilon^{\Gamma x}\phi(-\Gamma x,\Gamma y)]$	where $w = (u^2+v^2)^{1/2}$ and: $A_0 = 0,\ A_1 = w, \ldots A_n = \frac{1}{n}[wu^{n-1} - (n-1)v^2A_{n-2}]$ $\phi(0,v) = \frac{\pi}{2}[\boldsymbol{H}_0(v) - Y_0(v)]$ $\phi(\mp u,\delta) = \pm Ei(\pm u) - (1\mp 1)\left[i\frac{\pi}{2} + C - \log_\epsilon(2/\delta)\right]$ $\delta << 1, \quad C = 0.5772\ldots$
3 	$I(x) = \frac{J}{2}\frac{\rho G(\Gamma)}{2\pi}[\epsilon^{-\Gamma x}\phi(\Gamma x,\Gamma y) - \epsilon^{\Gamma x}\phi(-\Gamma x,\Gamma y)]$ $V(x) = \frac{J}{2}\frac{\rho\Gamma}{2\pi}[\epsilon^{-\Gamma x}\phi(\Gamma x,\Gamma y) + \epsilon^{\Gamma x}\phi(-\Gamma x,\Gamma y)]$ $V_e(x,y) =$ $\frac{J\rho}{2\pi}\left\{\frac{1}{R} - \frac{\rho G(\Gamma)K_0(\Gamma y)}{2\pi}\left[\frac{2}{R_1} - \Gamma\epsilon^{-\Gamma x}\phi(\Gamma x,\Gamma y_1) - \Gamma\epsilon^{\Gamma x} \times \phi(-\Gamma x,\Gamma y_1)\right]\right\}$	J_0 = Bessel function of the first kind, zero order Y_0 = Bessel function of the second kind, zero order $\boldsymbol{H}_0$ = Struve's associated Bessel function, zero order Ei = exponential integral, $Ei(u) = \int_{u}^{\infty}\frac{\epsilon^{-v}du}{u}$ $\mu = M(\Gamma)/Z(\Gamma)$ — a = conductor radius, meters $M(\Gamma) = 2i\omega 10^{-7}\log_\epsilon\frac{1.85}{y_1\alpha}$ — z = internal impedance, ohms per meter $Z(\Gamma) = z + 2i\omega 10^{-7}\log_\epsilon\frac{1.85}{a\alpha}$ — g = leakage conductance, mhos per meter
4 	$I(x) = \pm\frac{J}{2}\mu(1 - \epsilon^{-\Gamma\lvert x\rvert}), \quad \pm x > 0$ $V(x) = -\frac{J}{2}\mu\frac{\Gamma}{G(\Gamma)}\epsilon^{-\Gamma\lvert x\rvert}$ $V_e(x,y) = -\frac{J}{2}\mu\frac{\rho\Gamma}{2\pi}[\epsilon^{-\Gamma x}\phi(\Gamma x,\Gamma y) + \epsilon^{\Gamma x}\phi(-\Gamma x,\Gamma y)]$	$G(\Gamma) = \left[g^{-1} + \frac{\rho}{\pi}\log_\epsilon\frac{1.12}{a\Gamma}\right]^{-1}$ — ρ = earth resistivity, meter-ohms $\Gamma = \sqrt{G(\Gamma)Z(\Gamma)}$ $\alpha^2 = i\omega\nu\rho^{-1} + \Gamma^2, \quad \omega = 2\pi f$ $\nu = 4\pi\cdot 10^{-7}$ henries per meter y = separation at which earth potential is taken, meters y_1 = separation from leaky conductor of energized electrode or isolated wire, meters

Inserting this relation in the preceding expression and integrating,

$$E^0(x) = \frac{J}{2\pi}\int_{-\infty}^{\infty}\frac{1}{iu}\,p(u,y_1)\epsilon^{ixu}du$$

Hence

$$e(u) = \frac{J}{2\pi}\,\frac{1}{iu}\,p(u,y_1)$$

Comparison of Cases

The solutions for the preceding cases may be obtained in a number of other ways. Thus, case 2 may be obtained from case 3 and *vice versa*, by noting that by the reciprocal theorem the conductor potential with the electrode energized must equal the earth potential at the electrode with the conductor energized. Case 2 may also be obtained as follows: On the current obtained by letting in case 3 the electrode separation approach the conductor radius is superposed the current due to an impressed potential between the conductor and the earth adjacent to it. This potential is assumed to vanish except at $x = 0$. The result for case 1 may be obtained by assuming the current to be induced in the leaky conductor by energizing a small rectangular loop in its vicinity. Such a metallic loop may be obtained by superposing 4 insulated circuits of the kind used in case 4.

In the first and second cases there are discontinuities in voltage and current, respectively, at $x = 0$. In the third and fourth cases there are no discontinuities. In the first case the current is an even and the potential an odd function of x; in the other cases the opposite is true. The third case may appear to be a special condition of the second case in which the electrode separation is equal to the conductor radius. This is true only when the conductor is in perfect contact with the earth. The first case is, with $2V(0) = 1$, the fundamental case referred to in part I, in terms of which other solutions are obtained by means of equation 2.

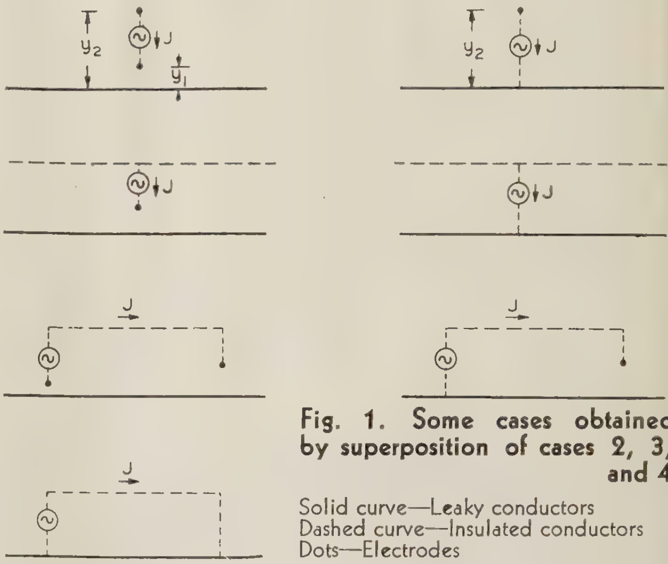
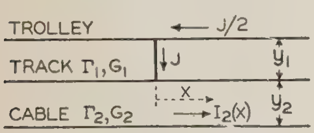
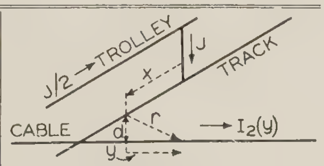


Fig. 1. Some cases obtained by superposition of cases 2, 3, and 4

Solid curve—Leaky conductors
Dashed curve—Insulated conductors
Dots—Electrodes

Table III—Leaky Conductors Near Electrified Tracks

	 Depth d small
$I_2(x) = \frac{J}{2}\frac{G_2(1-\mu)\Gamma_1^2}{\Gamma_1^2-\Gamma_2^2}[\psi(\Gamma_2x)-\psi(\Gamma_1x)]$ $U_2(x) = -\frac{J}{2}\frac{G_2(1-\mu)\Gamma_1^2}{g_2(\Gamma_1^2-\Gamma_2^2)}[\psi_x'(\Gamma_2x)-\psi_x'(\Gamma_1x)]$ $\psi(\Gamma x) = \pm Z_{12}\frac{1-\epsilon^{-\Gamma\|x\|}}{\Gamma^2}+\frac{\rho}{2\pi}[\epsilon^{-\Gamma x}\times\phi(\Gamma x,\Gamma_1y_2)-\epsilon^{\Gamma x}\phi(-\Gamma x,\Gamma_1y_2)]$ where $\pm x = 0$ $\psi_x'(\Gamma_1x) = Z_{12}\frac{\epsilon^{-\Gamma\|x\|}}{\Gamma}-\frac{\rho\Gamma}{2\pi}[\epsilon^{-\Gamma x}\times\phi(\Gamma x,\Gamma_1y_2)+\epsilon^{\Gamma x}\phi(-\Gamma x,\Gamma_1y_2)]$	$I_2(y) = \frac{J\rho G_2\Gamma_1}{4\pi\Gamma_2}(1-\mu)\epsilon^{-\Gamma_1\|x\|}\{\epsilon^{\Gamma_2y}Ei(\Gamma_2r)-\epsilon^{-\Gamma_2y}[Ei(-\Gamma_2r)+i\pi]\}$ where $\Gamma_2y<<1$ $U_2(y) = -\frac{J\rho G_2\Gamma_1}{4\pi g_2}(1-\mu)\epsilon^{-\Gamma_1\|x\|}\{\epsilon^{\Gamma_2y}\times Ei(\Gamma_2r)+\epsilon^{-\Gamma_2y}[Ei(-\Gamma_2r)+i\pi]\}$ where $\Gamma_2y<<1$

$U_2(x)$ = potential between cable and adjacent point in earth, in volts
$G_2 = \left[g_2^{-1}+\frac{\rho}{\pi}\log_\epsilon\frac{1.12}{a_2\Gamma_2}\right]^{-1}$, a_2 = outside radius of cable duct, in meters
Γ_2 = propagation constant of cable, per meter
g_2 = cable leakage conductance as measured to outside of cable duct, mhos per meter
Z_{12} = mutual impedance of track and cable, ohms per meter
$\phi(u,v)$, $Ei(u)$, ρ and μ as in table II.

Solutions have been published for the second and third cases for the special conditions of direct current and perfect contact between the conductor and the earth. Case 2 has been solved by F. Ollendorff[4] by a different method, and case 3 by F. Noether[5] by a method similar to that used here, both with substantially the same result as given here.

The approximate formulas given in table II for the current correspond to the first term $I_1(x)$ of equation 5. Using the method of approximation outlined in part I the next terms for the current and the conductor potential for cases 1 and 2, respectively, become,*

Case 1

$$I_{21}(x) = -I(0)\frac{G(\Gamma)\rho}{4\pi}\{(1+\Gamma x)\epsilon^{\Gamma x}Ei(\Gamma x)-(1-\Gamma x)\epsilon^{-\Gamma x}[Ei(-\Gamma x)+i\pi]\}$$

$$V_{21}(x) = I(0)\frac{\Gamma\rho}{4\pi}\{\Gamma x\epsilon^{\Gamma x}Ei(\Gamma x)-\Gamma x\epsilon^{-\Gamma x}[Ei(-\Gamma x)+i\pi]-2\}$$

where $x \geq 0$ and $I_{21}(-x) = I_{21}(x)$; $V_{21}(-x) = -V_{21}(x)$

Case 2

$$I_{21}(x) = \frac{J}{2}\frac{G(\Gamma)\rho}{4\pi}\{\Gamma x\epsilon^{\Gamma x}Ei(\Gamma x)-\Gamma x\epsilon^{-\Gamma x}[Ei(-\Gamma x)+i\pi]-2(1-\epsilon^{-\Gamma x})\}$$

* In deriving these expressions it is assumed that $\Gamma << \gamma$ so that $p(\Gamma,a) - p(u,a) = 0$. A term $I(0)\rho G(\Gamma)\epsilon^{-\Gamma|x|}/2\pi$ is also obtained in the current for case 1 but is omitted in order to satisfy the boundary condition $I_{21}(x) = 0$ at $x = 0$.

$$V_{21}(x) = \frac{J}{2}\frac{\Gamma\rho}{4\pi}\{(1-\Gamma x)\epsilon^{\Gamma x}Ei(\Gamma x)-(1+\Gamma x)\epsilon^{-\Gamma x}[Ei(-\Gamma x)+i\pi]+2\epsilon^{-\Gamma x}\}$$

where $x \geq 0$ and $I_{21}(-x) = -I_{21}(x)$; $V_{21}(-x) = V_{21}(x)$.

These terms may be neglected in comparison with the first terms in the range of most practical applications. For sufficiently large values of Γx, however, they become large in comparison with the first terms. For case 1 the bracket terms of $I_{21}(x)$ and $V_{21}(x)$ vanish as $-8/(\Gamma x)^3$ and $4/(\Gamma x)^2$, respectively, and for case 2 as $4/(\Gamma x)^2$ and $4/\Gamma x$, respectively.

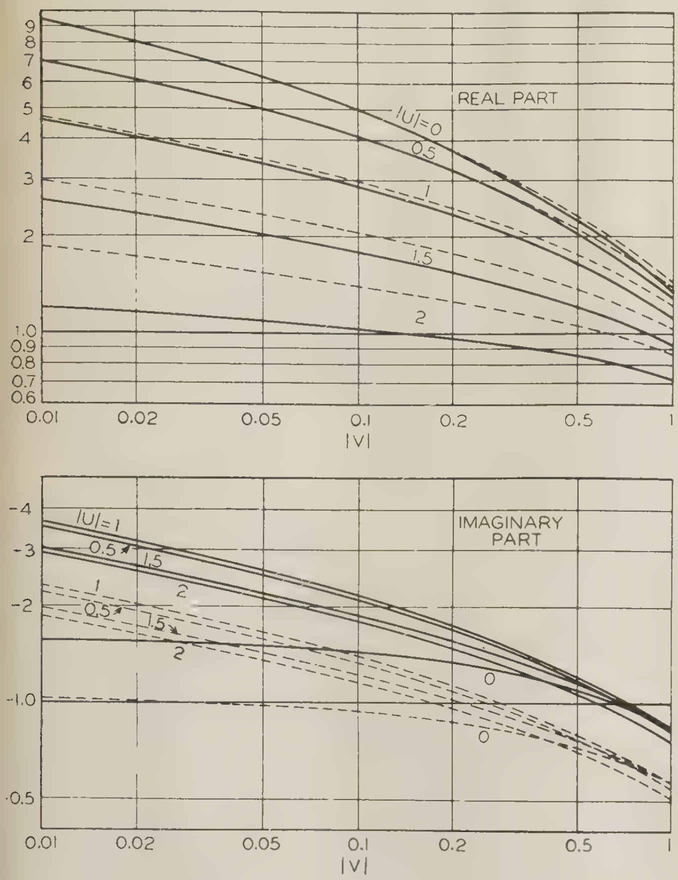


Fig. 2. Charts for values of the function $\epsilon^{-u}\phi(u, v) + \epsilon^{u}\phi(-u, v)$

Solid curve—45-degree angle of u and v
Dashed curve—30-degree angle of u and v

A strictly exponential current or potential propagation may be realized only under certain restricted conditions. If the conductor instead of being located at the surface of the earth is imbedded in a homogeneous medium of resistivity ρ and infinite extent in all directions, then $p(u,a)$ differs from $q(u,a)$ only by the factor $i\omega\nu/\rho$. With the conductor energized as in case 1, the propagation of the conductor potential will then be strictly exponential if $zg = i\omega\nu/\rho$ and the propagation constant will equal $\Gamma = \sqrt{zg}$. With the conductor energized as in case 2 the current propagation will be exponential under the same conditions. To have exponential propagation of both current and potential does not, however, appear possible under any conditions, except for the trivial case in which $\rho = 0$.

III—Practical Applications

From the last 3 cases considered in the preceding numerous others may be obtained by superposition, some of which are shown in figure 1. By superposition of the first and the other cases, the latter are extended to conductors which are discontinuous at some point.

Earth Potentials Near Electric Railways

Impressed voltages in earth-return circuits exposed to electric railways are due partly to induction and partly to earth potential differences between the circuit terminals. When the propagation constant of the track is determined by the method given herein, the usual methods[6] of calculating induced voltages are still appropriate. The calculation of earth potentials is considered in some detail in the following.

The case in which an insulated conductor (trolley) is energized to a parallel leaky conductor (track) is obtained by superposition of cases 2 and 4. The earth potential for this case is:

$$V_e(x,y) = \frac{J\rho\Gamma}{4\pi}(1-\mu)[\epsilon^{-\Gamma x}\phi(\Gamma x,\Gamma y)+\epsilon^{\Gamma x}\phi(-\Gamma x,\Gamma y)]$$

where the notation is the same as in table II. The function in the bracket is shown in table V and on figure 2 for a practical range of the variables.

When the trolley is energized between 2 points of the track at the distance s apart the earth potential is $V_e(x,y) - V_e(s - x,y)$.

In applying the above formulas μ is taken as the ratio of the earth-return mutual impedance between trolley and track to the earth-return self-impedance of the track.* In calculating the leakage conductance, G, an equivalent radius must be used for the track. A very accurate determination of this radius is not necessary, since G is only to a small degree dependent upon it. By assuming the leakage current to be uniformly distributed over the boundary surface of ballast and earth an equivalent radius equal to 0.37 times the width of the ballast is obtained.

The leakage conductance $G = G(\Gamma)$ is smaller than the ballast leakage conductance, g. As g increases $G(\Gamma)$ approaches the limiting value:

$$\lim_{g=\infty} G(\Gamma) = \frac{\pi}{\rho \log_\epsilon \frac{1.12}{a\Gamma}}$$

For electric railways the value of $a\Gamma$ may vary between 10^{-4} and 10^{-2}, as a result the maximum value of $G(\Gamma)$ varies between $0.34/\rho$ and $0.66/\rho$ mhos per meter, ρ being the earth resistivity in meter-ohms.

When there are several tracks the earth potentials are practically independent of the cross-bonding,

* When $\Gamma << \sqrt{i\omega\nu/\rho}$, as is usually the case for electric railways, $M(\Gamma) = M$ and $Z(\Gamma) = Z$ so that $\mu = M/Z$.

except for points near the tracks. The preceding formulas assume all the tracks to be cross-bonded at the point where the current enters the tracks. If there are 2 tracks and they are not cross-bonded at any point, the following term must be added to the formulas given in the preceding:

$$\Delta V_e(x,y) = \frac{J\rho\Gamma_1}{4\pi}\epsilon^{-\Gamma_1|x|}\log_\epsilon\frac{y''}{y'}$$

where

y' = separation to center of track which is energized to trolley
y'' = separation to center of other track
$\Gamma_1 = \sqrt{(Z-m)g}$
Z = self-impedance of 2 tracks in parallel
m = mutual impedance of the 2 tracks
g = ballast leakage conductance of the 2 tracks

The above expression shows that $\Delta V_e(x,y)$ vanishes rapidly with increasing separation y from the center of the 2 tracks.

Currents in Cables Near Electric Railways

In table III approximate formulas are given for the current and voltage to ground of a leaky conductor, such as a cable, which parallels an electrified track. These formulas are derived by assuming an earth potential along the cable as given by the preceding approximate formula and an induced voltage per unit length which is equal to the earth current times the mutual impedance of track and cable sheath. The reaction of the cable current on the track current is neglected. These approximate assumptions allow relatively simple formulas which should be sufficiently accurate for engineering use.

For the foregoing case of 2 parallel leaky conductors a formula has been published by R. Gibrat[7] for the special conditions of direct current and perfect contact between conductor and earth.

Similar formulas for the case of a cable crossing an electrified track at right angles are also given in table III. These formulas are based on the approximate assumption that the earth potential along the cable decreases logarithmically with the separation y from the track and that the reaction of the cable current on the track current may be neglected. The current may be obtained from equation 2 using the approximation for $h(x)$ stated in part I and taking $E^0(v) = 0$ in an infinitesimal interval at the crossing.

IV—General Derivation

Equation of Current Propagation

Let $V(x)$ and $V_e(x,y)$ be the resultant conductor and earth potentials, respectively, and $U(x) = V(x) - V_e(x,a)$ the resultant potential difference between the conductor and an adjacent point in the earth. If $A_w(x)$ and $A_e(x)$ are the x components of the resultant vector potentials at the surface of the wire and in the earth, respectively, then the x components of the electric force at the surface of the wire and in the earth are:

$$E_w(x) = -i\omega A_w(x) - \frac{d}{dx}V(x) \tag{10}$$

$$E_e(x) = -i\omega A_e(x) - \frac{d}{dx}V_e(x,a)$$

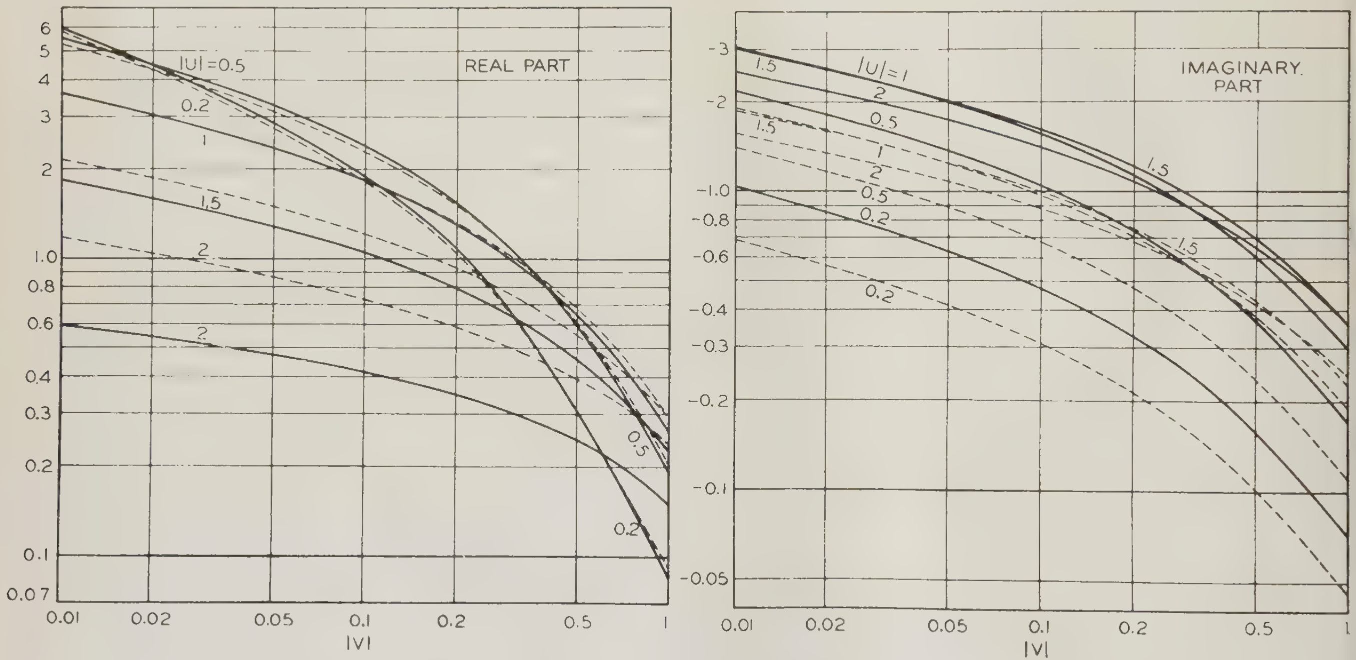


Fig. 3. Charts for values of the function $\epsilon^{-u}\phi(u, v) - \epsilon^{u}\phi(-u, v)$

Solid curve—45-degree angle of u and v
Dashed curve—30-degree angle of u and v

Table IV—Exponential Integral Functions Ei(±u) of Tables II and III for Angles θ of u Equal to 0, 30, and 45 Degrees*

$\|u\|$	$\theta = 0$		$\theta = \pi/6$		$\theta = \pi/4$	
	$Ei(u)$	$Ei(-u)+i\pi$	$Ei(u)$	$Ei(-u)+i\pi$	$Ei(u)$	$Ei(-u)+i\pi$
0.01	4.0379	4.0179	4.0366 − i0.5186	4.0193 − i0.5286	4.0350 − i0.7784	4.0209 − i0.7925
0.02	3.3547	3.3147	3.3521 − i0.5136	3.3175 − i0.5337	3.3489 − i0.7714	3.3207 − i0.7996
0.03	2.9591	2.8991	2.9552 − i0.5088	2.9033 − i0.5388	2.9506 − i0.7644	2.9081 − i0.8068
0.04	2.6813	2.6013	2.6761 − i0.5039	2.6068 − i0.5439	2.6699 − i0.7575	2.6134 − i0.8141
0.05	2.4679	2.3679	2.4615 − i0.4991	2.3749 − i0.5491	2.4539 − i0.7507	2.3832 − i0.8214
0.06	2.2953	2.1753	2.2877 − i0.4944	2.1834 − i0.5544	2.2786 − i0.7439	2.1938 − i0.8287
0.07	2.1508	2.0108	2.1421 − i0.4897	2.0208 − i0.5597	2.1315 − i0.7371	2.0326 − i0.8361
0.08	2.0269	1.8669	2.0170 − i0.4850	1.8785 − i0.5650	2.0051 − i0.7304	1.8920 − i0.8436
0.09	1.9187	1.7387	1.9077 − i0.4804	1.7518 − i0.5704	1.8943 − i0.7238	1.7671 − i0.8511
0.1	1.8229	1.6228	1.8108 − i0.4757	1.6375 − i0.5758	1.7962 − i0.7172	1.6547 − i0.8586
0.15	1.4645	1.1641	1.4470 − i0.4533	1.1872 − i0.6037	1.4258 − i0.6848	1.2140 − i0.8972
0.2	1.2227	0.8218	1.2004 − i0.4319	0.8540 − i0.6326	1.1733 − i0.6537	0.8911 − i0.9371
0.25	1.0443	0.5425	1.0178 − i0.4113	0.5848 − i0.6630	0.9853 − i0.6236	0.6330 − i0.9784
0.3	0.9057	0.3027	0.8753 − i0.3916	0.3557 − i0.6946	0.8379 − i0.5947	0.4158 − i1.0211
0.35	0.7942	0.0894	0.7605 − i0.3729	0.1543 − i0.7276	0.7186 − i0.5669	0.2270 − i1.0652
0.4	0.7024	−0.1048	0.6656 − i0.3549	−0.0272 − i0.7620	0.6197 − i0.5401	0 0590 − i1.1107
0.45	0.6253	−0.2849	0.5859 − i0.3377	−0.1935 − i0.7978	0.5363 − i0.5143	−0.0929 − i1.1578
0.5	0.5598	−0.4542	0.5180 − i0.3214	−0.3480 − i0.8352	0.4652 − i0.4895	−0.2320 − i1.2063
0.55	0.5034	−0.6153	0.4598 − i0.3055	−0.4929 − i0.8743	0.4039 − i0.4656	−0.3607 − i1.2564
0.6	0.4544	−0.7699	0.4088 − i0.2907	−0.6302 − i0.9148	0.3506 − i0.4427	−0.4807 − i1.3080
0.65	0.4115	−0.9194	0.3644 − i0.2764	−0.7610 − i0.9571	0.3042 − i0.4207	−0.5934 − i1.3613
0.7	0.3738	−1.0649	0.3254 − i0.2626	−0.8865 − i1.0010	0.2633 − i0.3996	−0.6993 − i1.4161
0.75	0.3403	−1.2073	0.2910 − i0.2500	−1.0074 − i1.0472	0.2272 − i0.3794	−0.7997 − i1.4726
0.8	0.3106	−1.3474	0.2604 − i0.2371	−1.1242 − i1.0945	0.1954 − i0.3599	−0.8950 − i1.5308
0.85	0.2840	−1.4857	0.2333 − i0.2252	−1.2377 − i1.1441	0.1671 − i0.3413	−0.9856 − i1.5906
0.9	0.2601	−1.6228	0.2090 − i0.2135	−1.3481 − i1.1955	0.1420 − i0.3234	−1.0721 − i1.6521
0.95	0.2387	−1.7591	0.1873 − i0.2031	−1.4559 − i1.2496	0.1197 − i0.3063	−1.1546 − i1.7154
1.0	0.2194	−1.8951	0.1679 − i0.1927	−1.5613 − i1.3055	0.0998 − i0.2900	−1.2334 − i1.7804
1.1	0.1860	−2.1674	0.1344 − i0.1735	−1.7660 − i1.4240	0.0664 − i0.2593	−1.3810 − i1.9156
1.2	0.1584	−2.4421	0.1073 − i0.1558	−1.9637 − i1.5518	0.0395 − i0.2313	−1.5160 − i2.0582
1.3	0.1355	−2.7214	0.0855 − i0.1402	−2.1551 − i1.6903	0.0188 − i0.2057	−1.6385 − i2.2078
1.4	0.1162	−3.0072	0.0675 − i0.1258	−2.3413 − i1.8393	0.0023 − i0.1825	−1.7497 − i2.3648
1.5	0.1000	−3 3013	0.0527 − i0.1127	−2.5225 − i1.9999	−0.0106 − i0.1613	−1.8495 − i2.5292
1.6	0.08631	−3.6053	0.0398 − i0.1009	−2.6998 − i2.1728	−0.0205 − i0 1431	−1.9377 − i2.7009
1.7	0.07465	−3.9210	0.0308 − i0.0903	−2.8706 − i2.3586	−0.0279 − i0.1248	−2.0142 − i2.8798
1.8	0.06471	−4.2499	0.0227 − i0.0806	−3.0373 − i2.5583	−0.0334 − i0.1091	−2.0786 − i3.0659
1.9	0.05620	−4.5937	0.0162 − i0.0719	−3.1988 − i2.7727	−0.0372 − i0.0951	−2.1305 − i3.2590
2.0	0.04261	−4.9542	0.0109 − i0.0640	−3.3545 − i3.0026	−0.0397 − i0.0824	−2.1693 − i3.4588

* The values for zero degrees are taken from Jahnke and Emde, Funktionentafeln, the other values are computed from the series:

$$Ei(u) = -C - \log|u| - i\theta - \sum_{n=1}^{\infty} \frac{(-1)^n|u|^n}{n \cdot n!}(\cos n\theta + i \sin n\theta) = \int_u^{\infty} \frac{\epsilon^{-u}}{u}\,du \qquad u = |u|\epsilon^{i\theta}; \quad C = 0.57722\ldots$$

and

$$E_w(x) - E_e(x) = -i\omega[A_w(x) - A_e(x)] - \frac{d}{dx}U(x) \tag{11}$$

If the shell surrounding the wire is sufficiently thin the bracket term in equation 11 may be neglected since the magnetic flux between the inner and outer surfaces of the shell becomes negligible. With $E_w(x) = zI(x)$ equation 11 then becomes:

$$zI(x) - E_e(x) = -\frac{d}{dx}U(x) \tag{12}$$

$E_e(x)$ and $U(x)$ each may be written as the sum of an impressed or primary and a secondary component

$$E_e(x) = E_1(x) + E_2(x)$$

$$U(x) = U_1(x) + U_2(x) \tag{13}$$

The secondary potential difference $U_2(x)$ has the following relation to the current leaving the wire:

$$U_2(x) = -g^{-1}\frac{dI(x)}{dx} \tag{14}$$

Inserting equations 13 and 14 in equation 12 the following equation is obtained:

$$g^{-1}\frac{d^2I(x)}{dx^2} - zI(x) + E_2(x) = \frac{d}{dx}U_1(x) - E_1(x) \tag{15}$$

The secondary electric force $E_2(x)$ due to the current in the wire is given by the expression:

$$E_2(x) = -i\omega\int_{-\infty}^{\infty} I(\tau)P(s)d\tau + \frac{d}{dx}\int_{-\infty}^{\infty} I'(\tau)Q(s)d\tau \tag{16}$$

When equation 16 is inserted in 15, equation 3 is obtained with:

$$E^0(x) = E_1(x) - \frac{d}{dx}U_1(x) \tag{17}$$

In case of several parallel leaky conductors the equations of current propagation may be derived in a similar way.

General Solution

The current is assumed as:

$$I(x) = \int_{-\infty}^{\infty} F(u)\epsilon^{iux}du \tag{18}$$

Inserting equation 18 in equation 3 the following integrals arise:

(a) $i\omega\int_{-\infty}^{\infty} I(\tau)P(s)d\tau = i\omega\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u)\epsilon^{i\tau u}P(s)du\,d\tau;$

With

$$\tau - x = v \qquad d\tau = dv \qquad s = (v^2 + a^2)^{1/2}$$

Table V—Functions $\epsilon^{-u}\phi(u,v) \pm \epsilon^{u}\phi(u,v)$ of Table II for Angles θ of u and v Equal to 30 and 45 Degrees

$\epsilon^{-u}\phi(u, v) + \epsilon^{u}\phi(-u, v)$

$\lvert v \rvert$	$\lvert u \rvert = 0$, $\theta = \pi/6$	$\lvert u \rvert = 0$, $\theta = \pi/4$	$\lvert u \rvert = 0.5$, $\theta = \pi/6$	$\lvert u \rvert = 0.5$, $\theta = \pi/4$	$\lvert u \rvert = 1.0$, $\theta = \pi/6$	$\lvert u \rvert = 1.0$, $\theta = \pi/4$	$\lvert u \rvert = 1.5$, $\theta = \pi/6$	$\lvert u \rvert = 1.5$, $\theta = \pi/4$	$\lvert u \rvert = 2.0$, $\theta = \pi/6$	$\lvert u \rvert = 2.0$, $\theta = \pi/4$
0.01	9.460 − i1.037	9.456 − i1.559	7.015 − i1.987	7.147 − i3.017	4.684 − i2.334	4.605 − i3.657	2.988 − i2.205	2.590 − i3.504	1.852 − i1.862	1.201 − i2.931
0.10	5.000 − i0.943	4.974 − i1.447	4.120 − i1.248	4.114 − i1.897	2.984 − i1.406	2.879 − i2.182	2.069 − i1.349	1.812 − i2.112	1.412 − i1.176	1.027 − i1.825
0.20	3.760 − i0.890	3.720 − i1.344	3.249 − i1.038	3.207 − i1.577	2.460 − i1.137	2.349 − i1.754	1.781 − i1.096	1.560 − i1.703	1.270 − i0.971	0.963 − i1.495
0.40	2.640 − i0.780	2.576 − i1.178	2.396 − i0.831	2.323 − i1.262	1.931 − i0.876	1.817 − i1.341	1.482 − i0.849	1.305 − i1.304	1.118 − i0.769	0.886 − i1.170
0.60	2.060 − i0.694	1.986 − i1.046	1.921 − i0.714	1.836 − i1.082	1.617 − i0.733	1.504 − i1.116	1.295 − i0.711	1.140 − i1.084	1.017 − i0.653	0.824 − i0.986
0.80	1.698 − i0.624	1.616 − i0.938	1.607 − i0.632	1.516 − i0.955	1.396 − i0.638	1.286 − i0.967	1.156 − i0.618	1.017 − i0.936	0.936 − i0.573	0.765 − i0.863
1.00	1.448 − i0.570	1.360 − i0.848	1.380 − i0.569	1.287 − i0.857	1.228 − i0.568	1.120 − i0.857	1.044 − i0.549	0.916 − i0.829	0.868 − i0.513	0.721 − i0.769

$\epsilon^{-u}\phi(u, v) - \epsilon^{u}\phi(-u, v)$

$\lvert v \rvert$	$\lvert u \rvert = 0.20$, $\theta = \pi/6$	$\lvert u \rvert = 0.20$, $\theta = \pi/4$	$\lvert u \rvert = 0.5$, $\theta = \pi/6$	$\lvert u \rvert = 0.5$, $\theta = \pi/4$	$\lvert u \rvert = 1.0$, $\theta = \pi/6$	$\lvert u \rvert = 1.0$, $\theta = \pi/4$	$\lvert u \rvert = 1.5$, $\theta = \pi/6$	$\lvert u \rvert = 1.5$, $\theta = \pi/4$	$\lvert u \rvert = 2.0$, $\theta = \pi/6$	$\lvert u \rvert = 2.0$, $\theta = \pi/4$
0.01	5.680 − i0.699	5.827 − i1.036	5.222 − i1.421	5.421 − i2.167	3.544 − i1.913	3.533 − i3.025	2.142 − i1.864	1.807 − i2.995	1.177 − i1.574	0.586 − i2.503
0.10	1.826 − i0.313	1.870 − i0.472	2.328 − i0.683	2.388 − i1.048	1.845 − i0.984	1.807 − i1.550	1.223 − i1.008	1.028 − i1.604	0.736 − i0.889	0.411 − i1.397
0.20	1.028 − i0.216	1.049 − i0.328	1.512 − i0.480	1.538 − i0.739	1.332 − i0.717	1.289 − i1.127	0.938 − i0.756	0.785 − i1.197	0.596 − i0.684	0.347 − i1.069
0.40	0.430 − i0.127	0.433 − i0.195	0.790 − i0.292	0.788 − i0.451	0.836 − i0.462	0.790 − i0.723	0.653 − i0.511	0.539 − i0.802	0.451 − i0.483	0.276 − i0.747
0.60	0.230 − i0.085	0.227 − i0.131	0.470 − i0.200	0.459 − i0.309	0.571 − i0.329	0.526 − i0.513	0.486 − i0.378	0.395 − i0.588	0.360 − i0.370	0.226 − i0.567
0.80	0.140 − i0.061	0.135 − i0.094	0.303 − i0.145	0.228 − i0.225	0.406 − i0.246	0.364 − i0.382	0.373 − i0.292	0.297 − i0.451	0.294 − i0.295	0.185 − i0.446
1.00	0.092 − i0.046	0.086 − i0.070	0.206 − i0.110	0.190 − i0.169	0.297 − i0.191	0.259 − i0.293	0.292 − i0.233	0.226 − i0.354	0.243 − i0.240	0.152 − i0.359

$$i\omega \int_{-\infty}^{\infty} I(\tau)\, P(s)\, d\tau = \int_{-\infty}^{\infty} F(u) p(u,a) \epsilon^{iux} du \tag{19}$$

where

$$p(u,y) = i\omega \int_{-\infty}^{\infty} \epsilon^{iuv} P(r) dv, \quad r = (v^2 + y^2)^{1/2} \tag{20}$$

(b) $$\frac{d}{dx} \int_{-\infty}^{\infty} I'(\tau) Q(s) d\tau = \frac{d}{dx} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} iuF(u) \epsilon^{i\tau u} Q(s) du d\tau$$

$$= -\int_{-\infty}^{\infty} u^2 F(u) q(u,a) \epsilon^{iux} du \tag{21}$$

where

$$q(u,y) = \int_{-\infty}^{\infty} \epsilon^{iuv} Q(r) dv \tag{22}$$

Inserting equations 19 and 21 in equation 3 the following equation is obtained:

$$\int_{-\infty}^{\infty} \Delta(u)\, F(u) \epsilon^{iux} du = E^0(x) \tag{23}$$

where

$$\Delta(u) = u^2[g^{-1} + q(u,a)] + z + p(u,a)$$

If $E^0(x)$ is now represented by a Fourier integral:

$$E^0(x) = \int_{-\infty}^{\infty} e(u) \epsilon^{iux} du \tag{24}$$

then the function $F(u)$ in equation 18 is obtained from equation 23 as:

$$F(u) = \frac{e(u)}{\Delta(u)}$$

which gives equation 1 as a general solution of equation 3.

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Calorimetric Measurement of Dielectric Losses in Solids

A new transient calorimetric method for measurement of dielectric losses in solids at 10,000 volts and 1,000 kilocycles, is described in this paper and experimental data obtained by this method are presented. A steady-state method also is described and a direct comparison between the results obtained by the 2 methods is given. The absolute value of loss factor (0.0009) determined for quartz shows exact agreement between the 2 methods. The transient method is shown to reduce the time of making calorimetric measurements on solid materials from hours to minutes. Results of tests on 15 samples, including 6 different low-loss materials, show that the absolute values of losses in these particular materials are of the order of magnitude determined by previous low-voltage high-frequency measurements. It is thought that the transient calorimeter technique can be readily extended to ultra-high frequencies.

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ACCURATE, sensitive, Schering-bridge methods have been used for measuring dielectric losses in insulating materials at frequencies of from 10 to 2,000 cycles per second and voltages up to 300 kv.[1,8,9,10] The upper limit of voltage is determined only by the supply equipment and the standard capacitor, so there is no inherent difficulty in raising the voltage limit of such measurements if desired. Also, excellent equipment has been developed for the same measurements at frequencies up to 10^7 cycles per second with low voltage (say 50 volts or less) on the sample.[2] Such methods have recently been extended to 5×10^8 cycles per second (60-centimeter wave length) by Rohde.[11]

That is, good methods are available for *low-frequency high-voltage* and *high-frequency low-voltage* measurements, but to the authors' knowledge no satisfactory short-time method has hitherto been reported for determining dielectric losses on solid dielectrics at *high frequency and high voltage.* Krutzsch[3] reports measurements on 3 solid dielectrics at audio frequencies up to 6 kilocycles and voltages up to 10 kv using a special high voltage bridge circuit. Vogler[4] determined the dielectric loss in a number of insulating liquids using a steady-state calorimeter at frequencies of from 2 to 12×10^6 cycles per second and voltages up to 3.5 kv. A double calorimeter was used by Owen[5] to measure dielectric loss in a number of solid dielectrics at 30–50 volts and 300–500 kilocycles. Similar double calorimeters[6] have also been used to calibrate resistors and capacitors at low voltage and high frequency. In the present investigation absolute measurements of dielectric loss were made on a number of solid dielectrics at 10^6 cycles per second and 10 kv. The authors see no inherent limitation to extending this method to higher frequencies and voltages.

In general, it has been found by practical experience that the high-voltage high-frequency characteristics of all materials cannot be extrapolated either from the low-voltage high-frequency measurements, or from the high-voltage low-frequency measurements. In other words, the dielectric losses of insulating materials may vary in an unpredictable manner with increases in frequency or voltage gradient. Therefore, measurements of dielectric loss in solid dielectrics at high voltage and high frequency are necessary and valuable from both the practical and theoretical viewpoints; from the practical viewpoint because in the case of short wave oscillators it is important to know the electrical properties of solid insulation subjected to high voltage and high frequency; from the theoretical viewpoint because the same information will aid in determining the mechanisms of dielectric loss in various classes of materials.

In the succeeding section of this paper are reported the methods used and the results obtained by C. F. Baldwin in 1926 which have not been published previously. Baldwin used a *steady-state calorimeter* for measuring dielectric loss in fused quartz, at frequencies from 0.5 to 1.5×10^6 cycles per second and voltage gradients from 7 to 12 kv per centimeter. The major disadvantage of Baldwin's method was the time necessary to reach steady state, which was approximately 8 hours. The method was therefore costly and impractical for routine or control measurements.

Over a year ago the authors decided to attempt to modify the calorimetric method so as to obtain

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1. For all numbered references see list at end of paper.

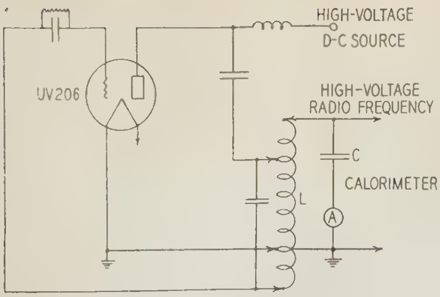

Fig. 1. Oscillator circuit used by Baldwin

measurements in much shorter time. This work resulted in the *transient calorimetric method* described in the remainder of the paper which requires only a few minutes per measurement. The results obtained to date with this method are reported.

Another reason for the present investigation is that calorimetric methods should be more suitable than bridge methods for extending dielectric loss measurements to still higher frequencies and voltages so as to evaluate materials under the conditions in which they are used in short-wave transmitters.

Steady-State Measurements on Fused Quartz

A simple Hartley oscillator was used by Baldwin in 1926 as a high-frequency power source. The wiring diagram of this oscillator and the method of measuring the output voltage are shown in figure 1. The power rectifier circuit used is not shown, but a full wave rectifier was used with a voltage regulator on the a-c input. The d-c output voltage to the plate of the oscillator tube could be adjusted by means of this regulator from 500 to 15,000 volts. The calorimeter was connected as indicated in figure 1. The high-frequency voltage across the calorimeter was determined by measuring the current through an accurately calibrated capacitor (C) by means of a hot wire ammeter (A).

The schematic layout of Baldwin's calorimeter is shown in figure 2. A double calorimeter consisting of outer and inner vessels of heavy copper, nickel-plated and polished, and so made as to include no sharp corners, contained the test sample. The outer vessel was 17 inches in diameter and 6 inches high while the inner vessel was 15 inches in diameter and 4 inches high. The outer ambient box (see figure 2) had 2 wooden walls, the space between the walls being filled with sawdust. The temperature inside this ambient box was held constant by means of a 200-watt lamp heater and a thermostat. The test sample used was 5 inches wide, 7 inches long, and 0.278 inch thick. This sample of quartz was floated on a pool of chemically cleaned mercury which was the ground electrode. The high-potential electrode consisted of a pool of mercury held in place by a metal ring with rounded edges (see figure 2). The high-potential lead was brought in through a special bushing made up of a small strip of Advance resistance wire sealed under vacuum in a small glass tube. For d-c calibration a grid type resistor sealed in glass was immersed in the mercury-pool ground electrode. Thermocouples were welded to both the inner and outer calorimeter walls, and the temperature of these walls was measured by mean of a standard potentiometer.

Measurements were made with Baldwin's calorimeter in the following manner:

1. A known high-frequency potential was applied to the samp and the temperatures of the inner and outer calorimeter walls we recorded at definite intervals until these temperatures reached constant value. This usually took 8 to 10 hours.

2. The high-frequency potential was then shut off and the calorimeter was allowed to cool to the temperature of the ambient bo A known amount of d-c power was then dissipated in the heat immersed in the ground electrode, and the foregoing procedure w repeated.

Several runs were taken at various values of d power dissipation in order to obtain a calibratio

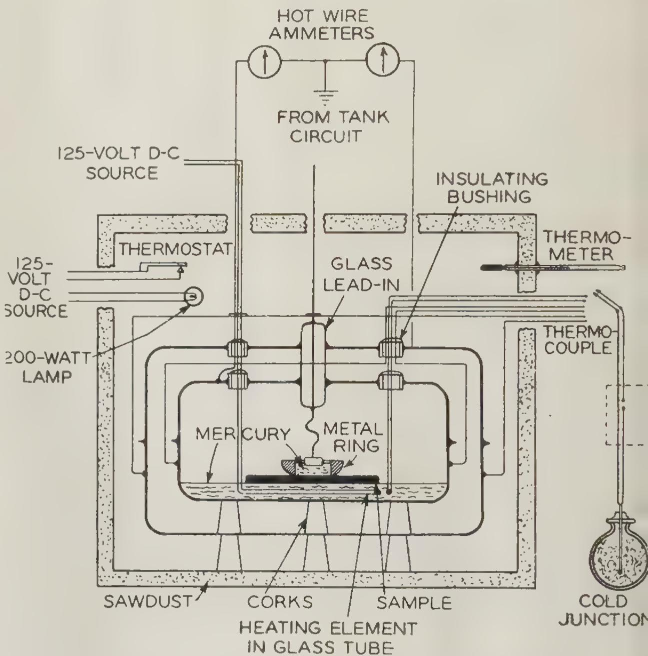

Fig. 2. Baldwin's steady-state calorimeter

curve. The difference in the temperatures of the calorimeter walls was used as an index of the pow loss. Thus from the temperature difference o tained at steady state in the high-frequency run value of watts loss could be read from the d-c ca bration curve (temperature difference versus wat loss) and the power factor calculated by means of t recorded values of high-frequency voltage and cu rent, as follows:

$$PF = \text{Watts loss}/E_{HF}I_{HF}$$

Table I shows the results of measurements ma by Baldwin on fused quartz. The results shown a the averages obtained from several runs at ea frequency specified. The maximum and minimu values of power factor, loss factor, and dielect constant obtained on any single run over this ran are also given in order to show the maximum dev tions recorded.

Baldwin made 26 runs in all on 3 different samp of quartz. Of these runs, 24 were made on o sample; the other 2 runs were made on 2 differe

samples as a check to insure that the results obtained were truly representative of fused quartz. The average values of loss factor (ϵ'') and power factor given in table I are stated by Baldwin to be accurate to ±10 per cent. The value (0.0009) for loss factor determined by Baldwin at 1,000 kilocycles was used as a basis for determining the accuracy of measurements made with the transient type calorimeter described in the succeeding section of this paper.

Transient Calorimetric Measurements

Calorimeter. The complete calorimeter assembly is shown in figure 3. The calorimeter is divided into 3 compartments: the inner or sample chamber; the middle chamber; and the large outer or ambient chamber. The walls separating the (inner, middle, and outer) compartments are made of fused quartz to reduce the residual losses in the calorimeter itself to a minimum. The outer or ambient chamber is enclosed by a Pyrex bell jar mounted in a deep circular groove cut in the heavy Bakelite base of the calorimeter. The inner quartz cylinder is ground to fit on a quartz disk, this disk being fastened to the copper ground lead (see figure 4). The outer quartz cylinder or middle wall of the calorimeter is supported on Pyrex mounted on the base of the calorimeter. Both quartz cylinders are closed at the top by

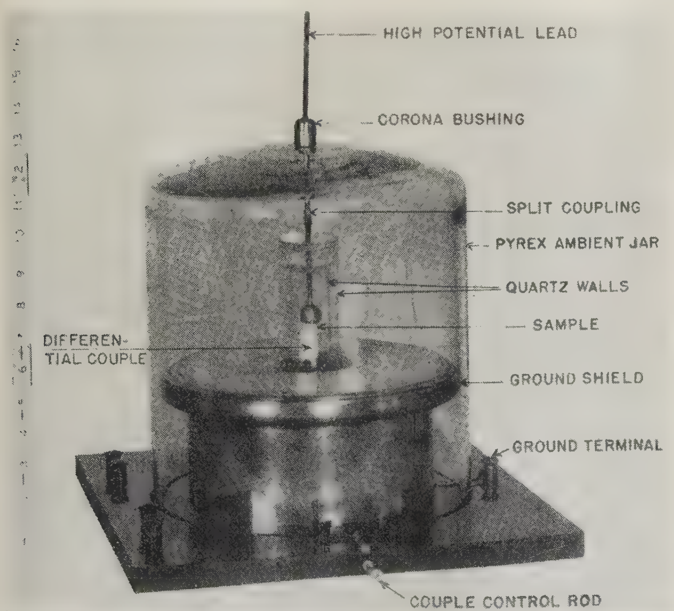

Fig. 3. Assembly of transient calorimeter

quartz disks (see figure 4). These cover disks are ground to fit the cylindrical walls. The ambient jar has a hole at the top, ground to accommodate the corona bushing on the high-potential lead. The ground lead, a copper tube $^3/_{16}$ inch in diameter, is brought in through the Pyrex support at the base and terminates in the inner sample chamber in the form of a split clamp-type copper cup machined to fit the ground electrode of the samples (see figure 4). The samples themselves are made up in the form of cylindrical rods, $^1/_2$ inch in diameter and $1^1/_2$ to 2 inches in length. Lead electrodes are molded on the ends of the samples as shown in figure 7; the high-potential sample lead, a copper tube $^3/_{16}$ inch in diameter and 3 inches long, is fastened on during the molding process. The external high-potential rod is brought in through the bushing at the top of the ambient jar and is fastened to the sample lead by means of a split coupling to facilitate the changing of samples. A ground shield of spun copper is located within the ambient chamber (see figure 3). The temperature indicating device is described in detail in the following section of the paper.

Movable Differential Thermocouple. In order to measure the small temperature differences inherent with the use of a transient method of calorimetry and to obtain consistent results, it was found advantageous to measure the temperature difference between the surface of the test sample and a point in space 3 millimeters away from the sample by using a differential thermocouple rather than to attempt to measure absolute values of temperature. The relation between temperature difference and watts input is obtained by low-voltage d-c calibrations using resistors of the same size and shape as the test samples.

The physical setup of this differential couple is shown in figures 3 and 4. The couple assembly is made up of 2 copper-Copnic junctions so connected that their electromotive forces cancel one another if both are at the same temperature. These 2 junctions are made of 0.005-inch wire and are mounted on a Polystyrol tip supported by a phosphor-bronze

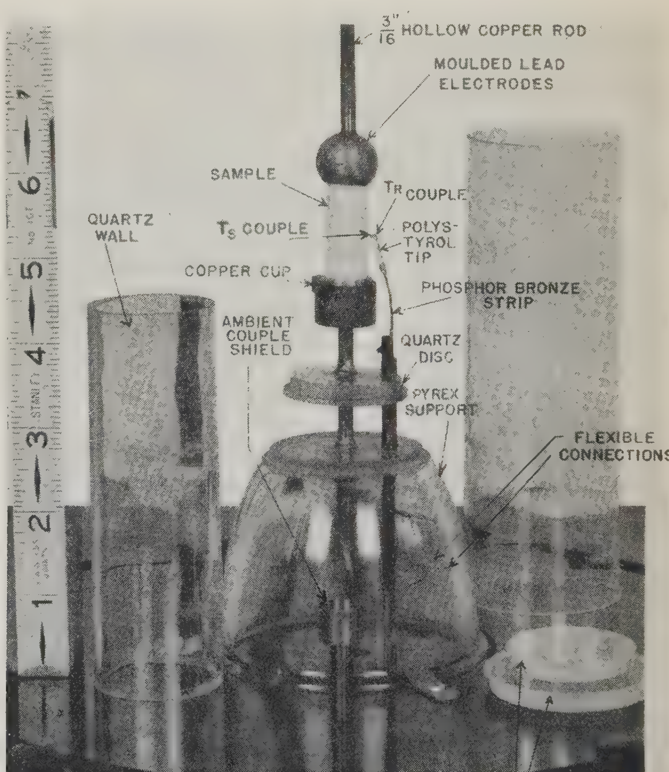

Fig. 4. Transient calorimeter assembly (without Pyrex jar cover)

spring. This spring is guided by a groove inside of a Bakelite tube $^3/_{16}$ inch in diameter which extends through the Pyrex support at the bottom of the calorimeter. The strip of bronze spring is so connected to the couple-control rod located in the calorimeter base that the couple assembly can be adjusted to any desired position along the sample

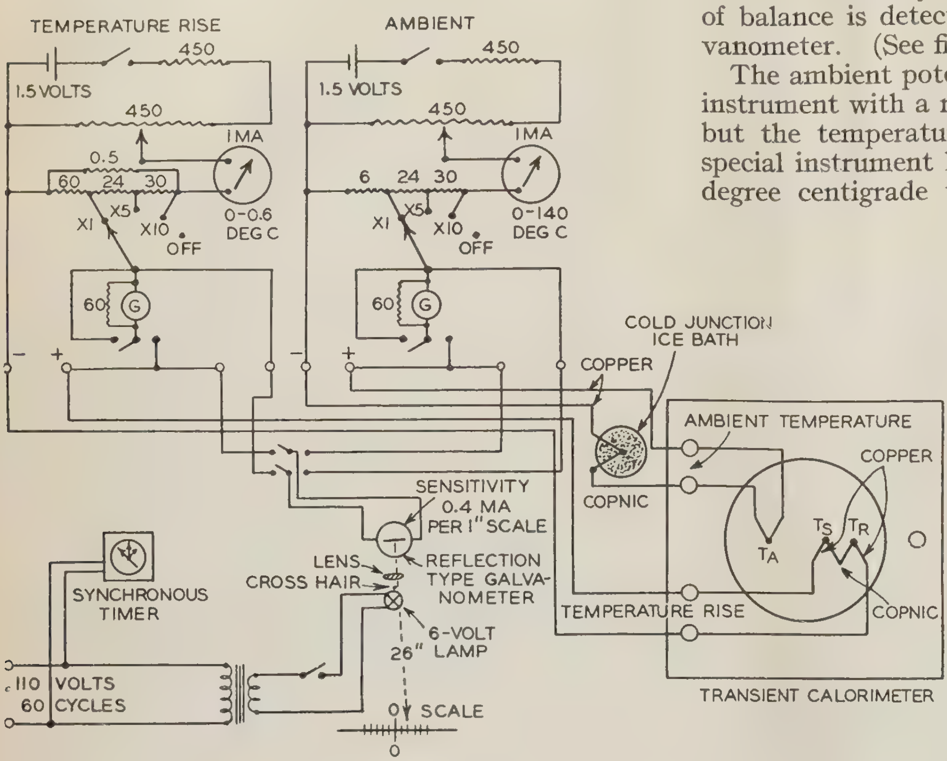

Fig. 5. Wiring diagram of temperature-measuring equipment for transient calorimeter

T_A—Ambient thermocouple (20-mil copper-Copnic)
T_S—Sample thermocouple (5-mil copper-Copnic)
T_R—Differential air thermocouple (5-mil copper-Copnic)
All resistances are in ohms
G—Galvanometer
IMA—Indicating milliammeter

length by simply pushing or pulling the control rod. This control also allows the couple to be drawn down inside the copper ground shield shown in figure 3. This procedure was found necessary in order to reduce radio-frequency pick-up, and the consequent heating of this couple unit to a negligible value during the application of high-frequency high-voltage to the sample. The 2 couple junctions are mounted slightly above the insulating tip and spaced 3 millimeters apart. The 2 Copnic leads from the junctions are joined just below the Polystyrol tip. The 2 copper leads are brought down the bronze strip and then out to the external binding posts on the base by means of the flexible connections shown in figure 4. The bronze strip not only allows adjustment of the couple position, but also acts as a spring insuring good contact between the sample couple (T_S) and the sample. This arrangement of couples provides a means of direct measurement of the temperature difference between the sample surface, measured by the sample couple (T_S), and a point in the air surrounding the sample 3 millimeters away, measured by the differential couple (T_R).

The temperature inside the ambient jar is measured by means of a copper-Copnic couple mounted in a copper shield cup on the calorimeter base (see figure 4).

The electrical connections of the differential couple and of the temperature-measuring equipment are shown in figure 5.

Temperature-Measuring Equipment. The electromotive forces developed in both the differential and ambient couples are measured by balanced potentiometer circuits and indicated by the milliameters in the potentiometer circuits. These instruments are calibrated directly in degrees centigrade. The point of balance is detected by means of a sensitive galvanometer. (See figure 5.)

The ambient potentiometer is a standard portable instrument with a range of 0–140 degrees centigrade but the temperature difference potentiometer is a special instrument having a full scale reading of 0.6 degree centigrade with a shunt for reading 0–6.0 degrees. With this instrument, including the galvanometer shown in figure 5 and the differential couple in the calorimeter itself, temperature differences can be read directly to 0.01 degree centigrade and estimated to about 0.001 degree.

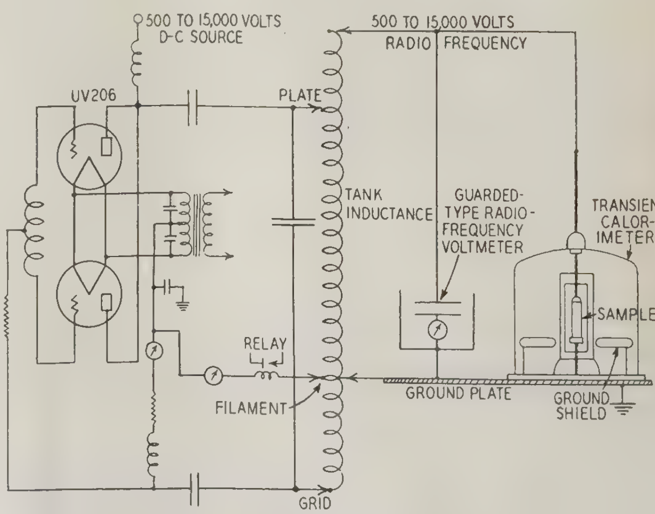

Fig. 6. Wiring diagram of high-frequency power supply and transient calorimeter

Calibration of the Calorimeter. For calibrating purposes a d-c resistance unit is made by coating a sample of insulating material between the electrodes with conducting paint leaving a path along the length of the sample 1/4 inch wide uncoated for the T_S couple to contact (see figure 7). The conducting paint consists of powdered graphite, lacquer, and thinner mixed in proper proportions to make a hard even coating having the desired d-c resistance. This unit is then baked at 50 degrees centigrade for 3 hours and an outer protective coating of white enamel is applied, after which the whole unit is baked thoroughly at 50 degrees centigrade for approximately 15 hours.

Two calibrating resistors were used to obtain the data reported in this paper, one having a fused-quartz body, and one having an Isolantite body. These 2 particular units are shown at the left in figure 7. The calibration runs are made in the following manner: The calibrating unit is placed in the calorimeter in place of the ordinary sample so that the differential couple will make contact along the center of the uncoated path. The stop on the couple-control rod is then adjusted so as to bring the (T_S) couple at the mid-position of the calibrating unit when the rod is pushed in. The calorimeter is assembled as in figure 3, and a d-c source of power is connected between the high-potential lead at the top of the calorimeter and the ground terminal at the base. A voltmeter and an ammeter are connected in the d-c circuit to indicate the amount of power dissipated in the unit. The temperature measuring equipment is now connected to the calorimeter as shown in figure 5 and the temperature difference at the mid-point of the calibrating unit is measured. If the calorimeter is at equilibrium, the value should be zero. However, accurate measurements can be made if this value is a small percentage of ΔT_0. When approximate equilibrium is obtained, the ambient temperature is measured and the differential couple is pulled down to its lowest position inside the ground shield. A known value of d-c power input (watts input) is then applied and held constant for exactly 2 minutes and then shut off. The 2-minute "power-on" period was shown experimentally to be the shortest period that would give an appreciable temperature difference on the best materials. The differential couple is then pushed up to the mid-position as quickly as possible and a cooling curve is taken, recording temperature difference in degrees centigrade versus time in seconds. Zero time is taken as the time at which the power is turned off. This curve is then plotted on semilog paper and extrapolated to zero time. This procedure is repeated with various known quantities of power input until enough runs have been made to cover the calibration range required for the high-frequency measurements. The cooling curves taken during the d-c calibration with a fused-quartz unit and an Isolantite unit are shown in figures 8 and 10. Before each run the initial temperature difference of the sample is recorded and these values are indicated on figures 8 and 10 (zero correction). In order to put the calibration curves into a more convenient form, the values of ΔT_0 extrapolated in figures 8 and 10 are corrected for the initial temperature difference and are replotted against power input in watts per unit volume of the calibrating units. These curves are shown in figures 9 and 11.

It may be noted that the cooling curves plotted on semilog paper (figures 8 and 10) are straight parallel lines within experimental limits, which indicates that they are exponential functions as predicted from the theory of free-cooling bodies. Figures 8 and 10 are also very interesting in that the experimental points taken during actual runs are plotted and show not only the sensitivity of the

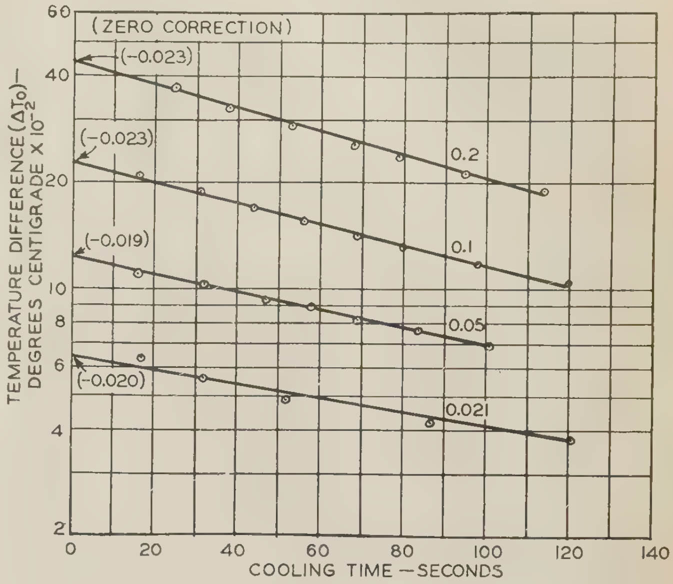

Fig. 8. Cooling curves taken during d-c calibration of transient calorimeter with a fused-quartz resistor; 2-minute runs

Watts input to fused-quartz unit is shown by figures at the right of the curves

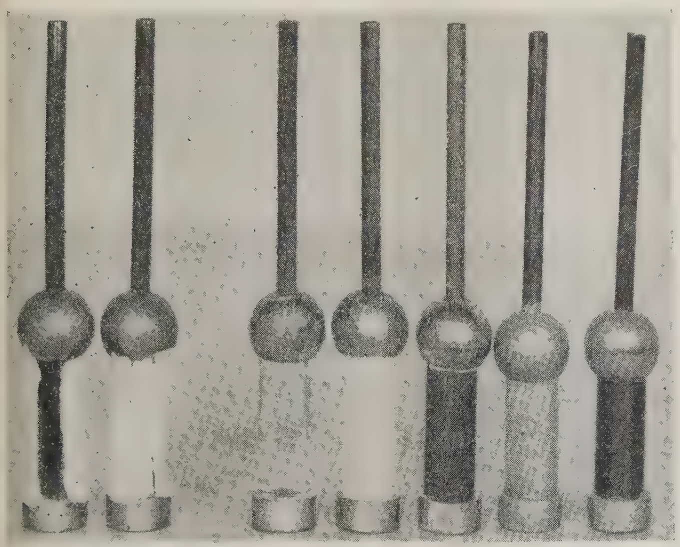

Fig. 7. Calibrating (left 2) and radio-frequency samples (right 5) tested in transient calorimeter

measuring equipment, but also the ability of the equipment to measure such small values of temperature differences with so little deviation from the average curves.

Figures 9 and 11 show that the temperature differences measured are directly proportional to the watts input.

Checks on the original d-c calibration curves after 3-weeks' use (of making high-frequency measurements) were within ±2 per cent.

High-Frequency Measurements. Figure 6 shows the wiring diagram of the high-frequency power oscillator and the method of connecting the calorimeter and the voltmeter in the circuit. The d-c power supply, not shown in this diagram, is of the conventional full-wave-rectifier type, the output voltage being adjusted by means of a voltage regulator on the a-c input.

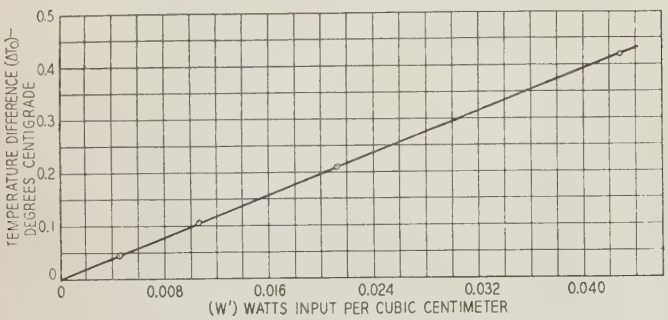

Fig. 9. D-c calibration curve for transient calorimeter with fused-quartz resistor; 2-minute runs

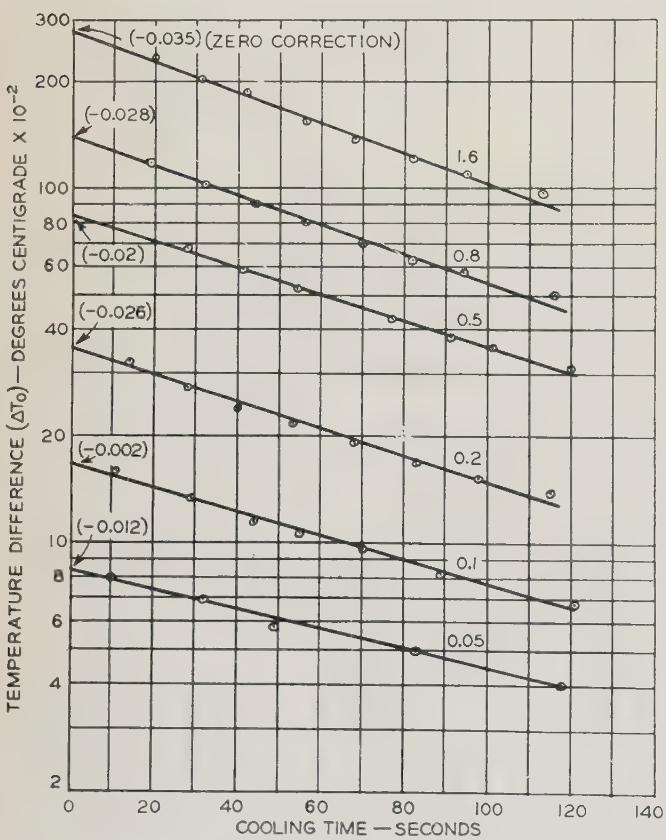

Fig. 10. Cooling curves taken during a-c calibration of transient calorimeter with Isolantite resistor; 2-minute runs

Watts input to Isolantite unit is shown by figures at the right of the curves

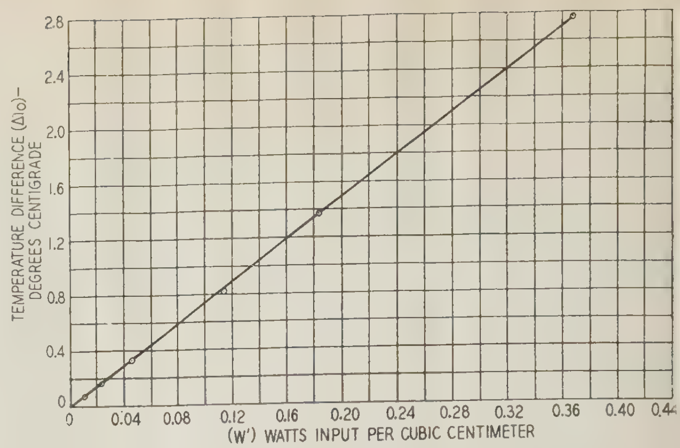

Fig. 11. D-c calibration of transient calorimeter with Isolantite resistor; 2-minute runs

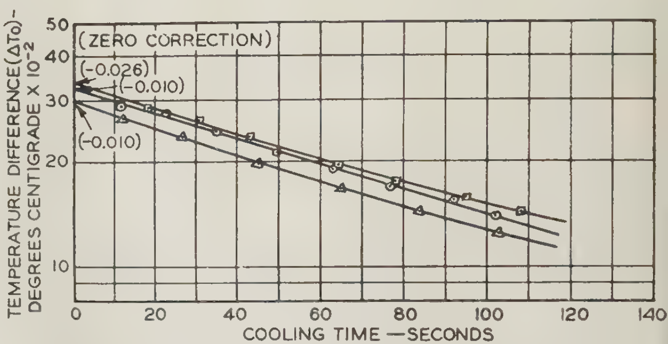

Fig. 12. Effect of thermocouple position on d-c calibration of transient calorimeter with Isolantite resistor; 0.2-watt input; 2-minute runs

⊙—Run 76, couple at mid-point
△—Run 77, couple 1/4 inch above mid-point
⊡—Run 78, couple 1/4 inch below mid-point

The high-frequency runs are made in the same manner as outlined for the d-c calibration runs except that the calibrating unit is replaced by a sample of insulation (see figure 7), and high-voltage high-frequency power is applied to the sample as indicated in figure 6.

Figures 15 to 18 show the actual cooling curves taken on various samples of insulating materials after 2-minute runs at 10,000 volts, 994 kilocycles. The loss factors of these samples can be calculated from the values of ΔT_0 obtained from these radio frequency curves and the d-c calibration curves as shown in the next section of the paper.

Calculations

Nomenclature

W = total watts dissipated
W' = watts per cubic centimeter dissipated (see figures 9 and 11)
I = current in amperes recorded in d-c calibration
R = resistance in ohms of calibrating unit

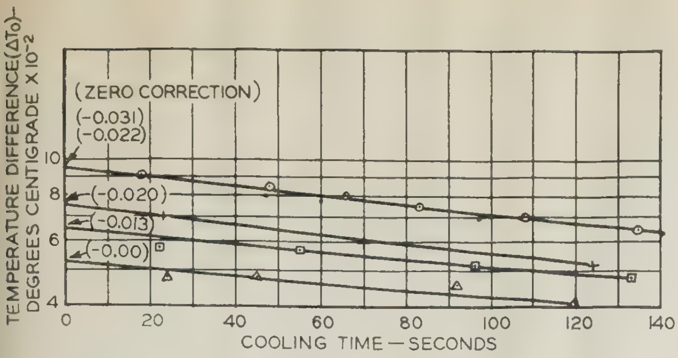

Fig. 13. Measurements on fused quartz at radio frequency; 2-minute runs; 10 kv; 994 kilocycles

•—Run 1; ⊙—Run 2; △—Run 5; ⊡—Run 6; +—Run 7
Average value of ΔT_0 = 0.059

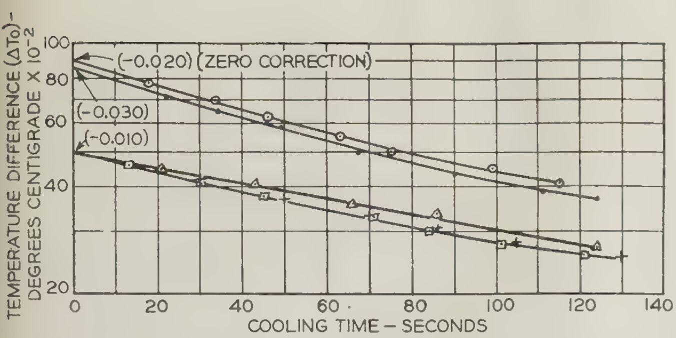

Fig. 14. Measurements on Isolantite resistor at radio frequency; 2-minute runs; 10 kv; 994 kilocycles

Symbol	Run	Sample
•	32	6
⊙	33	6
△	58	22
⊡	59	22
+	71	22

V = volume of calibrating body between electrodes in cubic centimeters
K = constant determined from slope of d-c calibration curves
ΔT_0 = temperature difference in degrees centigrade extrapolated to zero time and corrected for initial rise at start of run
f = frequency in cycles per second
E = total voltage in volts applied to sample
l = length of sample in centimeters between electrodes
ϵ'' = loss factor
A = constant of calorimeter determined experimentally
S = specific heat in calories per gram
D = density in grams per cubic centimeter
PF = power factor
ϵ' = dielectric constant

Case 1. When the sample being measured and the calibrator body are made of identical materials,

$$W = I^2R = W'V = KV(\Delta T_0) \tag{1}$$

or

$$W' = K(\Delta T_0)$$

From the definition of loss factor[7] (see appendix),

$$W' = 0.555f(E/l)^2\epsilon''(10)^{-12} \tag{2}$$

From equations 1 and 2,

$$\epsilon'' = 1.8(10)^{12}(l/E)^2K(\Delta T_0)/f \tag{3}$$

Case 2. When the sample being measured and the calibrator body are made of different materials, and the specific heat and density of the sample are known,

$$W' = A(SD)\Delta T_0$$

From data obtained with the fused-quartz calibrator,

$$A = W'/(SD)\Delta T_0 = 0.0428/(0.19 \times 2.15)0.42 = 0.25$$

(See figure 9.)

$$W' = 0.25(SD)\Delta T_0 \tag{4}$$

From equations 2 and 4,

$$\epsilon'' = 0.45(10)^{12}(l/E)^2(SD)\Delta T_0/f \tag{5}$$

Power factor (PF) can be determined from the loss factor (ϵ'') as follows:

$PF = \epsilon''/\epsilon'$ (for values of PF less than 0.1; see appendix)

The value of ϵ' must be determined independently by a conventional bridge or substitution method.

Typical Results

In order to determine the magnitude of the residual losses in the calorimeter itself, an air sample was devised by making up a set of dummy electrodes and placing them in the calorimeter in such a manner that the air space between them was equal to the space normally occupied by a sample. A regular 2-minute high-frequency run was then taken on this air sample, but no temperature difference could

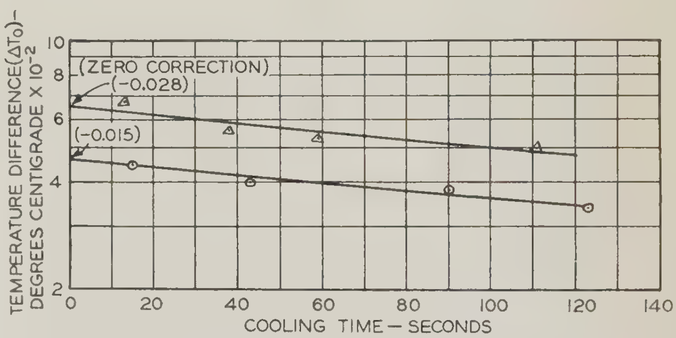

Fig. 15. Measurements on German Polystyrol at radio frequency; 2-minute runs; 10 kv; 994 kilocycles

⊙—Run 60 △—Run 61
Average value of ΔT_0 = 0.034

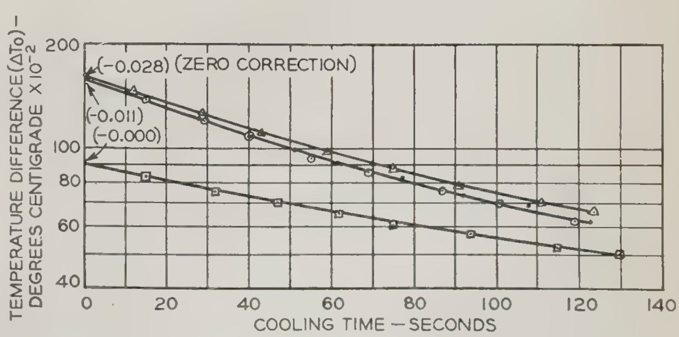

Fig. 16. Measurements on 1360 Mycalex at radio frequency; 2-minute runs; 10 kv; 994 kilocycles

Symbol	Run	Sample
•	29	9
⊙	31	9
△	62	25
⊡	66	24

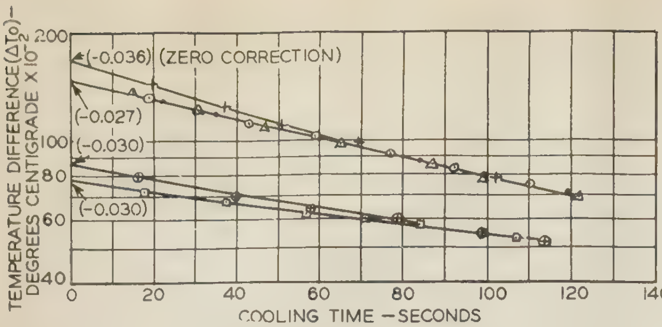

Fig. 17. Measurements on Pyrex at radio frequency; 2-minute runs; 10 kv; 994 kilocycles

Symbol	Run	Sample
•	36	11
⊙	45	11
△	47	11
□	37	12
⊕	38	12
+	41	13

be detected thus showing no appreciable heat radiation from the calorimeter walls.

Figure 12 shows d-c calibration runs taken on the Isolantite resistor unit at 0.2-watt input in order to check the effect of the couple position. Three runs were taken: run 76 at the mid-position, run 77 $^1/_4$ inch above the mid-point, and run 78 $^1/_4$ inch below the mid-point. The corrected extrapolated values of temperature difference at zero time show differences of the order of 0.01 degree out of 0.30 degree centigrade, which indicates a negligible error due to possible deviations from the mid-point of the couple position for different runs. Run 76 taken at the mid-point also checks within 0.01 degree centigrade the original 0.2-watt run on this unit (see figure 10).

Measurements on fused quartz were used as a means of standardizing the equipment at high frequency. The actual curves taken on the quartz sample used are shown in figure 13. The average value of loss factor (ϵ'') calculated from the 5 runs taken is 0.00091, which is a check of the average value determined by Baldwin for fused quartz at the same frequency. The deviation from this average value by any single run is much less than the single-run deviations reported by the steady-state method. (See tables I and II and also a previous section of this paper.)

The results of measurements on 2 Isolantite samples 6 and 22 are shown in figure 14. Sample 6 was $1^1/_2$ inches long over-all (3.32 centimeters between electrodes), and sample 22 was 2 inches in length (4.57 centimeters between electrodes). Measurements on samples of different lengths check quite closely (see table II). Run 33, figure 14, is a check on run 32 with the sample rotated 90 degrees so that the differential couple was measuring the temperature difference at an entirely different point on the surface of the sample. Run 59 is a check on run 58 in the same manner. Run 71 is an independent check of the measurements made on sample 22, being taken after a week's period of measuring other samples.

It may be noticed that the cooling curves shown in figures 14 to 18 are not all straight lines; changes in the slopes of these curves are evident in rechecks on the same sample. These deviations from straight lines and slight changes in slopes are believed to be caused by changes in the ambient temperature during the cooling period. If, for example, the ambient temperature rose 0.02 degrees centigrade during the cooling period of 2 minutes the curves probably would deviate from the theoretical straight line since the sample no longer could be considered a small free-cooling body radiating to a large body at constant temperature. The fact that the temperature-measuring equipment is sensitive enough to detect and consistently measure this small deviation from a straight line so that the temperature difference at zero time can be extrapolated as accurately as shown in figures 14 to 18 is more important than the fact that slight deviations are present. For instance, run 58 (figure 14) is a straight-line cooling curve, while runs 59 and 71 curved slightly, but the extrapolated values of ΔT_0 for these 3 runs check exactly.

Measurements on the German Polystyrol samples are extremely interesting since the cooling curves (figure 15) show an average value of ΔT_0 of approximately half that found for fused quartz. These curves also indicate the lower limit of the ability to measure temperature differences with the temperature-measuring equipment since the zero corrections are of the same order of magnitude as the measured values. However, it may be noticed that when the zero corrections are applied to the extrapolated values of ΔT_0 runs 60 and 61 agree within 0.006 degree centigrade.

Figure 16 shows the cooling curves taken on 3 Mycalex samples. Runs 29, 31, and 62 show very close agreement between the 2 short samples 9 and

Table I—Results of Measurements With Steady-State Calorimeter

Sample	Thickness, Centimeters	Area, Square Centimeters	Number of Runs	Frequency, Kilocycles per Second	Potential Gradient, Volts per Centimeter	Watts Loss per Cubic Centimeter	Loss Factor (ϵ'')	Dielectric Constant (ϵ')	Power Factor (PF)
Fused quartz	0.278	59	5	500	10,100	0.033	0.0010	4.3	0.00023
			11	750	10,200	0.040	0.0009	4.1	0.00022
			5	1,000	9,150	0.048	0.0009	4.1	0.00022
			5	1,500	10,300	0.083	0.0010	4.9	0.00020

In a Single Run

Maximum ϵ''	Maximum ϵ'	Maximum PF	Minimum ϵ''	Minimum ϵ'	Minimum PF
0.0013	5.3	0.00032	0.0008	3.9	0.00019

25. Run 31 is a check run on run 29 with the sample rotated 180 degrees. Sample 24 was longer than either sample 9 or 25 and its cooling curve falls relatively lower.

Measurements were made on 3 samples of Pyrex (figure 17); sample 11 was a $1^1/_2$-inch rod, sample 12 a 2-inch rod, and sample 13 a piece of tubing $1^1/_2$ inches long having an outside diameter of $^1/_2$ inch and an inside diameter of approximately $^1/_4$ inch. This sample of tubing was measured in order to find out if the heating was uniform throughout the cross section of the sample. Run 41 on this particular sample of tubing shows a ΔT_0 in close agreement with the values from runs 36, 45, and 47 on sample 11, a Pyrex rod of the same length and having the same external diameter. This check supports the theory that the temperature differences should be equal for the same voltage gradients, and same watts loss per unit volume of the sample. Runs 37 and 38 on the long sample 12 are check runs with the sample rotated 90 degrees.

The measurements on hard-rubber samples are shown in figure 18. The 3 runs on the 2-inch sample 23 show exact agreement in the extrapolated value of ΔT_0. This sample was rotated 90 degrees in each case. The 2 runs on the shorter sample 15 have a wider spread, but the ΔT_0 values agree within 0.03 degree out of a temperature difference of 1.7 degrees centigrade.

Table II shows the results of the transient-calorimetric measurements on 15 samples of 6 different materials; 80 runs were made in all including the d-c calibration runs. The values of ϵ'' given were calculated as shown under "Calculations."

The average value of ϵ'' obtained for fused quartz agrees with the average value obtained with the steady-state method (see table I). This fact indicates that absolute values of loss factor measured by the transient method are as reliable as those measured by the steady-state method.

Measurements on the German Polystyrol samples show loss factors of the order of that of fused quartz.

The loss factors for the other low-loss materials listed in table II have the same order of magnitude as previous values obtained by other methods at low voltages. The values themselves show good agreement between different samples of the same materials.

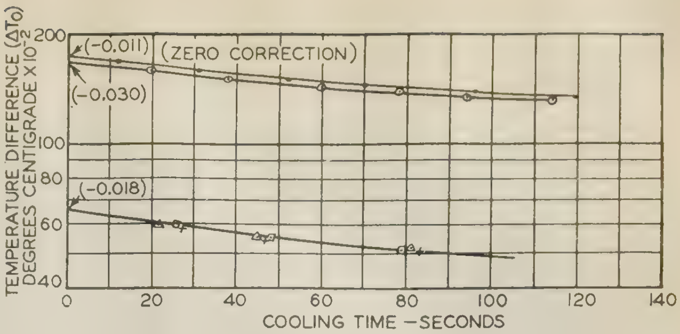

Fig. 18. Measurements on hard rubber at radio frequency; 2-minute runs; 10 kv; 994 kilocycles

Symbol	Run	Sample
•	51	15
⊙	52	15
△	48	23
⊡	49	23
+	50	23

Conclusions

1. In the authors' knowledge this is the first time that absolute values of loss factors of solid materials have been determined by calorimetric measurements on the transient part of the heating curve, with attendant savings in time and cost of measurement.

2. The average value of loss factor of fused quartz determined by the *transient method* agrees with the value previously determined by C. F. Baldwin using the *steady-state method* as reported in this paper (table I).

3. With the temperature measuring equipment developed, temperature differences can be measured directly to 0.01 degrees centigrade and the readings estimated to about 0.001 degree centigrade.

4. Measurements on other low-loss materials show that the values of loss factor of these materials are of the order of magnitude of the values determined by previous low-voltage high-frequency measurements.

5. The actual time of measurement by the *transient method* is four minutes as contrasted to 8 hours for the *steady-state method*. The time of calorimetric measurement has thus been decreased enormously without apparent sacrifice of accuracy.

6. With only one d-c calibration of the calorimeter a method is presented for calculating the value of loss factor for any material of known specific heat and density directly from short-time temperature measurements at high voltage and high frequency.

7. The authors believe that this *transient calorimetric method* for absolute measurements of dielectric loss in solids can be more easily

Table II—Results of Measurements With Transient Calorimeter at 10,000 Volts, 994 Kilocycles

Sample Number	Material	Sample Length Between Electrodes, Centimeters	ΔT_0 Degrees Centigrade	Potential Gradient, Volts per Centimeter	Loss Factor (ϵ'')	Dielectric Constant (ϵ')	Power Factor (PF)
1	Fused quartz	2.92	0.059	3,420	0.00091	4.1	0.00022
4	Isolantite	3.26	0.97	3,060	0.0250	6.0	0.0041
5	Isolantite	2.89	1.17	3,460	0.0236	6.0	0.0040
6	Isolantite	3.32	0.85	3,010	0.0215	6.0	0.0036
22	Isolantite	4.57	0.48	2,190	0.0246	6.0	0.0041
8	German Polystyrol	3.09	0.032	3,240	0.00103	2.4	0.00043
20	German Polystyrol	2.93	0.034	3,420	0.00095	2.4	0.00041
9	1360 Mycalex	3.15	1.57	3,170	0.0471	8.9	0.0053
24	1360 Mycalex	4.59	0.89	2,190	0.0568	8.9	0.0064
25	1360 Mycalex	3.29	1.58	3,040	0.0532	8.9	0.0060
11	Pyrex (rod)	3.29	1.45	3,040	0.0320	5.7	0.0056
12	Pyrex (rod)	4.29	0.79	2,340	0.0296	5.7	0.0052
13	Pyrex (tubing)	3.30	1.64	3,030	0.0365	5.7	0.0064
15	Hard rubber	3.16	1.70	3,160	0.0391	3.1	0.0126
23	Hard rubber	4.53	0.64	2,210	0.0302	3.1	0.0098

extended to ultra-high frequencies than can the conventional bridge or substitution methods.

Appendix

The loss factor ϵ'' is a scalar coefficient which is a direct measure of the rate of heat generation per unit volume of a material caused by the dielectric loss in this material, and can be determined either from an equivalent electric circuit or directly from the physical dimensions of the material being measured, the voltage applied, and the power dissipated.

It has been found convenient to interpret all bridge or substitution measurements on liquid, solid, and composite dielectrics in terms of *the equivalent parallel circuit of the measuring equipment.* The complex expression for the reciprocal impedance of the measuring circuit is given by the relation

$$1/Z = G_x + j\omega C_x = \omega C_v(\epsilon'' + j\epsilon') \tag{6}$$

where

G_x = the equivalent parallel conductance of the sample
C_x = the equivalent parallel capacitance of the sample
$\omega = 2\pi f$; f = frequency in cycles per second
$j = \sqrt{-1}$
$Cv = 0.08842 \times 10^{-12}(a/l)$ = capacitance of the same geometric arrangement of electrodes in vacuum, in which a = effective cross-sectional area of sample in square centimeters and l = effective length of sample in centimeters

From the equation 6,

$$\epsilon' = C_x/C_v \tag{7}$$

$$\epsilon'' = G_x/\omega C_v \tag{8}$$

$$W' = E^2 G_x/V \tag{9}$$

where

E = total applied potential in volts

$V = al$ = volume of the sample in cubic centimeters

From equations 8 and 9,

$$W' = 0.555(10)^{-12}\epsilon'' f(E/l)^2 \tag{10}$$

Equation 10 shows that the coefficient ϵ'' can be obtained directly from measurements of the heat dissipated per unit volume, voltage gradient, and frequency.

If it is desired to correlate the loss factor (ϵ'') with the dielectric constant (ϵ') and the power factor (PF), the following relations are used:

$$PF = \cos\cot^{-1}(\epsilon''/\epsilon') \tag{11}$$

and

$$W' = 0.555(10)^{-12} f(E/l)^2 \epsilon' \cot\cos^{-1}(PF) \tag{12}$$

For low-loss materials in which the cosine and cotangent of the loss angle are approximately equal, equations 11 and 12 reduce to the more familiar forms:

$$PF = (\epsilon''/\epsilon') \tag{11a}$$

and

$$W' = 0.555(10)^{-12} f(E/l)^2 \epsilon'(PF) \tag{12a}$$

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Complex Vectors in 3-Phase Circuits

To provide additional bridges between the fields of mathematics and electrical engineering is the main object of this paper, which is a sequel to a previous work on dyadic algebra. The paper includes an application of Gibbs's directional ellipse to 3-phase circuits, a treatment of symmetrical components by using the isoclinic unit vector and 2 circular complex vectors, and an exposition of dyadics as appplied to in 3-phase symmetrical machines.

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IN THE development of an engineering science, it often happens that some abstract idea originally introduced into pure mathematics has been put to good use. Thus, for example, complex numbers since the pioneering work of A. E. Kennelly have become so versatile in electrical engineering problems that the symbol j standing for th

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imaginary unit has acquired much more than an imaginary existence in the minds of all electrical engineers. In a manner quite analogous to the introduction of the complex number into electrical engineering, it has been recently found that the entity, complex vector, is extremely useful in the study of polyphase or multibranched a-c networks. In order to become proficient in the application of complex vectors to his problems, an electrical engineer, it is believed, should know as much as possible about the formal mathematical properties of a complex vector. A systematic treatment of these properties exemplified by simple circuit problems should, therefore, be helpful to those following this phase of the development of the electrical science. However, such a discussion is still lacking in the literature. The present paper will attempt to fill the desired rôle by making a study of some of the properties of complex vectors not from the standpoint of a pure mathematician but with a view to their immediate application to 3-phase circuits.

Definitions and Notation

It is rather unfortunate that in electrical engineering literature the terms "complex scalar" and "vector" have been used synonymously. For the purpose of the present study, it is absolutely essential that they be distinguished. In fact, it would be best to recognize 4 different entities, which will be defined as follows in order to avoid misunderstanding:

1. A real scalar is a real number, rational or irrational, having only magnitude but no direction.

2. A complex scalar is a number having a real part as well as an imaginary part. The usual symbol j, printed in italics, will be used to distinguish the imaginary part from the real part of a complex scalar when necessary. Thus the complex scalar

$$m = m' + j\,m'' \tag{1}$$

has a real part equal to m' and an imaginary part equal to m''. Since a real scalar may be considered as a complex scalar with its imaginary part equal to zero, no special symbolic type will be used to distinguish them and both will be printed in italics. Unless it is desirable to emphasize the complex or the real nature of the quantity, the word "scalar" will be used to denote either, leaving the use of a qualifying adjective for the purpose of emphasis.

3. A real vector is a quantity having both magnitude and direction. In a 3-dimensional space the length of a line may be taken as the magnitude of a real vector whose direction is to coincide with that of the line. By direction one may mean an actual geometrical direction or something created arbitrarily for the problem at hand but possessing no physical significance. In analytic discussions it is sometimes more convenient to denote 3-dimensional vectors by their projections on 3 non-coplanar lines. In particular, if the 3 lines are chosen to be mutually perpendicular to each other and the symbols $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ (printed in bold-faced italic type) used to denote unit lengths on them, a real space vector can be written as:

$$\boldsymbol{A} = A_i\boldsymbol{i} + A_j\boldsymbol{j} + A_k\boldsymbol{k} \tag{2}$$

in which $A_i\,\boldsymbol{i}$, $A_j\,\boldsymbol{j}$, $A_k\,\boldsymbol{k}$ will be called the "components" of the vector $\boldsymbol{A}$ in the directions $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ and the scalar coefficients A_i, A_j, A_k will be called the "measure numbers" of the vector $\boldsymbol{A}$ with respect to $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ as "base" or "reference vectors."

4. A complex vector is defined to be a vector whose components have a real as well as an imaginary part. Thus the complex vector $\boldsymbol{A}$ may also be written as in equation 2, provided the measure numbers A_i, A_j, A_k are understood to be complex scalars, each of which may have a real and an imaginary part like that given by equation 1, that is

$$A_i = A_i' + j\,A_i''; \quad A_j = A_j' + j\,A_j''; \quad A_k = A_k' + j\,A_k'' \tag{3}$$

Using the above values for the measure numbers, a complex vector may also be written as

$$\begin{aligned}\boldsymbol{A} &= (A_i'\,\boldsymbol{i} + A_j'\,\boldsymbol{j} + A_k'\,\boldsymbol{k}) + j\,(A_i''\,\boldsymbol{i} + A_j''\,\boldsymbol{j} + A_k''\,\boldsymbol{k})\\ &= \boldsymbol{A}' + j\,\boldsymbol{A}''\end{aligned} \tag{4a}$$

wherein

$$\boldsymbol{A}' = A_i'\,\boldsymbol{i} + A_j'\,\boldsymbol{j} + A_k'\,\boldsymbol{k} \tag{4b}$$

$$\boldsymbol{A}'' = A_i''\,\boldsymbol{i} + A_j''\,\boldsymbol{j} + A_k''\,\boldsymbol{k} \tag{4c}$$

are 2 real vectors which may be called respectively the real and the imaginary parts of the complex vector $\boldsymbol{A}$. Since a real vector is a special case of the complex vector, the word vector will be used to denote either one. Following common practice bold-faced italic letters will be used to denote vectors.

Algebra of Complex Vectors

The algebraic rules of operations with complex vectors are combinations of rules defined in vector algebra and in the algebra of complex scalars. Thus from the rule for the equality of 2 complex scalars the definition is derived that 2 complex vectors are equal when their real and imaginary parts are separately equal, that is, if $\boldsymbol{A} = \boldsymbol{A}' + j\boldsymbol{A}''$ and $\boldsymbol{B} = \boldsymbol{B}' + j\boldsymbol{B}''$, then the equality

$$\boldsymbol{A} = \boldsymbol{B} \tag{5}$$

means

$$\boldsymbol{A}' = \boldsymbol{B}' \quad \text{and} \quad \boldsymbol{A}'' = \boldsymbol{B}'' \tag{5a}$$

Further, if $\boldsymbol{A}'$, $\boldsymbol{A}''$, $\boldsymbol{B}'$, $\boldsymbol{B}''$ are decomposed into their components in any 3 non-coplanar directions such as, for example, given by equation 4*b* and 4*c*, then from the rule of the equality of 2 real vectors, the following are taken to be true:

$$\left.\begin{aligned}A_i' &= B_i'; \quad & A_j' &= B_j'; \quad & A_k' &= B_k'\\ A_i'' &= B_i''; \quad & A_j'' &= B_j''; \quad & A_k'' &= B_k''\end{aligned}\right\} \tag{6}$$

In other words the single equality (5) is equivalent to the 6 separate equations (6).

The sum, the difference, the scalar (or dot), and the vector (or cross) products of complex vectors are defined in the same manner as for real vectors. In fact, all the laws of vector algebra including the invalidity of the commutative law in cross multiplication will be taken as being true for operations involving complex vectors. Thus in symbols one finds:

$$m\boldsymbol{A} = \boldsymbol{A}m = (m'\boldsymbol{A}' - m''\boldsymbol{A}'') + j\,(m''\boldsymbol{A}' + m'\boldsymbol{A}'') \tag{7}$$

$$\begin{aligned}\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{B} + \boldsymbol{A} &= (\boldsymbol{A}' + \boldsymbol{B}') + j\,(\boldsymbol{A}'' + \boldsymbol{B}'')\\ &= (\boldsymbol{B}' + \boldsymbol{A}') + j\,(\boldsymbol{B}'' + \boldsymbol{A}'')\end{aligned} \tag{8}$$

$$\begin{aligned}\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{B}\cdot\boldsymbol{A} &= (\boldsymbol{A}'\cdot\boldsymbol{B}' - \boldsymbol{A}''\cdot\boldsymbol{B}'') + j\,(\boldsymbol{A}'\cdot\boldsymbol{B}'' + \boldsymbol{A}''\cdot\boldsymbol{B}')\\ &= (\boldsymbol{B}'\cdot\boldsymbol{A}' - \boldsymbol{B}''\cdot\boldsymbol{A}'') + j\,(\boldsymbol{B}''\cdot\boldsymbol{A}' + \boldsymbol{B}'\cdot\boldsymbol{A}'')\end{aligned} \tag{9}$$

$$\begin{aligned}\boldsymbol{A}\times\boldsymbol{B} = -\boldsymbol{B}\times\boldsymbol{A} &= (\boldsymbol{A}'\times\boldsymbol{B}' - \boldsymbol{A}''\times\boldsymbol{B}'') + j(\boldsymbol{A}'\times\boldsymbol{B}'' + \boldsymbol{A}''\times\boldsymbol{B}')\\ &= (\boldsymbol{B}''\times\boldsymbol{A}'' - \boldsymbol{B}'\times\boldsymbol{A}') - j(\boldsymbol{B}''\times\boldsymbol{A}' + \boldsymbol{B}'\times\boldsymbol{A}'')\end{aligned} \tag{10}$$

The scalar triple product of 3 complex vectors $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ will be denoted by writing them in their correct order within square brackets as usual, that is

$$\begin{aligned}\boldsymbol{A}\cdot\boldsymbol{B}\times\boldsymbol{C} = \boldsymbol{A}\times\boldsymbol{B}\cdot\boldsymbol{C} &= [\boldsymbol{ABC}] = [\boldsymbol{BCA}] = [\boldsymbol{CAB}]\\ &= -[\boldsymbol{ACB}] = -[\boldsymbol{CBA}] = -[\boldsymbol{BAC}], \text{etc.}\end{aligned} \tag{11}$$

Conjugate Quantities

Because a complex quantity has a real as well as an imaginary part, the operation known as conjugation is oftentimes useful. Two complex quantities are said to be conjugate when their real parts are equal and their imaginary parts are the negative of each other. Thus the conjugate of a real scalar or vector is the scalar or the vector itself. To denote conjugate value an asterisk (*) will be written after the quantity. (This special notation is used for a reason given in a later section.) For instance, if m is a complex scalar given by equation 1, then its conjugate is

$$m^* = m' - jm'' \tag{12}$$

Also the conjugate of a complex vector $\boldsymbol{A}$ may be written in one of the following ways:

$$\boldsymbol{A}^* = \boldsymbol{A}' - j\boldsymbol{A}'' = A_i^* \boldsymbol{i} + A_j^* \boldsymbol{j} + A_k^* \boldsymbol{k} \tag{13}$$

From equations 7 to 10 it can be shown easily by performing the indicated operations that

$$(m\boldsymbol{A})^* = m^* \boldsymbol{A}^* \tag{14a}$$

$$(\boldsymbol{A} + \boldsymbol{B})^* = \boldsymbol{A}^* + \boldsymbol{B}^* \tag{14b}$$

$$(\boldsymbol{A}\cdot\boldsymbol{B})^* = \boldsymbol{A}^*\cdot\boldsymbol{B}^* \tag{14c}$$

$$(\boldsymbol{A}\times\boldsymbol{B})^* = \boldsymbol{A}^* \times \boldsymbol{B}^* \tag{14d}$$

$$\boldsymbol{A}^{**} = \boldsymbol{A} \tag{14e}$$

In other words, the operation of conjugation is commutative and distributive with the other operations such as addition, subtraction, and dot or cross multiplication. It is *reflexive* with itself as shown by equation 14*e*.

Although the absolute value of a complex scalar m is usually defined in terms of the real and the imaginary parts, it would be better to bring in the conjugate value m^* and call the square of the absolute value of m (denoted by enclosing m within bars) as the product of m and its conjugate thus:

$$|m|^2 = mm^* = (m')^2 + (m'')^2 \tag{15}$$

By using the absolute value of m, one gets the following alternate ways of writing equations 1 and 12:

$$m = |m| (\cos\theta + j\sin\theta) = |m| \angle\theta \tag{16a}$$

in which $\tan\theta = m''/m'$, and

$$m^* = |m| (\cos\theta - j\sin\theta) = |m| \angle -\theta \tag{16b}$$

Generalizing the idea of absolute value above given, the absolute value of a complex vector $\boldsymbol{A}$ will be defined as the positive square root of the dot product of $\boldsymbol{A}$ and its conjugate, that is,

$$|\boldsymbol{A}|^2 = \boldsymbol{A}\cdot\boldsymbol{A}^* = \boldsymbol{A}'\cdot\boldsymbol{A}' + \boldsymbol{A}''\cdot\boldsymbol{A}'' \tag{17}$$

This product is never negative since it is the sum of the squares of 2 real numbers. Hence the absolute value above defined for a complex vector is always real. It is zero only when the vector is zero. The product given by equation 17 should be carefully distinguished from the following:

$$\boldsymbol{A}\cdot\boldsymbol{A} = (\boldsymbol{A}'\cdot\boldsymbol{A}' - \boldsymbol{A}''\cdot\boldsymbol{A}'') + 2j\,\boldsymbol{A}'\cdot\boldsymbol{A}'' \tag{18}$$

which is a complex scalar unless $\boldsymbol{A}'$ be perpendicula[r] to $\boldsymbol{A}''$. In case $\boldsymbol{A}'$ is perpendicular to $\boldsymbol{A}'$, $\boldsymbol{A}\cdot\boldsymbol{A}$ wi[ll] be real and will be zero when the lengths of $\boldsymbol{A}'$ an[d] $\boldsymbol{A}''$ are equal. The vanishing of $\boldsymbol{A}\cdot\boldsymbol{A}$, which fac[t] for real vectors means the vanishing of the vector $\boldsymbol{A}$ is not sufficient to insure the vanishing of the complex vector $\boldsymbol{A}$. The necessary and sufficient con[di]tion for a complex vector $\boldsymbol{A}$ to be zero is tha[t] equation 17 equal zero. As for the significance o[f] the vanishing of the product $\boldsymbol{A}\cdot\boldsymbol{A}$, it will be take[n] up later on. It is interesting to note here that th[e] cross product of a complex vector by its conjugat[e] gives a vector having no real part, that is,

$$\boldsymbol{A} \times \boldsymbol{A}^* = -2j\boldsymbol{A}' \times \boldsymbol{A}'' \tag{19}$$

Representation of 3-Phase Sinusoidal Quantities

The above preliminary discussions on the defini[ti]tions and the elementary operations with comple[x] vectors will now be applied to the study of th[e] steady-state conditions in a 3-phase system in whic[h] harmonics are not present. For the purpose a[t] hand it will be recalled that in single-phase a-[c] theory, a sinusoidal function of time such a[s]

$$e = \sqrt{2}\,|E| \cos(\omega t + \theta) \tag{20a}$$

is representable by a complex scalar

$$E = |E| (\cos\theta + j\sin\theta) = E' + jE'' = |E| \angle\theta \tag{20b}$$

Generalizing this notion it is evident that the rea[l] vector, proposed previously for the representatio[n] of 3-phase systems,[1,2]

$$\boldsymbol{e} = \sqrt{2}\{\,|E_i| \cos(\omega t + \alpha)\,\boldsymbol{i} + |E_j| \cos(\omega t + \beta)\,\boldsymbol{j} + |E_k| \cos(\omega t + \gamma)\,\boldsymbol{k}\} \tag{21}$$

in which $|E_i|$, $|E_j|$, and $|E_k|$ denote the root mean-square values of the phase quantities and α, β, and γ the initial phase angles, may be represente[d] by the complex vector

$$\begin{aligned} \boldsymbol{E} &= |E_i| \angle\alpha\,\boldsymbol{i} + |E_j| \angle\beta\,\boldsymbol{j} + |E_k| \angle\gamma\,\boldsymbol{k} \\ &= E_i \boldsymbol{i} + E_j \boldsymbol{j} + E_k \boldsymbol{k} \\ &= (E_i' + jE_i'')\,\boldsymbol{i} + (E_j' + jE_j'')\,\boldsymbol{j} + (E_k' + jE_k'')\,\boldsymbol{k} \\ &= (E_i' \boldsymbol{i} + E_j' \boldsymbol{j} + E_k' \boldsymbol{k}) + j(E_i'' \boldsymbol{i} + E_j'' \boldsymbol{j} + E_k'' \boldsymbol{k}) \\ &= \boldsymbol{E}' + j\boldsymbol{E}'' \end{aligned} \tag{22}$$

which shows several different ways of writing a com[-]plex vector.

It is thus evident that one way of visualizing [a] complex vector such as $\boldsymbol{E}$ of equation 22 is to mak[e] the correlation between equations 21 and 22 [a] strictly rigid affair. In other words, whenever [a] complex vector is given in one of the forms shown i[n] equation 22, a corresponding real vector (varyin[g] with time) is set up in accordance with equation 21 so as to enable one whose sense of perception i[s] limited to 3-dimensions to see how a real locus i[n] space may be used to describe all the essentials of [a] complex vector.

Directional Ellipse of a Complex Vector

As the end of the vector $\boldsymbol{e}$ of equation 21 describe[s] an elliptical locus in space with center at the origin

1. For all numbered references see list at end of paper.

it is permissible then to state that to every complex vector there corresponds a real ellipse. Following Gibbs-Wilson,[3] this ellipse will be called the "*directional ellipse*" of the vector.

In general, the real and the imaginary parts of a complex vector $\boldsymbol{E} = \boldsymbol{E}' + j\boldsymbol{E}''$ are not parallel to each other, that is, $\boldsymbol{E}' \neq c\boldsymbol{E}''$. If they are parallel, then it can be shown that the directional ellipse degenerates into a straight line and the complex vector is said to have a "real direction." (The term "real direction" is due to Gibbs.) The necessary and sufficient condition for a complex vector to have a real direction is that it be reducible to the product of a real vector and a scalar, real or complex. The complex vector does not need to be a real vector in order to have a real direction. Thus if $\boldsymbol{E}' = c\,\boldsymbol{E}''$ in equation 22, then the complex vector $\boldsymbol{E}$ may be written as

$$\boldsymbol{E} = m\boldsymbol{E}'' = m(E_i''\,\boldsymbol{i} + E_j''\,\boldsymbol{j} + E_k''\,\boldsymbol{k}) \tag{23}$$

in which the complex scalar m is $(c + j) = |m|\angle\theta$, say. The real vector corresponding to equation 23, according to the correlation above established, is then:

$$\boldsymbol{e} = \sqrt{2}\,|m|\,\{E_i''\cos(\omega t + \theta)\,\boldsymbol{i} + E_j''\cos(\omega t + \theta)\,\boldsymbol{j} + E_k''\cos(\omega t + \theta)\,\boldsymbol{k}\} \tag{24}$$

in which E_i'', E_j'', and E_k'' are real scalars. Interpreted from the standpoint of 3-phase circuits, a complex vector having a real direction signifies that the 3 phase quantities are in time phase with each other. Note that such a system generally contains a positive, a negative, and a zero-sequence component, unless the 3 values E_i'', E_j'', and E_k'' are equal. In the latter case a balanced zero-sequence system obtains and the direction of $\boldsymbol{E}$ or $\boldsymbol{e}$ coincides with the isoclinic direction $\boldsymbol{u}$, defined as $\boldsymbol{u} = (1/\sqrt{3})(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$.

Conjugate Diameters of Directional Ellipse

Instead of using equation 21 to define the directional ellipse of a complex vector such as $\boldsymbol{E}$ of equation 22, the 2 real vectors $\boldsymbol{E}'$ and $\boldsymbol{E}''$ denoting the real and the imaginary parts respectively of $\boldsymbol{E}$ may be used to the same effect. Consider, for the present, the real vector $\boldsymbol{E}'$ from the standpoint of a complex vector. Then according to what has just been discussed, its directional ellipse is a straight line coincident with $\boldsymbol{E}'$. The varying vector correlated to $\boldsymbol{E}'$ is

$$\boldsymbol{e}' = \sqrt{2}(\cos\omega t)(E_i'\,\boldsymbol{i} + E_j'\,\boldsymbol{j} + E_k'\,\boldsymbol{k}) = \sqrt{2}(\cos\omega t)\,\boldsymbol{E}' \tag{25}$$

Similarly the vector $j\boldsymbol{E}''$ has a real direction coincident with $\boldsymbol{E}''$ and its correlated varying vector is:

$$\begin{aligned}\boldsymbol{e}'' &= \sqrt{2}\cos(\omega t + 90°)(E_i''\,\boldsymbol{i} + E_j''\,\boldsymbol{j} + E_k''\,\boldsymbol{k})\\ &= -\sqrt{2}(\sin\omega t)\,\boldsymbol{E}''\end{aligned} \tag{26}$$

Considering next $\boldsymbol{e}'$ and $\boldsymbol{e}''$ as the components of $\boldsymbol{e}$ in the directions $\boldsymbol{E}'$ and $\boldsymbol{E}''$, it may be seen that the elliptical locus of the end of $\boldsymbol{e}$ may be compounded from 2 simple harmonic motions taking place along the directions $\boldsymbol{E}'$ and $\boldsymbol{E}''$ with a time phase difference of 90 degrees. This then shows that $\boldsymbol{E}'$ and $\boldsymbol{E}''$ are 2 conjugate semidiameters of the directional ellipse.

It should be noted that if the positive direction of rotation is to be associated with increasing values of t, then as t increases, the point describing the real ellipse will rotate from the direction of $\boldsymbol{E}''$ to that of $\boldsymbol{E}'$ through an angle of less than 180 degrees as shown in figure 1, which is drawn on the plane of the ellipse. It is thus evident that a complex vector can be defined completely by 2 conjugate semidiameters of a directional ellipse. Since every ellipse has an infinity of pairs of conjugate diameters, a given directional ellipse corresponds to an infinite number of vectors. It now becomes of interest to investigate what relationship exists analytically among the different vectors having the same directional ellipse.

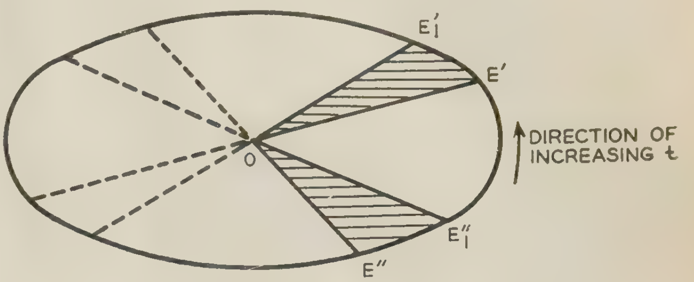


Fig. 1. Conjugate diameters of directional ellipse

Elliptical Rotation

Any root of unity, namely, a complex scalar having an absolute value of unity such as

$$\cos\theta + j\sin\theta = \angle\theta \tag{27}$$

in which θ is a real angle, has been called by Gibbs-Wilson[4] a "cyclic factor." When a complex vector $\boldsymbol{E}$ is multiplied by a cyclic factor, its directional ellipse remains unchanged while the corresponding conjugate semidiameters will be rotated in a positive direction, as defined in the last paragraph, *through a sector whose area is to the total area of the ellipse as θ is to 360 degrees.* (For proof of this statement, see appendix I.) This sort of rotation may be said to be an "elliptical rotation through a sector θ."

The significance of an elliptical rotation through a sector θ is interesting from the standpoint of 3-phase circuits. Thus, if a general unbalanced system of sinusoidal voltages is impressed on a linear static network consisting of 3 identical impedances, without mutual effects, connected in star and with power-factor angle θ, the directional ellipses of the current and the voltage vectors will lie on the same plane while the vectors themselves are obtainable one from the other by an elliptical rotation through a sector proportional to the power-factor angle θ, in addition to a stretching or shortening of their lengths. In case of a balanced positive- or negative-sequence system, the directional ellipses become circles and the voltage and the current vectors will make a constant angle θ with each other.

Circular Complex Vectors

Although a balanced system of voltages or currents has a circular locus as its directional ellipse, it does not follow that systems with circular directional ellipses are balanced in the usual sense of the word. It can be shown (see appendix II) that the necessary and sufficient condition for a complex vector $\boldsymbol{A}$ to have directional circles instead of ellipses is that the dot product of the vector with itself vanish; that is,

$$\boldsymbol{A}\cdot\boldsymbol{A} = 0 \tag{28}$$

Complex vectors satisfying the condition of equation 28 are called "circular complex vectors." As familiar and important examples of circular complex vectors there may be mentioned the 2 following unit vectors occurring in 3-phase circuit theory:

$$\boldsymbol{f} = (1/\sqrt{3})(\boldsymbol{i} + a^2\boldsymbol{j} + a\boldsymbol{k}) \tag{29a}$$

$$\boldsymbol{b} = (1/\sqrt{3})(\boldsymbol{i} + a\boldsymbol{j} + a^2\boldsymbol{k}) \tag{29b}$$

in which a and a^2 are the 2 complex cube roots of unity. The properties of $\boldsymbol{f}$ and $\boldsymbol{b}$ will be studied later on.

Representation of Power in 3-Phase Circuits

As steady-state power (real and reactive) in a single-phase circuit can be represented by the product of the complex voltage E and the conjugate of the complex current I, namely, I^*, and as power in a 3-phase circuit is equal to the sum of powers in each of the phases, it is evident that the steady-state power of a 3-phase circuit is representable by:

$$W = W' + jW'' = \boldsymbol{E}\cdot\boldsymbol{I}^* = E_iI_i^* + E_jI_j^* + E_kI_k^* \tag{30}$$

in which W' is the real or active power and W'' is the wattless or reactive power. Since the line-to-line voltage vector $\boldsymbol{E}_{jk}$ is related to the line-to-neutral voltage vector $\boldsymbol{E}_i$ by[1,2]

$$\boldsymbol{E}_{jk} = \sqrt{3}\,\boldsymbol{u} \times \boldsymbol{E}_i \tag{31a}$$

the identity $\boldsymbol{E}_i = \boldsymbol{u} \times \boldsymbol{E}_i \times \boldsymbol{u} + \boldsymbol{u}(\boldsymbol{E}_i\cdot\boldsymbol{u})$ can be put as:

$$\boldsymbol{E}_i = (1/\sqrt{3})(\boldsymbol{E}_{jk} \times \boldsymbol{u}) + \boldsymbol{u}(\boldsymbol{E}_i\cdot\boldsymbol{u}) \tag{31b}$$

wherein

$$\boldsymbol{u} = (1/\sqrt{3})(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) \tag{31c}$$

is the isoclinic unit vector defined before. Likewise, the line and the delta currents are related by:

$$\boldsymbol{I}_i = -\sqrt{3}\boldsymbol{u} \times \boldsymbol{I}_{jk} \tag{32a}$$

or

$$\boldsymbol{I}_{jk} = (1/\sqrt{3})(\boldsymbol{u} \times \boldsymbol{I}_i) + \boldsymbol{u}(\boldsymbol{I}_{jk}\cdot\boldsymbol{u}) \tag{32b}$$

Using equation 32*b* it is readily shown that the power in a delta connected 3-phase circuit is:

$$W = \boldsymbol{E}_{jk}\cdot\boldsymbol{I}_{jk}^* = (1/\sqrt{3})[\boldsymbol{u}\boldsymbol{I}_i^*\boldsymbol{E}_{jk}] \tag{33a}$$

and if the delta network is replaced by its equivalent star, equation 33*a* may also be written as:

$$W = \boldsymbol{E}_i\cdot\boldsymbol{I}_i^* \tag{33b}$$

by substituting $\boldsymbol{E}_{jk}$ from equation 31*a* and remembering that $\boldsymbol{u}\cdot\boldsymbol{I}_i = 0$.

Since power expended in a network under a given set of impressed voltages has a definite value independent of how the currents and voltages are actually measured, it appears from the above considerations that if the voltages have been measured in a specified way, the values of the currents should be measured in a definite *corresponding* way in order that the power in the circuit may be calculated simply as the sum of 3 terms. The question now is how the corresponding ways can be formulated analytically. To answer this question satisfactorily it will be necessary to inquire first how the measure numbers of a vector change as the base vectors are changed. Since the voltages and currents in a-c networks are usually measured through the use of instrument transformers, which procedure presupposes a strict proportionality between the secondary and the primary quantities on the 2 sides of the transformer, the discussion will be limited to a linear homogeneous transformation of the base vectors, which transformation corresponds physically to a change in the instrument transformer connections.

Covariant and Contravariant Measure-Numbers[9,10]

Let $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$ be any 3 non-coplanar real vectors. The components of a vector, $\boldsymbol{E}$ say, in the directions $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$ are to be found geometrically by the construction of a parallelopiped whose 3 adjacent sides are parallel to $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$ and whose main diagonal is $\boldsymbol{E}$. To find these components or the measure numbers r, s, t in

$$\boldsymbol{E} = r\,\boldsymbol{x}_a + s\,\boldsymbol{x}_b + t\,\boldsymbol{x}_c \tag{34}$$

analytically, it would be necessary to introduce the idea of reciprocal systems of vectors. Two systems of non-coplanar vectors $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, $\boldsymbol{x}_c$ and $\boldsymbol{x}^a$, $\boldsymbol{x}^b$, $\boldsymbol{x}^c$ are called reciprocal systems† when they satisfy the conditions:

$$\left.\begin{array}{lll} \boldsymbol{x}_a\cdot\boldsymbol{x}^a = 1, & \boldsymbol{x}_a\cdot\boldsymbol{x}^b = 0, & \boldsymbol{x}_a\cdot\boldsymbol{x}^c = 0, \\ \boldsymbol{x}_b\cdot\boldsymbol{x}^a = 0, & \boldsymbol{x}_b\cdot\boldsymbol{x}^b = 1, & \boldsymbol{x}_b\cdot\boldsymbol{x}^c = 0, \\ \boldsymbol{x}_c\cdot\boldsymbol{x}^a = 0, & \boldsymbol{x}_c\cdot\boldsymbol{x}^b = 0, & \boldsymbol{x}_c\cdot\boldsymbol{x}^c = 1 \end{array}\right\} \tag{35}$$

It can be shown that given $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$, the reciprocal system can be calculated from the following relations:

$$\boldsymbol{x}^a = \frac{\boldsymbol{x}_b \times \boldsymbol{x}_c}{[\boldsymbol{x}_a\boldsymbol{x}_b\boldsymbol{x}_c]},\ \boldsymbol{x}^b = \frac{\boldsymbol{x}_c \times \boldsymbol{x}_a}{[\boldsymbol{x}_a\boldsymbol{x}_b\boldsymbol{x}_c]},\ \boldsymbol{x}^c = \frac{\boldsymbol{x}_a \times \boldsymbol{x}_b}{[\boldsymbol{x}_a\boldsymbol{x}_b\boldsymbol{x}_c]} \tag{36}$$

In terms of $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$, the dyadic known as the "idemfactor" can be written as:

$$\mathbf{I} = \boldsymbol{x}_a\boldsymbol{x}^a + \boldsymbol{x}_b\boldsymbol{x}^b + \boldsymbol{x}_c\boldsymbol{x}^c = \boldsymbol{x}^a\boldsymbol{x}_a + \boldsymbol{x}^b\boldsymbol{x}_b + \boldsymbol{x}^c\boldsymbol{x}_c \tag{37}$$

The effect of the idemfactor used as a dot multiplier on a vector or dyadic is to leave that vector or dyadic unchanged. It is analogous to **1** in scalar algebra. Thus the following identity holds:

$$\begin{aligned} \boldsymbol{E} = \mathbf{I}\cdot\boldsymbol{E} &= (\boldsymbol{x}_a\boldsymbol{x}^a + \boldsymbol{x}_b\boldsymbol{x}^b + \boldsymbol{x}_c\boldsymbol{x}^c)\cdot\boldsymbol{E} \\ &= \boldsymbol{x}_a(\boldsymbol{x}^a\cdot\boldsymbol{E}) + \boldsymbol{x}_b(\boldsymbol{x}^b\cdot\boldsymbol{E}) + \boldsymbol{x}_c(\boldsymbol{x}^c\cdot\boldsymbol{E}) \end{aligned} \tag{38}$$

† Superscripts will be used to denote the system of vectors reciprocal to the system having the same subscripts. Contravariancy and covariancy are also distinguished by superscripts and subscripts. Superscripts should not be confused with exponents.

Comparing equations 38 and 34, it may be seen that the measure numbers r, s, and t are respectively:

$$r = \boldsymbol{x}^a \cdot \boldsymbol{E} \qquad s = \boldsymbol{x}^b \cdot \boldsymbol{E} \qquad t = \boldsymbol{x}^c \cdot \boldsymbol{E} \tag{39}$$

This equation shows that r, s, and t vary directly (or transform cogrediently) with $\boldsymbol{x}^a$, $\boldsymbol{x}^b$, and $\boldsymbol{x}^c$, which form the system reciprocal to $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$, while equation 35 or 36 shows that $\boldsymbol{x}^a$, $\boldsymbol{x}^b$, and $\boldsymbol{x}^c$ and $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$ vary inversely (or transform contragrediently) with each other. Hence with respect to $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$, the measure numbers r, s, and t transform contragrediently and may be called the "contra-variant measure-numbers" of $\boldsymbol{E}$. Instead of writing $\boldsymbol{E}$ as in equation 38, its components in the $\boldsymbol{x}^a$, $\boldsymbol{x}^b$, $\boldsymbol{x}^c$ directions might have been used, since the reciprocal system $\boldsymbol{x}^a$, $\boldsymbol{x}^b$, $\boldsymbol{x}^c$ is just as good a base-system as the given system $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, $\boldsymbol{x}_c$. In that case one writes:

$$\boldsymbol{E} = \boldsymbol{x}^a(\boldsymbol{x}_a \cdot \boldsymbol{E}) + \boldsymbol{x}^b(\boldsymbol{x}_b \cdot \boldsymbol{E}) + \boldsymbol{x}^c(\boldsymbol{x}_c \cdot \boldsymbol{E}) \tag{40}$$

wherein the scalar coefficients $(\boldsymbol{x}_a \cdot \boldsymbol{E})$, $(\boldsymbol{x}_b \cdot \boldsymbol{E})$, $(\boldsymbol{x}_c \cdot \boldsymbol{E})$ may be called the "covariant measure-numbers" of $\boldsymbol{E}$ with respect to $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, $\boldsymbol{x}_c$, since they vary directly or transform cogrediently with each other. Thus it may be seen that the covariant and the contra-variant measure-numbers are 2 alternate ways of specifying a vector with respect to a given base-system. In order to distinguish these 2 sets of measure numbers the usual convention is to write an upper index (superscript) to denote contra-variancy and a lower index (subscript) to indicate covariancy. This distinction will not be necessary in case the base system consists of 3 orthogonal unit vectors such as $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, because such a system is reciprocal to itself. As a matter of notation the following will then be used whenever necessary:

$$\boldsymbol{E} = E^a\boldsymbol{x}_a + E^b\boldsymbol{x}_b + E^c\boldsymbol{x}_c = E_a\boldsymbol{x}^a + E_b\boldsymbol{x}^b + E_c\boldsymbol{x}^c \tag{41a}$$

Returning to the consideration of power expression, it is now evident that the scalar product $\boldsymbol{E} \cdot \boldsymbol{I}^*$ can be calculated as the sum of 3 terms, provided one of the vectors be expressed through its contra-variant and the other through its covariant measure-numbers. In other words, if in addition to equation 41a,

$$\begin{aligned} \boldsymbol{I}^* &= (I^*)^a\boldsymbol{x}_a + (I^*)^b\boldsymbol{x}_b + (I^*)^c\boldsymbol{x}_c \\ &= (I^*)_a\boldsymbol{x}^a + (I^*)_b\boldsymbol{x}^b + (I^*)_c\boldsymbol{x}^c \end{aligned} \tag{41b}$$

then the power in the 3-phase circuit is:

$$\begin{aligned} \boldsymbol{E} \cdot \boldsymbol{I}^* &= E^a(I^*)_a + E^b(I^*)_b + E^c(I^*)_c \\ &= E_a(I^*)^a + E_b(I^*)^b + E_c(I^*)^c \end{aligned} \tag{42}$$

Transformation to Complex Base-Systems

In the above considerations on the transformation to new co-ordinate axes, real axes have been tacitly assumed in order to fix ideas. It is evident that if $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$ are noncoplanar complex vectors, equation 35 and the following would be still valid. Of particular interest and importance in 3-phase circuit theory is the system of unit vectors defined by:

$$\begin{aligned} \boldsymbol{u} &= (1/\sqrt{3})(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) \\ \boldsymbol{f} &= (1/\sqrt{3})(\boldsymbol{i} + a^2\boldsymbol{j} + a\boldsymbol{k}) = \boldsymbol{b}^* \\ \boldsymbol{b} &= (1/\sqrt{3})(\boldsymbol{i} + a\,\boldsymbol{j} + a^2\boldsymbol{k}) = \boldsymbol{f}^* \end{aligned} \tag{43}$$

which are the isoclinic unit vector and 2 special circular complex vectors, respectively. It can be readily shown that

$$\left.\begin{aligned} &\boldsymbol{u}\cdot\boldsymbol{u} = 1, && \boldsymbol{u}\cdot\boldsymbol{f} = 0, && \boldsymbol{u}\cdot\boldsymbol{b} = 0, \\ &\boldsymbol{f}\cdot\boldsymbol{u} = 0, && \boldsymbol{f}\cdot\boldsymbol{f} = 0, && \boldsymbol{f}\cdot\boldsymbol{b} = 1, \\ &\boldsymbol{b}\cdot\boldsymbol{u} = 0, && \boldsymbol{b}\cdot\boldsymbol{f} = 1, && \boldsymbol{b}\cdot\boldsymbol{b} = 0 \end{aligned}\right\} \tag{44}$$

and

$$\boldsymbol{f} \times \boldsymbol{b} = j\boldsymbol{u} \qquad \boldsymbol{b} \times \boldsymbol{u} = j\boldsymbol{b} \qquad \boldsymbol{u} \times \boldsymbol{f} = j\boldsymbol{f}$$

while

$$[\boldsymbol{u}\,\boldsymbol{f}\,\boldsymbol{b}] = j \tag{45}$$

so that by equation 36, the system reciprocal to $\boldsymbol{u}$, $\boldsymbol{f}$, $\boldsymbol{b}$ is simply $\boldsymbol{u}$, $\boldsymbol{b}$, $\boldsymbol{f}$, which is the first system taken in a different order. In other words, if the first system is considered to be right-handed, the reciprocal system will be left-handed. In virtue of equation 44, neither system can be considered as orthogonal. However, if r, s, and t are the contra-variant measure-numbers of a complex vector $\boldsymbol{A}$ with respect to $\boldsymbol{u}$, $\boldsymbol{f}$, and $\boldsymbol{b}$ as base vectors, the covariant measure-numbers of $\boldsymbol{A}$ also with respect to $\boldsymbol{u}$, $\boldsymbol{f}$, and $\boldsymbol{b}$ will be r, t, and s, since

$$\boldsymbol{A} = r\boldsymbol{u} + s\boldsymbol{f} + t\boldsymbol{b} = r\boldsymbol{u} + t\boldsymbol{b} + s\boldsymbol{f} \tag{46}$$

This shows that when a vector is referred to $\boldsymbol{u}$, $\boldsymbol{f}$, and $\boldsymbol{b}$ as base vectors, it will not be necessary to use upper or lower indices to distinguish the contra-variant and the covariant measure-numbers. If upper and lower indices are to be retained in this case, the following relations must be true:

$$A_u = A^u, \qquad A_b = A^f, \qquad A_f = A^b \tag{47}$$

Because of this set of relations the following expression for power in terms of symmetrical components may appear to be paradoxical at first but is true nonetheless:

$$\begin{aligned} W = \boldsymbol{E} \cdot \boldsymbol{I}^* &= E_u(I^*)^u + E_f(I^*)^f + E_b(I^*)^b \\ &= E_u(I^*)_u + E_f(I^*)_b + E_b(I^*)_f \\ &= E_u(I_u)^* + E_f(I_f)^* + E_b(I_b)^* \end{aligned} \tag{48}$$

wherein

$$\left.\begin{aligned} E_u &= \boldsymbol{u}\cdot\boldsymbol{E} = (1/\sqrt{3})(E_i + E_j + E_k) \\ E_b &= \boldsymbol{b}\cdot\boldsymbol{E} = (1/\sqrt{3})(E_i + a\,E_j + a^2E_k) \\ E_f &= \boldsymbol{f}\cdot\boldsymbol{E} = (1/\sqrt{3})(E_i + a^2E_j + a\,E_k) \end{aligned}\right\} \tag{49}$$

and

$$\left.\begin{aligned} (I^*)^u &= (I^*)_u = (I_u)^* = (1/\sqrt{3})(I_i^* + I_j^* + I_k^*) \\ (I^*)^b &= (I^*)_f = \boldsymbol{f}\cdot\boldsymbol{I}^* = (1/\sqrt{3})(I_i^* + a^2I_j^* + aI_k^*) = (\boldsymbol{f}^*\cdot\boldsymbol{I})^* \\ &= (I_b)^* \\ (I^*)^f &= (I^*)_b = \boldsymbol{b}\cdot\boldsymbol{I}^* = (1/\sqrt{3})(I_i^* + aI_j^* + a^2I_k^*) = (\boldsymbol{f}\cdot\boldsymbol{I})^* \\ &= (I_f)^* \end{aligned}\right\} \tag{50}$$

The paradoxical nature of the foregoing will disappear when it is remembered that the operation of conjugation is not commutative with the operation of obtaining the positive-sequence or the negative-sequence components. In other words, the conjugate of the positive-sequence component is equal to the negative-sequence component of the conjugate, and the conjugate of the negative-sequence component is equal to the positive-sequence component of the conjugate. The reason for using a separate symbol, such as the asterisk, *after instead of above* the letter to denote its conjugate is now apparent.

Transformation Formulas for Dyadics

With the preceding discussions on how the measure numbers of a vector change with a linear homogenous transformation of base-vectors, it is an easy matter to extend the same method to dyadics, since by definition a dyadic consists of the sum of a number of dyads, each of which contains 2 vectors written side by side without any sign in-between. Thus let the nonion form of an impedance dyadic, for example, be written out in full as:

$$\boldsymbol{\mathfrak{Z}} = \left\{ \begin{aligned} & Z_{ii}\,\boldsymbol{ii} + Z_{ij}\,\boldsymbol{ij} + Z_{ik}\,\boldsymbol{ik} \\ & + Z_{ji}\,\boldsymbol{ji} + Z_{jj}\,\boldsymbol{jj} + Z_{jk}\,\boldsymbol{jk} \\ & + Z_{ki}\,\boldsymbol{ki} + Z_{kj}\,\boldsymbol{kj} + Z_{kk}\,\boldsymbol{kk} \end{aligned} \right\} \tag{51}$$

and a set of non-coplanar vectors $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, $\boldsymbol{x}_c$ be specified as base. Then there will be 4 alternate ways of expressing the measure numbers of the dyadic with respect to $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$. They are:

$$\boldsymbol{\mathfrak{Z}} = \left\{ \begin{aligned} & Z^{aa}\,\boldsymbol{x}_a\boldsymbol{x}_a + Z^{ab}\,\boldsymbol{x}_a\boldsymbol{x}_b + Z^{ac}\,\boldsymbol{x}_a\boldsymbol{x}_c \\ & + Z^{ba}\,\boldsymbol{x}_b\boldsymbol{x}_a + Z^{bb}\,\boldsymbol{x}_b\boldsymbol{x}_b + Z^{bc}\,\boldsymbol{x}_b\boldsymbol{x}_c \\ & + Z^{ca}\,\boldsymbol{x}_c\boldsymbol{x}_a + Z^{cb}\,\boldsymbol{x}_c\boldsymbol{x}_b + Z^{cc}\,\boldsymbol{x}_c\boldsymbol{x}_c \end{aligned} \right\} \tag{52a}$$

$$\boldsymbol{\mathfrak{Z}} = \left\{ \begin{aligned} & Z^{a\cdot}_{\cdot a}\,\boldsymbol{x}_a\boldsymbol{x}^a + Z^{a\cdot}_{\cdot b}\,\boldsymbol{x}_a\boldsymbol{x}^b + Z^{a\cdot}_{\cdot c}\,\boldsymbol{x}_a\boldsymbol{x}^c \\ & + Z^{b\cdot}_{\cdot a}\,\boldsymbol{x}_b\boldsymbol{x}^a + Z^{b\cdot}_{\cdot b}\,\boldsymbol{x}_b\boldsymbol{x}^b + Z^{b\cdot}_{\cdot c}\,\boldsymbol{x}_b\boldsymbol{x}^c \\ & + Z^{c\cdot}_{\cdot a}\,\boldsymbol{x}_c\boldsymbol{x}^a + Z^{c\cdot}_{\cdot b}\,\boldsymbol{x}_c\boldsymbol{x}^b + Z^{c\cdot}_{\cdot c}\,\boldsymbol{x}_c\boldsymbol{x}^c \end{aligned} \right\} \tag{52b}$$

$$\boldsymbol{\mathfrak{Z}} = \left\{ \begin{aligned} & Z_{a\cdot}^{\cdot a}\,\boldsymbol{x}^a\boldsymbol{x}_a + Z_{a\cdot}^{\cdot b}\,\boldsymbol{x}^a\boldsymbol{x}_b + Z_{a\cdot}^{\cdot c}\,\boldsymbol{x}^a\boldsymbol{x}_c \\ & + Z_{b\cdot}^{\cdot a}\,\boldsymbol{x}^b\boldsymbol{x}_a + Z_{b\cdot}^{\cdot b}\,\boldsymbol{x}^b\boldsymbol{x}_b + Z_{b\cdot}^{\cdot c}\,\boldsymbol{x}^b\boldsymbol{x}_c \\ & + Z_{c\cdot}^{\cdot a}\,\boldsymbol{x}^c\boldsymbol{x}_a + Z_{c\cdot}^{\cdot b}\,\boldsymbol{x}^c\boldsymbol{x}_b + Z_{c\cdot}^{\cdot c}\,\boldsymbol{x}^c\boldsymbol{x}_c \end{aligned} \right\} \tag{52c}$$

$$\boldsymbol{\mathfrak{Z}} = \left\{ \begin{aligned} & Z_{aa}\,\boldsymbol{x}^a\boldsymbol{x}^a + Z_{ab}\,\boldsymbol{x}^a\boldsymbol{x}^b + Z_{ac}\,\boldsymbol{x}^a\boldsymbol{x}^c \\ & + Z_{ba}\,\boldsymbol{x}^b\boldsymbol{x}^a + Z_{bb}\,\boldsymbol{x}^b\boldsymbol{x}^b + Z_{bc}\,\boldsymbol{x}^b\boldsymbol{x}^c \\ & + Z_{ca}\,\boldsymbol{x}^c\boldsymbol{x}^a + Z_{cb}\,\boldsymbol{x}^c\boldsymbol{x}^b + Z_{cc}\,\boldsymbol{x}^c\boldsymbol{x}^c \end{aligned} \right\} \tag{52d}$$

the 9 scalar coefficients in each of which may be called respectively the contra-contravariant, the contra-covariant, the co-contravariant, and the co-covariant measure-numbers with respect to $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$. In case $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, and $\boldsymbol{x}_c$ are orthogonal unit vectors, these 4 forms will be all identical. Because of the reciprocal nature of $\boldsymbol{x}_a$, $\boldsymbol{x}_b$, $\boldsymbol{x}_c$ to $\boldsymbol{x}^a$, $\boldsymbol{x}^b$, $\boldsymbol{x}^c$, the different measure-numbers may be easily found. Thus, using m and n to stand for any one of the indices, one gets:†

$$\begin{aligned} Z^{mn} &= \boldsymbol{x}^m\cdot\boldsymbol{\mathfrak{Z}}\cdot\boldsymbol{x}^n \\ Z^{m\cdot}_{\cdot n} &= \boldsymbol{x}^m\cdot\boldsymbol{\mathfrak{Z}}\cdot\boldsymbol{x}_n \\ Z_{m\cdot}^{\cdot n} &= \boldsymbol{x}_m\cdot\boldsymbol{\mathfrak{Z}}\cdot\boldsymbol{x}^n \\ Z_{mn} &= \boldsymbol{x}_m\cdot\boldsymbol{\mathfrak{Z}}\cdot\boldsymbol{x}_n \end{aligned} \tag{53}$$

As a particular case of the above transformation, take the impedance dyadic of a completely symmetrical machine[5,6,7] which may be written as:

$$\boldsymbol{\mathfrak{Z}} = \left\{ \begin{aligned} & A\boldsymbol{ii} + C\boldsymbol{ij} + B\boldsymbol{ik} \\ & + B\boldsymbol{ji} + A\boldsymbol{jj} + C\boldsymbol{jk} \\ & + C\boldsymbol{ki} + B\boldsymbol{kj} + A\boldsymbol{kk} \end{aligned} \right\} \tag{54}$$

and transform it to $\boldsymbol{u}$, $\boldsymbol{f}$, and $\boldsymbol{b}$ as base vectors. It is then found that

$$\boldsymbol{\mathfrak{Z}} = Z_0\boldsymbol{uu} + Z_1\boldsymbol{fb} + Z_2\boldsymbol{bf} \tag{55}$$

in which

$$\begin{aligned} Z_0 &= \boldsymbol{u}\cdot\boldsymbol{\mathfrak{Z}}\cdot\boldsymbol{u} = A + B + C \\ Z_1 &= \boldsymbol{b}\cdot\boldsymbol{\mathfrak{Z}}\cdot\boldsymbol{f} = A + aB + a^2C \\ Z_2 &= \boldsymbol{f}\cdot\boldsymbol{\mathfrak{Z}}\cdot\boldsymbol{b} = A + a^2B + aC \end{aligned} \tag{55a}$$

As $\boldsymbol{u}$,$\boldsymbol{f}$, and $\boldsymbol{b}$ form the system reciprocal to $\boldsymbol{u}$, $\boldsymbol{b}$, and $\boldsymbol{f}$ and *vice versa*, calculation for all 4 forms of equation 54 as indicated by equation 52 using this particular base-system will result in the same expression already given in equation 55. In passing, it should also be noted that the 3 measure-numbers Z_0, Z_1, and Z_2 can be obtained by multiplying the dyadic of equation 54 by the unitary dyadics $\mathfrak{M}$ and $\mathfrak{M}^{-1}$ defined by:‡

$$\mathfrak{M} = \frac{1}{\sqrt{3}}\left\| \begin{matrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{matrix} \right\| \qquad \mathfrak{M}^{-1} = \frac{1}{\sqrt{3}}\left\| \begin{matrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{matrix} \right\| \tag{56}$$

because

$$\mathfrak{M}\cdot\boldsymbol{\mathfrak{Z}}\cdot\mathfrak{M}^{-1} = Z_0\,\boldsymbol{ii} + Z_1\,\boldsymbol{jj} + Z_2\,\boldsymbol{kk} = \boldsymbol{\mathfrak{Z}}', \text{ say} \tag{57}$$

The difference between equations 55 and 57 in their relations with equation 54 should be carefully noted. Equation 55 gives the same dyadic as given by equation 54 referred to new co-ordinate axes while equation 57 gives a new dyadic $\boldsymbol{\mathfrak{Z}}'$ after an affine transformation of space with origin fixed, the co-ordinate axes being unchanged. There are 2 ways of regarding a linear homogenous transformation.

Dyadic Analogues of 1, a, a^2

In the application of dyadic algebra to 3-phase circuits, it has already been noted that the cross product of the isoclinic vector, that is, $(\boldsymbol{u}\times)$, and another vector is quite similar to j in scalar multiplication with a complex scalar, both denoting rotations through 90 degrees. Instead of speaking of $(\boldsymbol{u}\times)$ as $\boldsymbol{u}$ in cross multiplication, it is sometimes more convenient to consider the same operation as a dyadic $(\boldsymbol{u}\times\mathfrak{I})$ in dot multiplication, wherein $\mathfrak{I}$ is the idemfactor

$$\mathfrak{I} = \boldsymbol{ii} + \boldsymbol{jj} + \boldsymbol{kk} = \boldsymbol{x}_a\boldsymbol{x}^a + \boldsymbol{x}_b\boldsymbol{x}^b + \boldsymbol{x}_c\boldsymbol{x}^c = \boldsymbol{uu} + \boldsymbol{fb} + \boldsymbol{bf}$$

already mentioned before. Just as $j^2 = -1$, $j^3 = -j$, $j^4 = 1$, $j^5 = j$, etc., the dyadic $(\boldsymbol{u}\times\mathfrak{I})$ has similar properties, which, however, must be expressed in somewhat different symbolism, because cross-multiplying any vector by $\boldsymbol{u}$ nullifies the isoclinic component of that vector altogether. Thus it can be easily shown that if $\boldsymbol{R}$ is any vector,

$$\boldsymbol{u}\times(\boldsymbol{u}\times\boldsymbol{R}) = -(\mathfrak{I} - \boldsymbol{uu})\cdot\boldsymbol{R}$$

or

$$(\boldsymbol{u}\times\mathfrak{I})\cdot(\boldsymbol{u}\times\mathfrak{I}) = (\boldsymbol{u}\times\mathfrak{I})^2 = -(\mathfrak{I} - \boldsymbol{uu}) \tag{58a}$$

Further from $\boldsymbol{u}\times\{\boldsymbol{u}\times(\boldsymbol{u}\times\boldsymbol{R})\} = -\boldsymbol{u}\times\boldsymbol{R}$ one gets

$$(\boldsymbol{u}\times\mathfrak{I})^3 = -(\boldsymbol{u}\times\mathfrak{I}) \tag{58b}$$

Likewise,

$$(\boldsymbol{u}\times\mathfrak{I})^4 = (\mathfrak{I} - \boldsymbol{uu}) \tag{58c}$$

which, when operated on any vector, gives that vector without its isoclinic component. To complete the analogy between $(\boldsymbol{u}\times\mathfrak{I})$ and j, it may be stated that

$$(\boldsymbol{u}\times\mathfrak{I})^5 = \boldsymbol{u}\times\mathfrak{I} \tag{58d}$$

and that the other higher powers follow equations 58*a*, *b*, *c*, and *d*. The dyadic $(\boldsymbol{u}\times\mathfrak{I})$ may, therefore, be considered to be the square root of $-(\mathfrak{I} - \boldsymbol{uu})$ just as j can be considered as the square root o

† Note that the dots before and after indices m and n in $Z^{m\cdot}_{\cdot n}$ or $Z_{m\cdot}^{\cdot n}$ are to show which index comes first and do not have any mathematical meaning as the dots used in dot multiplication.

‡ In equation 56 the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ and the plus signs have all bee omitted to bring out the arrangement of the coefficients in the matrices. Ther should be no confusion as to the meaning of the coefficients as the position c any one in the square array will show at once which 2 unit vectors are associate with it. See reference 2.

−1. The utility of $(\boldsymbol{u} \times \mathfrak{I})$ in star-delta voltage or current transformations has been noted before, while the usefulness of $(\mathfrak{I} - \boldsymbol{uu})$ in eliminating, and $\boldsymbol{uu}$ in obtaining, the zero-sequence component should be apparent to those who have occasion to do so.

Coming to the investigation of the dyadic analogues of the 2 complex cube roots of unity, namely,

$$a = \cos 120° + j \sin 120° \qquad a^2 = \cos 240° + j \sin 240°$$

it may be noted that the impedance dyadic of a symmetrical 3-phase machine as given by equation 54 can be written as:

$$\mathfrak{Z} = A\mathfrak{I} + B\mathfrak{J} + C\mathfrak{K} \tag{59}$$

in which

$$\mathfrak{I} = \boldsymbol{ii} + \boldsymbol{jj} + \boldsymbol{kk} \qquad \mathfrak{J} = \boldsymbol{ji} + \boldsymbol{ik} + \boldsymbol{kj} \qquad \mathfrak{K} = \boldsymbol{ki} + \boldsymbol{ij} + \boldsymbol{jk}$$

or

$$\mathfrak{I} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} \qquad \mathfrak{J} = \begin{Vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{Vmatrix} \qquad \mathfrak{K} = \begin{Vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{Vmatrix} \tag{60}$$

The dyadic $\mathfrak{I}$ is simply the idemfactor—the analogue of 1. The dyadics $\mathfrak{J}$ and $\mathfrak{K}$ may be considered to be formed from $\mathfrak{I}$ by moving each row of the matrix of $\mathfrak{I}$ down and up once, respectively. The dot product of $\mathfrak{I}$ and $\mathfrak{J}$ or $\mathfrak{K}$ leaves $\mathfrak{J}$ or $\mathfrak{K}$ unchanged, while

$$\mathfrak{J}\cdot\mathfrak{J} = \mathfrak{K} \qquad \mathfrak{K}\cdot\mathfrak{K} = \mathfrak{J} \qquad \mathfrak{J}\cdot\mathfrak{K} = \mathfrak{J}^3 = \mathfrak{K}\cdot\mathfrak{J} = \mathfrak{K}^3 = \mathfrak{I} \tag{61}$$

These are analogous to

$$aa = a^2 \qquad a^2a^2 = a \qquad aa^2 = a^3 = a^2a = (a^2)^3 = 1 \tag{62}$$

Moreover, corresponding to

$$(a - a^2) = \sqrt{3}\,j, \quad (a^2 - 1) = \sqrt{3}\,ja, \; (1 - a) = \sqrt{3}\,ja^2 \tag{63}$$

one finds:

$$\left.\begin{aligned} \mathfrak{J} - \mathfrak{K} &= \sqrt{3}(\boldsymbol{u} \times \mathfrak{I}) = \sqrt{3}(\mathfrak{I} \times \boldsymbol{u}) \\ \mathfrak{K} - \mathfrak{I} &= \sqrt{3}(\boldsymbol{u} \times \mathfrak{J}) = \sqrt{3}(\mathfrak{J} \times \boldsymbol{u}) \\ \mathfrak{I} - \mathfrak{J} &= \sqrt{3}(\boldsymbol{u} \times \mathfrak{K}) = \sqrt{3}(\mathfrak{K} \times \boldsymbol{u}) \end{aligned}\right\} \tag{64}$$

Thus $\mathfrak{J}$ is analogous to a and $\mathfrak{K}$ analogous to a^2.

As for the dyads $\boldsymbol{uu}$, $\boldsymbol{fb}$, and $\boldsymbol{bf}$, which appear in equation 55 for machine impedance, they are quite simply related to $\mathfrak{I}$, $\mathfrak{J}$, and $\mathfrak{K}$ as follows:

$$\left.\begin{aligned} 3\boldsymbol{uu} &= \mathfrak{I} + \mathfrak{J} + \mathfrak{K} \\ 3\boldsymbol{fb} &= \mathfrak{I} + a^2\mathfrak{J} + a\mathfrak{K} \\ 3\boldsymbol{bf} &= \mathfrak{I} + a\mathfrak{J} + a^2\mathfrak{K} \end{aligned}\right\} \tag{65}$$

In concluding it may be noted that the determinant formed from the matrix of $\mathfrak{I}$, $\mathfrak{J}$, or $\mathfrak{K}$ as given in equation 60 is in each case unity, that $\mathfrak{J}$ is the transpose as well as the reciprocal of $\mathfrak{K}$ and *vice versa*, and that all 3 quantities are commutable with each other in dot multiplication. These properties facilitate greatly calculations involving them. The practical usefulness of expressing machine impedance in terms of $\mathfrak{I}$, $\mathfrak{J}$, and $\mathfrak{K}$ lies in the fact that with constants A, B, and C known, a completely symmetrical 3-phase machine can be treated as a static 3-phase network though with peculiar mutual impedances. Thus in systems where several dis-symmetrical static impedances coexist with symmetrical machines, the time-honored Kirchhoff's laws can be applied in a straightforward manner without resorting to the use of symmetrical components, which method, interesting as it may be, sometimes actually transforms simple relations into longer ones.

Appendix I

To show the nature of elliptical rotation, multiply the vector $\boldsymbol{E} = \boldsymbol{E}' + j\boldsymbol{E}'' = |E_i| \angle\alpha\, \boldsymbol{i} + |E_j| \angle\beta \boldsymbol{j} + E_k \angle\gamma \boldsymbol{k}$ of equation 22 by the cyclic factor of equation 27 so as to result in:

$$\boldsymbol{E}_1 = \boldsymbol{E}_1' + j\boldsymbol{E}_1'' = |E_i| \angle(\alpha + \theta)\, \boldsymbol{i} + |E_j| \angle(\beta + \theta)\boldsymbol{j} + |E_k| \angle(\gamma + \theta)\boldsymbol{k} \tag{66}$$

The real varying vector correlated to $\boldsymbol{E}_1$ is then:

$$\boldsymbol{e}_1 = \sqrt{2}\{|E_i| \cos(\omega t + \alpha + \theta)\, \boldsymbol{i} + |E_j| \cos(\omega t + \beta + \theta)\, \boldsymbol{j} + |E_k| \cos(\omega t + \gamma + \theta)\boldsymbol{k}\} \tag{67}$$

Comparing this with equation 21, it may be seen that they represent the same ellipse, because the value of $\boldsymbol{e}_1$ at any (ωt) is the same as that of $\boldsymbol{e}$ at θ degrees later. The positions of $\boldsymbol{E}_1'$ and $\boldsymbol{E}_1''$ relative to $\boldsymbol{E}'$ and $\boldsymbol{E}''$ will, therefore, be as shown in figure 1. It remains next to show that the shaded area between $\boldsymbol{E}'$ and $\boldsymbol{E}_1'$ and that between $\boldsymbol{E}''$ and $\boldsymbol{E}_1''$ are equal and that each is equal to $\theta/360$ degrees of the total area of the ellipse. Consider the parametric equations of an ellipse referred to its major and minor axes as

$$\left.\begin{aligned} x &= A \cos \omega t \\ y &= B \sin \omega t = A \cos \phi \sin \omega t \end{aligned}\right\} \tag{68}$$

These show that any point P (figure 2*a*) on the ellipse may be found from the circumscribed and the inscribed circles having radii A and B by drawing a line OM to intersect the circles at M and M' and then projecting vertically and horizontally to meet at P. Referring to figure 2*a*, it may be seen that

$$SM = OM \sin \omega t = A \sin \omega t$$
$$SP = OM' \sin \omega t = B \sin \omega t$$

or for any position of the point P

$$SM/SP = A/B = 1/(\cos \phi) \tag{69}$$

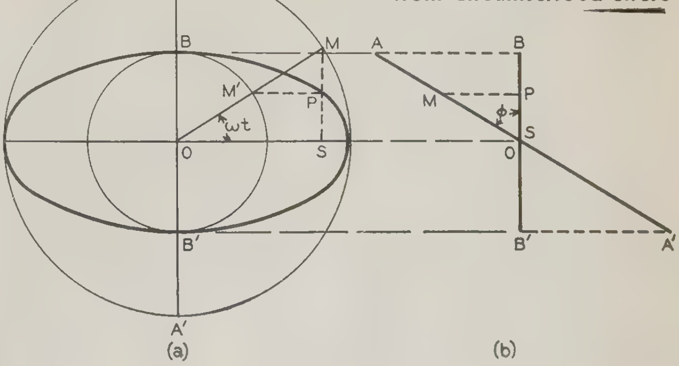
This shows that if the plane of the circumscribed circle is tilted so as to make an angle of ϕ with the plane of the ellipse (figure 2*b*), then every point P on the ellipse can be considered as the projection of a point M of the circle. Hence, as the line OM rotates through an angle θ, the sector swept through by OP of the ellipse will be proportional to θ, the factor of proportionality being determined by the fact that when OM makes a complete revolution of 360 degrees, the total area of the ellipse will have been swept through.

Appendix II

The condition $\boldsymbol{A}\cdot\boldsymbol{A} = 0$ is easily seen to be sufficient for the complex vector $\boldsymbol{A}$ to have a circle as its directional ellipse, because by equation 18, the vanishing of $\boldsymbol{A}\cdot\boldsymbol{A}$ requires the vanishing of $\boldsymbol{A}'\cdot\boldsymbol{A}''$ and the equality of $\boldsymbol{A}'\cdot\boldsymbol{A}'$ to $\boldsymbol{A}''\cdot\boldsymbol{A}''$. Since $\boldsymbol{A}'$ and $\boldsymbol{A}''$ are the conjugate semidiameters of the ellipse, the condition $\boldsymbol{A}'\cdot\boldsymbol{A}'' = 0$ shows them to be perpendicular to each other and hence semimajor and minor axes. The equality of the major and the minor axes shows the ellipse to be a circle.

To show that the given condition is necessary, let a cyclic factor

Fig. 2. Constructing an ellipse from circumscribed circle



$\angle\theta$ be found such that when the original vector $\boldsymbol{E}$ is multiplied by $\angle\theta$, namely $\boldsymbol{E}_1 = \boldsymbol{E}\angle\theta$, the new vector $\boldsymbol{E}_1$ will have conjugate diameters in the directions of the major and the minor axes of the ellipse. This cyclic factor $\angle\theta$ will have a determinate value if the directional ellipse is a true ellipse but will be indeterminate if the ellipse becomes a circle. Now

$$\boldsymbol{E}_1 = \boldsymbol{E}\angle\theta \quad \text{or } \boldsymbol{E} = \boldsymbol{E}_1\angle(-\theta) \tag{70}$$

Since the real and the imaginary parts of $\boldsymbol{E}_1$ are perpendicular,

$$\boldsymbol{E}\cdot\boldsymbol{E} = (\boldsymbol{E}_1'\cdot\boldsymbol{E}_1' - \boldsymbol{E}_1''\cdot\boldsymbol{E}_1'')\angle(-2\theta) \tag{71}$$

Since the quantity within parentheses is a real scalar, equation 71 shows that the angle θ depends only on the ratio of the real and the imaginary parts of $\boldsymbol{E}\cdot\boldsymbol{E}$. In other words, if $m = \boldsymbol{E}\cdot\boldsymbol{E} = m' + jm''$, then $\tan 2\theta = -m''/m'$. However, this value of θ will be indeterminate if $\boldsymbol{E}\cdot\boldsymbol{E} = 0$, for then $m' = m'' = 0$. Hence the proposition is true as stated.

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(References 9 and 10 give very good treatments on transformation theory and are recommended to supplement reference 3.)

Amplification Loci of Resistance-Capacitance Coupled Amplifiers

This paper applies the method of circular loci to the determination of the vector amplification of resistance-capacitance coupled amplifiers. It is shown that the locus for both exact and approximate equivalent circuits is the sum of 2 circles, whereas for the approximate circuits at high or low frequency the locus is a single circle. The locus of vector amplification for this type of amplifier may be drawn directly from the circuit constants and the magnitude and phase angle of amplification at any frequency read directly.

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THE VERSATILITY of the resistance-capacitance coupled amplifier has been increased greatly by the proper choice of shunt capacitances to secure either a flat or peaked amplification-frequency characteristic, as desired. For certain purposes it is necessary to reduce the shunt capacitances to a minimum, that is, the shunt capacitances of the tube and associated connections. In other cases, however, the required performance characteristic dictates much larger values of shunt capacitances, which are obtained by adding external capacitances to the grid and to the plate. Adjustment of the shunt capacitances, the coupling capacitance and the resistances in the circuit controls the sharpness of the response curve as well as its position in the frequency spectrum.

The large internal plate impedance and the practical elimination of grid-to-plate capacitance in the screen-grid tube has brought about a considerable advantage in favor of the resistance-capacitance coupled amplifier as compared to the transformer-coupled amplifier in voltage amplification work. This condition has been described by L. B. Arguimbau,[1] who has shown, "that the gain of a transformer-coupled amplifier is proportional to the effective impedance built up in the secondary circuit but is reduced by the step-up ratio of the transformer."

Although the design of the resistance-capacitance coupled amplifier is quite straightforward from the analytical standpoint, the accurate calculation of the response curve, when all the shunt capacitances are

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1. For all numbered references see list at end of paper.

ıken into account, becomes laborious and time-onsuming. When the equivalent circuit can be pproximated by neglecting shunt capacitances, the olution becomes somewhat simpler. However, if ıe frequency range is so broad that no circuit con-cants may be neglected, or if external shunt capaci-ınces are added to modify the shape of the response ırve, it is obvious that the exact equivalent circuit annot be avoided. D. G. C. Luck[2] has given an ccellent method of design which simplifies to a onsiderable extent the work entailed in selecting the rcuit constants to give a predetermined response ırve.

The author has found that the method of circular ci can be applied very conveniently to the deter-ination of the vector amplification as a function of ıe frequency, or at a fixed frequency, as a function f any one of the circuit constants. The method be described makes no assumption other than that ıe tube characteristic is linear, which is the quali-cation necessary to transform the resistance-ıpacitance coupled amplifier into an equivalent rcuit. The exact equivalent circuit will be eated first, then in turn, the so-called approximate quivalent circuit, the approximate circuit at low equency and the approximate circuit at high fre-uency. It will be shown that with variable fre-uency the amplification vector for the exact and r the approximate equivalent circuits follows a locus hich is the sum of 2 circles, whereas for the low ıd high frequency approximations the loci are ngle circles. These loci being drawn in the complex ane, the plots give not only the variation of the agnitude of the amplification but also of its phase ıgle.

XACT EQUIVALENT CIRCUIT

The exact equivalent circuit of a single stage is ıown in figure 1. The shunt capacitances C_p and $_g$ consist of tube and lead capacitances as well as ıy external capacitances added to control the re-onse curve. The resistances R_c and R_g are the oupling and grid resistances, and if necessary for me work, their values may be modified by parallel akage conductances of the insulating members. he resistance R_p is the internal plate resistance of ıe tube. Taking circulating currents around the ırious meshes as shown in figure 1, the following uations for electromotive force are obtained:

$$\left(R_p - \frac{j}{\omega C_p}\right) + I_2 \frac{j}{\omega C_p} \qquad = -\mu e_1$$

$$\frac{j}{\omega C_p} + I_2\left(R_c - \frac{j}{\omega C_p}\right) - I_3 R_c \qquad = 0$$

$$- I_2 R_c + I_3\left(R_c + R_g - \frac{j}{\omega C_c}\right) - I_4 R_g = 0$$

$$- I_3 R_g + I_4\left(R_g - \frac{j}{\omega C_g}\right) \qquad = 0$$

olving these equations simultaneously for I_4 gives

$$= \frac{\dfrac{j\mu e_1 R_c R_g}{\omega C_p}}{\text{denominator}}$$

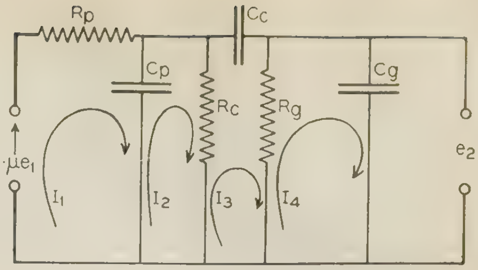


Fig. 1. Exact equivalent circuit of amplifier

in which the denominator arranged in descending powers of ω is

$$\omega^3(-jR_cR_pR_gC_p)(C_cC_g + C_cC_p + C_pC_g)$$
$$+ \omega^2\{-R_cR_gC_p(C_c+C_g) - R_pC_p[R_c(C_p+C_c)+R_g(C_g+C_c)]\}$$
$$+ \omega(+jC_p)(R_c + R_p)$$

The output voltage of the stage may be written as

$$e_2 = I_4\left(\frac{j}{\omega C_g}\right)$$

The expression for vector amplification, $\boldsymbol{A}$, is then

$$\boldsymbol{A} = |A| \angle \theta_A = \frac{e_2}{e_1}$$

which gives

$$\boldsymbol{A} = \frac{\dfrac{-j\mu C_c\omega}{R_p(C_cC_g + C_pC_g + C_pC_c)}}{\omega^2 + \dfrac{-j\omega[R_g(R_c+R_p)(C_g+C_c)+R_pR_c(C_p+C_c)] - (R_p+R_c)}{R_cR_pR_g(C_cC_g + C_pC_g + C_pC_c)}} \qquad (1)$$

This expression is linear with respect to the angular velocity in the numerator and is arranged as a quadratic of ω in the denominator. Condensing the scalar factors of ω into single constants, the equation is of the form

$$\boldsymbol{A} = \frac{-jk\omega}{\omega^2 - jb\omega - c} \qquad (2)$$

in which

$$\left.\begin{aligned} k &= \frac{\mu C_c}{R_p(C_cC_g + C_pC_g + C_pC_c)} \\ b &= \frac{R_g(R_c + R_p)(C_g + C_c) + R_pR_c(C_p + C_c)}{R_cR_pR_g(C_cC_g + C_pC_g + C_pC_c)} \\ c &= \frac{R_p + R_c}{R_cR_pR_g(C_cC_g + C_pC_g + C_pC_c)} \end{aligned}\right\} \qquad (3)$$

The equation of a circular locus is

$$\boldsymbol{S} = \frac{\alpha + \beta\rho}{\gamma + \delta\rho} \qquad (4)$$

in which $\boldsymbol{S}$ is a vector describing a circle as the scalar variable ρ ranges from minus to plus infinity and α, β, γ, δ are complex constants. The vector to the center of the circle is

$$\boldsymbol{C} = \frac{\alpha\bar{\delta} - \beta\bar{\gamma}}{\gamma\bar{\delta} - \delta\bar{\gamma}} \qquad (5)$$

and the radius of the circle is

$$R = \left|\frac{\alpha\delta - \beta\gamma}{\delta\bar{\gamma} - \gamma\bar{\delta}}\right| \qquad (6)$$

in which the vinculum ($^-$) indicates the conjugate of the vector to which it is attached.

The variable scalar in the present case is the angular velocity and the expression is not linear with respect to ω in the denominator, but is a quadratic. The amplification therefore, does not follow a circular locus, but by the expedient of partial fractions, may be handled conveniently as the locus of the sum of 2 circles. Thus equating the denom nator to zero, 2 roots are obtained, ω_1 and ω_2, which are complex and will be assumed to be unequal. Furthermore, in most cases, $b^2 > 4c$ making the roots pure imaginaries which will be the conditions assumed here.

$$\omega_{1,2} = \frac{+jb \pm \sqrt{-b^2 + 4c}}{2}$$

The amplification may then be written as the sum of 2 linear fractional transformations

$$A = \frac{-jk\omega}{\omega^2 - jb\omega - c} = \frac{M_1}{\omega - \omega_1} + \frac{M_2}{\omega - \omega_2}$$

in which M_1 and M_2 are constants free of ω. Clearing of fractions, equating coefficients of corresponding powers of ω, and solving for M_1 and M_2, remembering that $(\omega - \omega_1)(\omega - \omega_2) = \omega^2 - jb\omega - c$,

$$M_1 = \frac{-jk\omega_1}{\omega_1 - \omega_2} \qquad M_2 = \frac{jk\omega_2}{\omega_1 - \omega_2} \tag{7}$$

The amplification is then

$$A = \frac{\dfrac{-jk\omega_1}{\omega_1 - \omega_2}}{\omega - \omega_1} + \frac{\dfrac{jk\omega_2}{\omega_1 - \omega_2}}{\omega - \omega_2} = S_1 + S_2$$

where the 2 circles are

$$S_1 = \frac{\dfrac{-jk\omega_1}{\omega_1 - \omega_2}}{\omega - \omega_1} \qquad S_2 = \frac{\dfrac{jk\omega_2}{\omega_1 - \omega_2}}{\omega - \omega_2}$$

Thus the locus of amplification is the sum of circles, S_1 and S_2. The invariant points are, wh $\omega = 0$,

$$S_{1(0)} = \frac{jk}{\omega_1 - \omega_2} \qquad S_{2(0)} = \frac{-jk}{\omega_1 - \omega_2}$$

and when $\omega = \infty$,

$$S_{1(\infty)} = 0 \qquad S_{2(\infty)} = 0$$

From equation 5 the center vector of circle S_1 is

$$C_{s_1} = \frac{\dfrac{-jkj\,|\omega_1|}{j(|\omega_1| - |\omega_2|)}}{-j\,|\omega_1| - j\,|\omega_1|} = \frac{k}{2(|\omega_1| - |\omega_2|)}$$

and the center vector of circle S_2 is

$$C_{s_2} = \frac{\dfrac{jkj\,|\omega_2|}{j(|\omega_1| - |\omega_2|)}}{-j\,|\omega_2| - j\,|\omega_2|} = \frac{-k}{2(|\omega_1| - |\omega_2|)}$$

Similarly from equation 6 the radius of S_1 is

$$R_{s_1} = \left| \frac{\dfrac{-jkj\,|\omega_1|}{j(|\omega_1| - |\omega_2|)}}{j\,|\omega_1| + j\,|\omega_1|} \right| = \frac{k}{2(|\omega_1| - |\omega_2|)} \quad ($$

and the radius of S_2 is

$$Rs_2 = \left| \frac{\dfrac{+jkj\,|\omega_2|}{j(|\omega_1| - |\omega_2|)}}{j\,|\omega_2| + j\,|\omega_2|} \right| = \frac{k}{2(|\omega_1| - |\omega_2|)} \quad ($$

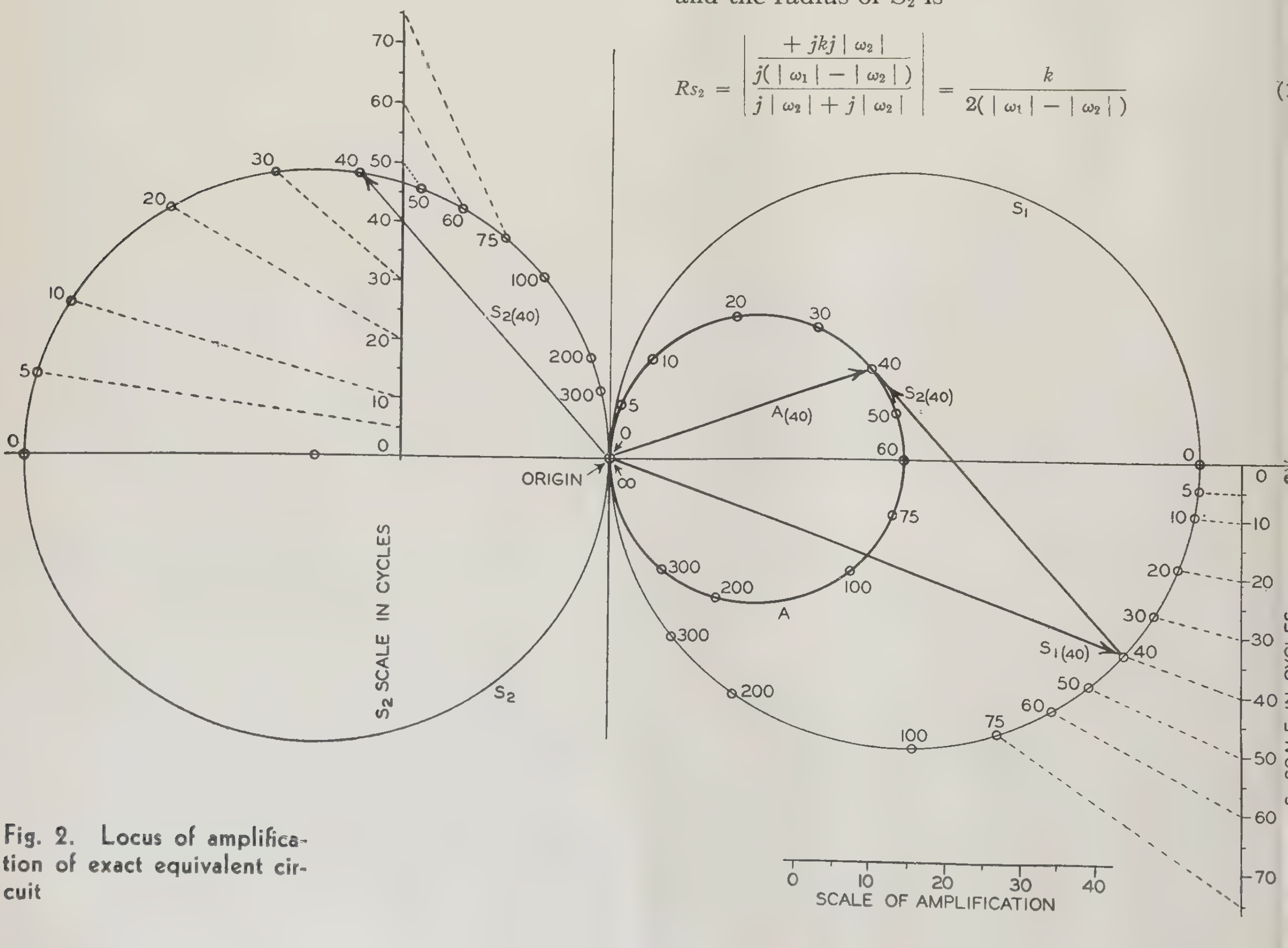


Fig. 2. Locus of amplification of exact equivalent circuit

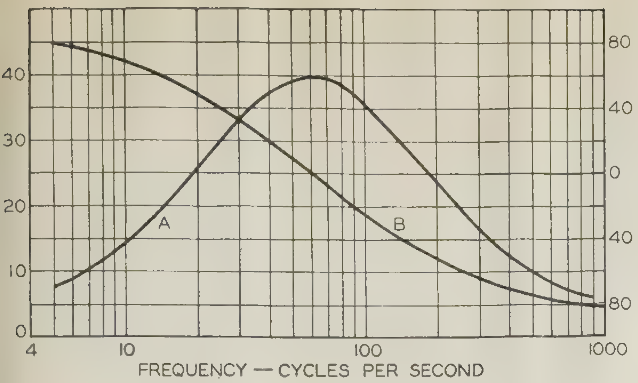


Fig. 3. Variation of magnitude and phase angle of amplification with frequency for exact equivalent circuit

A—Magnitude *B*—Phase angle

'hus both circles have equal diameters, $k/(\mid \omega_1 \mid - \mid \omega_2 \mid)$. As the frequency varies from zero to in-nity, S_1 travels from the real point $k/(\mid \omega_1 \mid - \mid \omega_2 \mid)$ to the origin along the circle in the fourth uadrant. At the same time S_2 travels from the real oint $-k/(\mid \omega_1 \mid - \mid \omega_2 \mid)$ to the origin along its ircle in the second quadrant.

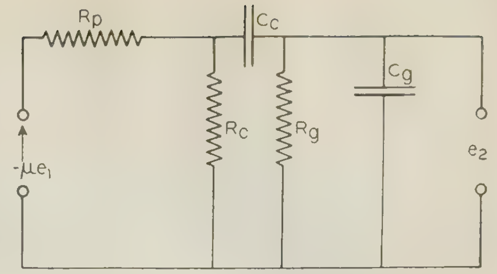
Both circles pass through the origin and have their enters on the real axis. The values of S_1 and S_2 are ero at infinite frequency and are of equal magnitude ut displaced by 180 degrees at zero frequency. 'hus the resultant curve passes through the origin t both of these points, that is, the amplification is ero at both zero and infinite frequencies. This is he condition required by the presence of shunt apacitances across the input and output terminals f the circuit and the series coupling capacitance. 'he amplification locus is a closed curve with a ouble point at the origin and belongs to the family f bicircular quartics. In case the roots of the enominator are equal, the procedure is the same s indicated above, with the exception that when reaking down the expression for amplification into artial fractions, one of the denominators will be $\omega - \omega_d)^2$ where ω_d is the double root. This form s the square of a circle. Its square root is therefore circle and it is plotted in the conventional manner. sufficient number of points on this square root ircle are then squared graphically, thereby giving he locus of the square of the circle. A solution of his type in connection with a bridge network may e found in a previous paper.[3]

PPLICATION TO A TYPICAL CIRCUIT

To illustrate the use of this method, the locus of mplification of the circuit called "type III" in the aper by Luck[2] will be determined. This particular ircuit was designed for a peaked response curve at frequency of 60 cycles in order to avoid stray effects ithout resorting to magnetic shielding. The con-tants of this circuit are as follows:

$_p$ = 0.75 megohm	C_p = 0.0085 microfarad
$_c$ = 0.24 megohm	C_g = 0.0085 microfarad
$_g$ = 0.18 megohm	C_c = 0.0085 microfarad
	μ = 652.5

Fig. 4. Approximate equivalent circuit of amplifier



Substituting these values into the equations of the constants of the bicircular quartic, equations 3, the constants of the locus become

$k = 34{,}120 \qquad b = 867.1 \qquad c = 141{,}000$

Inserting these constants into equation 2 the expression for amplification is

$$A = \frac{-j\,34{,}120\,\omega}{\omega^2 - j\,867.1\omega - 141{,}000}$$

The roots of the denominator are

$\omega_1 = j\,650.4 \qquad \omega_2 = j\,216.8$

The constants M_1 and M_2 from equations 7 are:

$M_1 = -j\,51{,}180 \qquad M_2 = j\,17{,}060$

The amplification may now be written as the sum of 2 circles, or

$$A = \frac{-j\,51{,}180}{\omega - j\,650.4} + \frac{j\,17{,}060}{\omega - j\,216.8}$$

The 2 circles are:

$$S_1 = \frac{-j\,51{,}180}{\omega - j\,650.4} \qquad S_2 = \frac{j\,17{,}060}{\omega - j\,216.8}$$

The invariant points are, when $\omega = 0$,

$S_{1(0)} = 78.7 \qquad S_{2(0)} = -78.7$

and when $\omega = \infty$

$S_{1(\infty)} = 0 \qquad S_{2(\infty)} = 0$

For the third point necessary to determine the linear scale line, a convenient value of ω is 377 corresponding to a frequency of 60 cycles. This gives

$S_{1(60)} = 68.1 \angle -30° \qquad S_{2(60)} = 39.2 \angle 120°$

The diameters of the 2 circles are equal to 78.7 and are coincident with the vectors S_1 and S_2 at zero frequency. The points for $\omega = 0$ and $\omega = 377$ determine the linear scale lines. A scale line is any line drawn perpendicular to the line joining the center of the circle with the point on the circle corresponding to $\omega = \infty$. Lines drawn from the latter point to any other points on the circle, such as $\omega = 0$ and $\omega = 377$, intercept a segment of the scale line proportional to the increment of the variable; in this case to 377 radians per second. The length of this segment fixes the scale of angular velocity or frequency along the scale line, which may now be subdivided according to any desired unit. In figure 2 the lines are shown subdivided in tens of cycles. Any line drawn from the infinity

point on the circle then intersects the scale and the circle at corresponding values of the variable, in this case the frequency. In figure 2 portions of such lines are shown dotted for both circles, for $f = 5, 10, 20, 30, 40, 50, 60$, and 75 cycles. Vectors of corresponding frequency on S_1 and S_2 are then added giving the resultant locus of vector amplification. This construction is shown in figure 2. The magnitude of the amplification at any frequency may be read off directly by means of the scale provided; the phase angle of the amplification may be obtained by measuring the angle the amplification vector makes with the real axis. The response curves plotted in figure 3 were obtained in this manner.

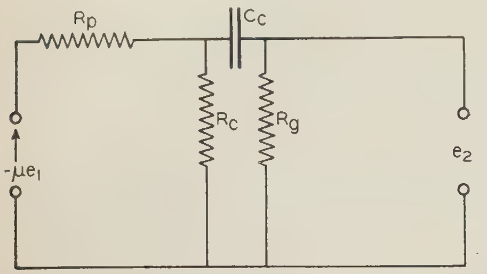


Fig. 5. Approximate equivalent circuit at low frequencies

Approximate Equivalent Circuit

In many cases where there is no added external capacitance across the plate of the tube, the plate-to-filament capacitance as well as the stray capacitances of the leads at the output end are small enough to be neglected, even for fairly high frequencies. The circuit then may be called the approximate equivalent circuit and it covers a large fraction of amplifiers of this type. This circuit is shown in figure 4. The solution for amplification for the exact circuit being available from the preceding work, the equation of amplification of the approximate circuit may be derived therefrom by merely making C_p zero throughout the expression. When this is done, the amplification becomes

$$A = \frac{\dfrac{-j\mu\omega}{R_pC_g}}{\omega^2 + \dfrac{-j\omega[R_g(R_c+R_p)(C_g+C_c)+R_pR_cC_c]}{R_cR_pR_gC_cC_g} - \dfrac{R_p+R_c}{R_cR_pR_gC_cC_g}} \quad (12)$$

This expression is identical in form with the preceding case and may be written as

$$A = \frac{-jk\omega}{\omega^2 - jb\omega - c}$$

in which

$$\left.\begin{aligned} k &= \frac{\mu}{R_pC_g} \\ b &= \frac{R_g(R_c+R_p)(C_g+C_c)+R_pR_cC_c}{R_cR_pR_gC_cC_g} \\ c &= \frac{R_p+R_c}{R_cR_pR_gC_cC_g} \end{aligned}\right\} \quad (13)$$

Inasmuch as the expressions for amplification i both cases are the same with the exception that tl scalar constants differ slightly, whatever has bee said concerning the characteristics of the locus fo the exact circuit is equally valid for the approxima equivalent circuit. Once the constants of the b circular quartic for the approximate circuit, k, and c, have been determined, the remaining wor is precisely the same as that for the exact equivaler circuit. The constants k, b, and c in both cases a all sums, products, and quotients of scalar quantiti

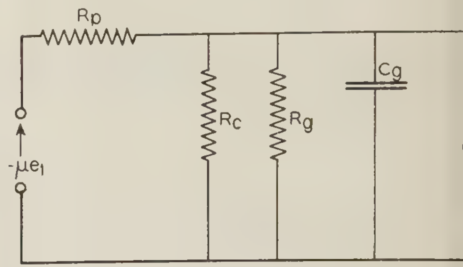


Fig. 6. Approximate equivalent circuit at high frequencies

and hence may be evaluated for the exact circu almost as quickly as for the approximate circui For this reason, as far as the method of circular lo is concerned, it is immaterial which circuit is used

Approximate Low-Frequency Circuit

At low frequencies when no externally adde shunt capacitances are employed, the input an output parallel capacitances of the tube play negligible rôle. This brings about a considerab simplification in the circuit which is drawn in figure The equation for amplification in this case may b obtained from the exact amplification equation b making C_p and C_g zero, whereupon the amplificatio for the low-frequency circuit becomes:

$$A = \frac{\mu\omega R_cR_gC_c}{-j(R_p+R_c)+\omega C_c(R_gR_c+R_cR_p+R_pR_g)} \quad (1$$

which is of the form

$$A = \frac{b\omega}{-jc+d\omega} \quad (1$$

in which

$$\left.\begin{aligned} b &= \mu R_cR_gC_c \\ c &= R_p+R_c \\ d &= C_c(R_gR_c+R_cR_p+R_pR_g) \end{aligned}\right\} \quad (1$$

The amplification, now being a linear fraction transformation, follows a circular locus. The cent vector and the radius are, from equations 5 and 6,

$$C = \frac{-jbc}{-jcd-jcd} = \frac{b}{2d} \quad (1$$

$$R = \left|\frac{-jbc}{jcd+jcd}\right| = \frac{b}{2d} \quad (1$$

The invariant points are, for $\omega = 0$ and $\omega =$ respectively,

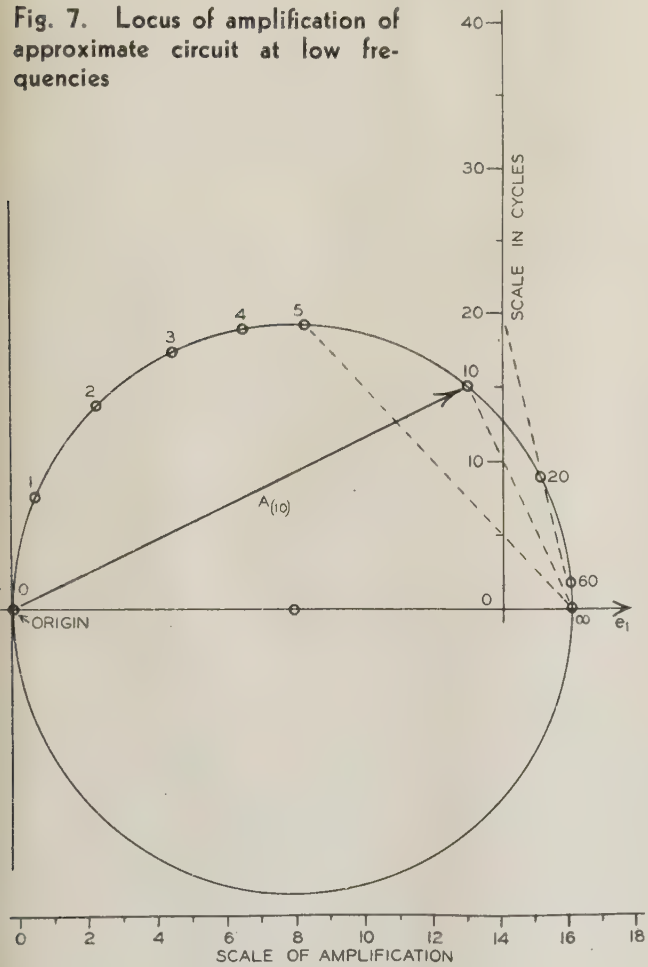
$$A_{(0)} = 0 \qquad A_{(\infty)} = b/d$$

The circle passes through the origin and has its center on the real axis at a distance $b/2d$. As the frequency varies from zero to infinity, the amplification vector travels from the origin to the real point b/d along the circle in the first quadrant.

Approximate High-Frequency Circuit

The usual circuit approximation at high frequencies neglects the plate shunt capacitance and the series coupling capacitance as shown in figure 6. This, of course, means that there is no external capacitance added across the plate, and that the frequency, although high, is not sufficient to make the magnitude of the plate-to-filament capacitance appreciable. Again reverting to the equation of

Fig. 7. Locus of amplification of approximate circuit at low frequencies



amplification for the exact circuit and making C_c infinite and C_p zero, the amplification for the approximate high-frequency circuit becomes

$$A = \frac{j\mu R_c R_g}{+j(R_g R_c + R_c R_p + R_p R_g) - \omega R_c R_p R_g C_g} \tag{19}$$

This being a linear fractional transformation is a circle in the form

$$A = \frac{ja}{jc - d\omega} \tag{20}$$

where

$$\left.\begin{aligned} a &= \mu R_c R_g \\ c &= R_g R_c + R_c R_p + R_p R_g \\ d &= R_c R_p R_g C_g \end{aligned}\right\} \tag{21}$$

The circle in this case has a radius, equation 6,

$$R = \left| \frac{-jad}{+jcd + jcd} \right| = \frac{a}{2c} \tag{22}$$

and a center vector, equation 5,

$$C = \frac{-jad}{-jcd - jcd} = \frac{a}{2c} \tag{23}$$

The invariant points are, for $\omega = 0$ and $\omega = \infty$ respectively,

$$A_{(0)} = \frac{a}{c} \qquad A_{(\infty)} = 0$$

The circle therefore passes through the origin and has a diameter coincident with the real axis, terminating at a/c. As the frequency varies from zero

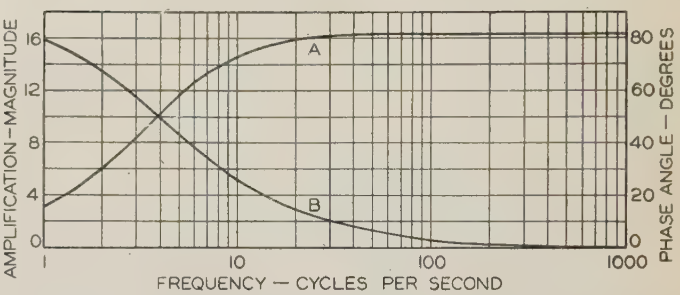


Fig. 8. Variation of magnitude and phase angle of amplification with frequency for approximate equivalent circuit at low frequencies

A—Magnitude *B*—Phase angle

to infinity the amplification vector travels from the real point to the origin along the circle in the fourth quadrant.

Example of the Low-Frequency Case

The following design is a typical application over a range of frequencies where both the grid and plate capacitances may be considered negligible. The circuit constants are:

R_p = 0.1 megohm	C_c = 0.05 microfarad
R_c = 0.15 megohm	μ = 30
R_g = 0.6 megohm	

Using these values the circle constants from equations 16 become

$$b = 0.1350 \times 10^6 \qquad c = 0.25 \times 10^6 \qquad d = 0.00825 \times 10^6$$

The expression for amplification as a linear fractional transformation is then

$$A = \frac{0.1350\omega}{-j\,0.25 + 0.00825\,\omega}$$

The invariant points are at $\omega = 0$ and $\omega = \infty$ respectively,

$$A_{(0)} = 0 \qquad A_{(\infty)} = 16.4$$

A convenient third point is at 60 cycles which gives

$$\mathbf{A}_{(60)} = 16.3\angle 4.5°$$

The center vector of the circle is, equation 17,

$$C = \frac{0.1350 \times 10^6}{2 \times 0.00825 \times 10^6} = 8.2$$

The radius has the same value, equation 18.

This locus is drawn in figure 7, together with the linear scale line as determined by the zero and 60-cycle points. The variation of magnitude and phase angle of the amplification as obtained from this plot is given in figure 8.

In both the low-frequency and high-frequency approximate circuits, it should be observed that only that portion of the circle is valid which falls in the frequency range over which the neglected capacitances are negligible.

For the convenience of the designer and computer a chart is given in table I, which lists in compact form the various constants and equations necessary for determining the amplification loci of the several circuits treated herein. Each circuit has its definite region of usefulness in the frequency spectrum as determined by the reactances of the capacitances. When external capacitances are added to the plate and grid to obtain a particular shape of response curve, it is always necessary to use the exact equivalent circuit. Otherwise when the shunt capacitanc are those of the tube itself and associated leads, tl approximate circuits give equally accurate resul over appropriate portions of the frequency spectrui

Amplifier Circuit at Constant Frequency

If it is desired to study the vector amplification a single frequency as any one of the circuit constan varies, the expression for amplification, equation may be rewritten with the variable element factor out in the numerator and the denominator. That the numerator and the denominator would th consist of a group of constant terms, plus the variab times another grouping of constant terms. Tl variable resistance or capacitance will always occ to the first power in both the numerator and tl denominator and the resulting locus will alwa therefore be a circle. This follows from the gener theorem concerning the circularity of loci, namel that in a network containing any number of line and bilateral self- and mutual-impedance elemen connected in any manner with constant sinusoid electromotive forces of like frequency connected any arms, all currents and all voltages follow circul loci, when any one self-impedance is varied along straight line in the complex plane.[4] Thus whenev any resistance or capacitance in the circuit is vari at constant frequency, the resulting locus of ampl fication is a single circle and is treated exactly as tl approximate circuits at low and high frequenc already described for variable frequency, with tl

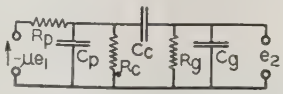
Table I—Equations for Determination of Loci of Resistance-Capacitance Coupled Amplifiers at Variable Frequen

Amplification = $A = e_2/e_1$

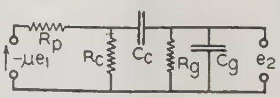
Circuit	Exact Equivalent Circuit	Approximate Equivalent Circuit	Approximate Circuit at Low Frequencies	Approximate Circuit a High Frequencies
Assumptions	None	$C_p = 0$	$C_p = 0 \quad C_g = 0$	$C_p = 0 \quad C_c = \infty$
Equation	$A = \frac{-jk\omega}{\omega^2 - jb\omega - c}$ for both exact and approximate circuits. Both cases the same except for constants		$A = \frac{b\omega}{-jc + d\omega}$	$A = \frac{ja}{jc - d\omega}$
Constants	$k = \frac{\mu C_c}{R_p(C_gC_c+C_cC_p+C_pC_g)}$ $b = \frac{R_g(R_c+R_p)(C_g+C_c)+R_pR_c(C_c+C_p)}{R_gR_cR_p(C_gC_c+C_cC_p+C_pC_g)}$ $c = \frac{R_c+R_p}{R_gR_cR_p(C_gC_c+C_cC_p+C_pC_g)}$	$k = \frac{\mu}{R_pC_g}$ $b = \frac{R_g(R_c+R_p)(C_g+C_c)+R_cR_pC_c}{R_gR_cR_pC_gC_c}$ $c = \frac{R_c+R_p}{R_gR_cR_pC_gC_c}$	$b = \mu R_gR_cC_c$ $c = R_c + R_p$ $d = C_c(R_gR_c+R_cR_p+R_pR_g)$	$a = \mu R_gR_c$ $c = R_gR_c+R_cR_p+R_p$ $d = R_gR_cR_pC_g$
Locus	Sum of 2 circles, that is, bicircular quartic		One circle	One circle

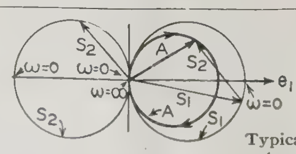
	Exact Equivalent Circuit
Roots	$\omega_{1,2} = \frac{+jb \pm\sqrt{-b^2 + 4c}}{2}$
Equation as sum of circles	$A = S_1 + S_2$ for $b^2 > 4c$ and $\omega_1 \neq \omega_2$ $S_1 = \frac{\frac{-k\omega_1}{\lvert\omega_1\rvert - \lvert\omega_2\rvert}}{\omega - \omega_1}$; $S_2 = \frac{\frac{k\omega_2}{\lvert\omega_1\rvert - \lvert\omega_2\rvert}}{\omega - \omega_2}$
Radius	$\frac{k}{2(\lvert\omega_1\rvert - \lvert\omega_2\rvert)}$ for both circles
Center vector	for S_1 $\frac{k}{2(\lvert\omega_1\rvert - \lvert\omega_2\rvert)}$; for S_2 $\frac{-k}{2(\lvert\omega_1\rvert - \lvert\omega_2\rvert)}$
Invariant points	$\omega = 0$; $S_1 = \frac{k}{\lvert\omega_1\rvert - \lvert\omega_2\rvert}$; $S_2 = \frac{-k}{\lvert\omega_1\rvert - \lvert\omega_2\rvert}$ $\omega = \infty \quad S_1 = 0 \quad S_2 = 0$

Exact equivalent circuit



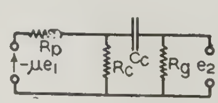
Approximate equivalent circuit

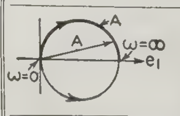




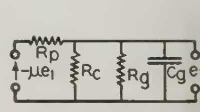
Typical locus

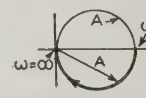
Approximate Circuit at Low Frequencies		Approximate Circuit a High Frequencies
$\frac{b}{2d}$	Radius	$\frac{a}{2c}$
$\frac{b}{2d}$	Center vector	$\frac{a}{2c}$
$\omega = 0 \quad A = 0$ $\omega = \infty \quad A = \frac{b}{d}$	Invariant points	$\omega = 0 \quad A = \frac{a}{c}$ $\omega = \infty \quad A = 0$





Typical locus





Typica locu

R_g = Grid resistance
R_c = Coupling resistance
R_p = Internal plate resistance
C_g = Grid capacitance
C_c = Coupling capacita
C_p = Plate capacitance
μ = Amplification fac

variable frequency replaced by the variable resistance or capacitance.

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New Developments in Ignitron Welding Control

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In this paper the use of resistance-welding timers using igniter-type mercury-cathode tubes is reviewed briefly and a seam-welding control unit containing a new design of inductive timer is described. An "ignition delay" form of control of welding heat or current magnitude is presented in which one set of power tubes controls both current magnitude and time duration. Comparison is made with the tap-changing method; operation of several welding machines from one timer is discussed.

PREVIOUS LITERATURE during the past several years has described the advantages of short and accurate timing for special and exacting resistance welding[1] as well as electronic welding control means for such work[2–7]. The purpose of this paper is to describe recent developments and refinements in the design and use of so-called ignitron-tube welding control apparatus and particularly to introduce a new "heat control" feature applied to the same power tubes heretofore used only to time the welding operation. To this end, a brief summarizing description of short-timing and electronic control is given, together with mention of trends in resistance welding which have resulted.

Resistance welding, particularly in the branches known as spot and seam welding, is rapidly assuming increased importance in old and in new fields. Contributing largely to wider application of the process are improved methods for dealing with one of the most difficult factors, weld timing. Timing-control devices are now available which, in operating from 60-cycle power sources, will accurately deliver less than $^1/_2$ cycle of welding power. Such timing, when applied with a welding machine of high current output, localizes the necessary welding heat. This localizing of the welding heat has been found to permit strong and consistent welding of structures demanding little surface deformation or burning, and, when desired, accurate fit with other parts. In addition, and of considerable importance, short-time resistance welding has in recent years been extended to a variety of nonferrous metals and heat-processed alloys, even when these are of low electrical resistance and sharp fusion point. Examples of the recent highly successful application of accurately controlled spot and seam welding are the welding of refrigeration evaporator units from copper alloy sheets, the butt welding of handles to pots and pans for enamel ware, the vacuum-tight welding of metal radio tubes and the welding of stainless steel and aluminum for numerous uses, as in the new high-speed trains and aircraft.

Extremely accurate control means in the form of electronic devices were generally introduced within the last few years. This form of control, applied to the primary of the welding transformer, functions to deliver current in terms of an exact number of half cycles. Transformer primary currents of 7,000 amperes and more (with corresponding secondary currents of many times this value) are carried, and interrupted, entirely by electronic tubes without dependence upon contactors.

Since the introduction of electronic welding control, a very marked improvement naturally has taken place in contactors and their auxiliary timing devices for use in short-time spot welding. As a result, a considerable number of such auxiliary timing devices, as well as faster-acting contactors, are on the market

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For all numbered references see list at end of paper.

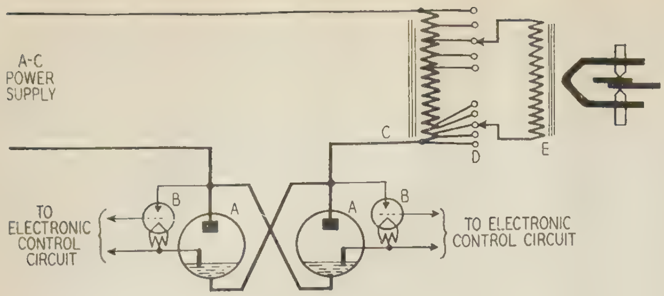


Fig. 1. Schematic diagram of spot-welding control circuit using ignitron weld timing

A—Ignitron tubes   *B*—Ignition control tubes
C—Autotransformer   *D*—Tap changers
E—Welding machine

at the present time. In some cases, the timing device is built as an integral part of the welding-contactor assembly.

The apparent ultimate accuracy of these metallic contacting devices is somewhat limited, however. Some manufacturers of such equipment, for the lower kilovolt-ampere ratings, guarantee accuracies of plus or minus $^1/_2$ cycle. Although this represents only a 5 per cent error in 10-cycle weld timing, it represents an error of 25 per cent in 2-cycle welding. (In addition, nonrepeating transients are generally present.) Accuracies, however, of plus or minus 1 cycle or even 2 cycles are probably more typical especially where appreciable welding loads are handled. The superior inherent accuracy and speed of electronic timing has, therefore, justified its greater cost for the more exacting modern high-speed spot welding. For the control of seam welding, no metallic contact device, particularly when appreciable welding power is involved, can equal the performance of electronic control.

The ignitron tube has been described in other papers[4,5,6] as well as has its use in the control of spot and seam welding.[7] Therefore, only the essential characteristics of the tube, as applied to resistance welding, will be reviewed here.

The tube consists of an evacuated vessel containing a graphite anode and a mercury-pool cathode, very similar to those used in conventional mercury-arc rectifiers, and, in addition, a boron-carbide ignition electrode dipping into the mercury pool. A small ignition current is passed into the pool through this igniter at the beginning of each desired half cycle of power current. This "fires" the tube in a manner somewhat similar to the firing of an automobile cylinder for each power stroke. The tube has proved ideally adapted to resistance-welding control since it embodies both the advantages of the mercury-pool cathode for heavy current duty, and a control feature such as is ordinarily supplied by the grid of hot-cathode tubes. During conduction of current, the arc drop of the tube is between 10 and 20 volts. The cathode consumes no energy during nonconducting periods.

When employed for timing resistance welding, 2 tubes (*A*) in parallel-inverse relation, as shown in figure 1, are necessary to control a-c power, since each individual tube conducts current in but or direction. For control of the ignition current, sma auxiliary hot-cathode tubes (*B*) are employed. Tl timing control circuit, whether for spot or sea welding, is then applied to the grids of these tube Welding-current magnitude, according to prese practice, is controlled either by taps (*D*) on a sep rate autotransformer (*C*) as shown in figure 1, or t taps on the welding machine transformer itself.

Simplified Control Unit for Seam Welding

The modern ignitron seam-welding control uni as illustrated by figures 2 and 3 and the circuit di gram in figure 4, represents considerable simplific tion and improvement over the former unit of tl same rating as described in 1934 by Stoddard[7] ar illustrated by figures 6, 7, and 8 of his paper. Tl principal improvements of the new unit over tl previously described unit are greater convenienc lessened maintenance, a new inductive-type timir device, and inclusion of the heat-control featur described later in this paper, besides portability ar much smaller size.

The new seam-welding control unit consists of separate assemblies mounted in a common enclosir cabinet with removable sides as shown in figure These 2 assemblies are the 24- by 36-inch contr panel visible in figure 2 and a power unit consistir of 2 steel tank-type ignitron tubes together with vacuum pump mounted on the bedplate behind tl control panel. Mounted on the front of the pan are ignition-control tubes and fuses, a timing dis (better shown in figure 3), a permanent magn equipped with coils (showing directly below tl disk in figure 2), an instrument for indicating ar d-c components in welding current, a weld-initiatir relay, and 2 dial-type knobs for adjusting "heat and "balance" respectively. On the rear of tl

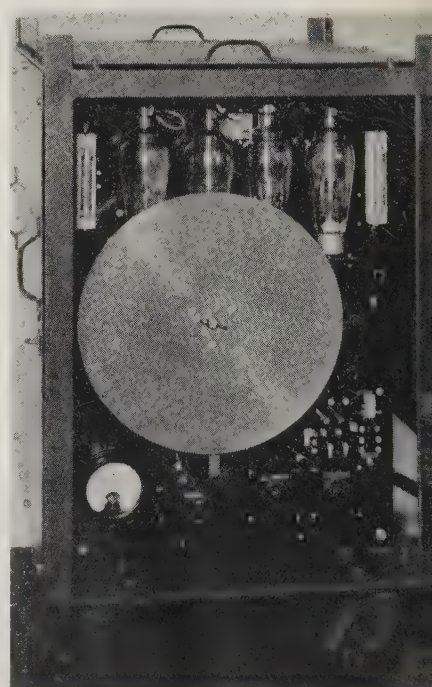


Fig. 2. The simplified ignitron seam - welding control unit

panel are mounted small transformers, resistors, and other circuit auxiliaries. The ignitron power unit, lthough it has been somewhat improved and simplified, requires no description in addition to that given n the earlier papers referred to previously.

A synchronously driven inductive timer replaces he earlier, and somewhat similar, photoelectric levice. The function of this inductive timer is to ontrol both the number of cycles of current delivered at each weld and the number of cycles in the nterval between welds. An aluminum-alloy timing lisk, as shown in figure 3, contains 120 holes evenly paced about the rim. This disk, in 60-cycle equipments, rotates once per second and therefore each nole represents one half-cycle. Accurately fitting teel pins are then plugged into each hole corresponding to which welding current is desired. These pins pass between the poles of a small permanent magnet upon which coils are mounted. Voltage generated n these coils by passage of the pins then activates he ignition control tubes and thereby controls the low of welding current. Various timing duties in cycles "on" and cycles "off" are set up by altering he pin arrangement. All pins are secured in place by a common rubber-faced clamping disk placed over he timing disk and secured by a wing nut.

The point of ignition of the tubes with respect to he voltage wave is not determined by the inductive imer but by the "heat control" circuit. The inductive timer, therefore, selects the proper half cycles of ine voltage and the heat-control feature selects the ctive portion of the half cycle of line voltage necessary to deliver the desired current magnitude. A description of circuit operation is given in the appendix.

Advantages of the inductive timer are its simplicity, reliability, and the definitely known timing n cycles "on" and "off" obtained by its use. The ptical and amplifying systems of the former photoelectric timer with their attendant maintenance such s replacement of lamps, phototubes, and amplifiers, nd cleaning and adjustment of optical parts, are liminated.

The seam-welding control unit shown in figure 2 s the smallest of 3 sizes now available. Upper limits

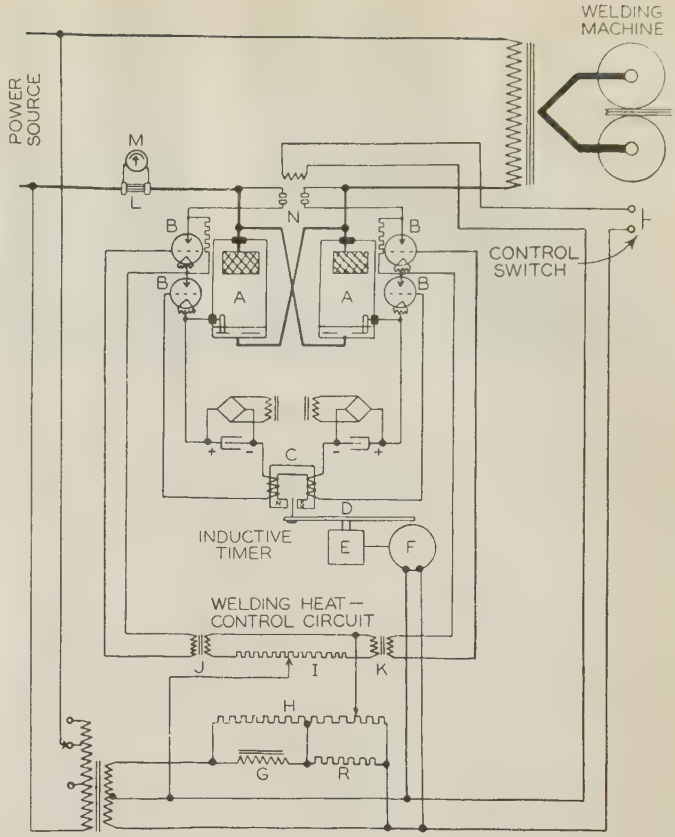


Fig. 4. Schematic diagram of the ignitron seam-welding control unit

Detailed description of operation is given in the appendix

A—Ignitron tubes
B—Ignition control tubes
C—Permanent magnet
D—Timing disk and pins
E—Worm gear speed reducer
F—Synchronous motor
G—Reactor
H—"Heat" adjustment potentiometer
I—"Balance" adjustment potentiometer
J—Grid transformer
K—Grid transformer
R—Resistor
L—Shunt
M—Instrument for indicating any d-c components in welding current
N—Control relay

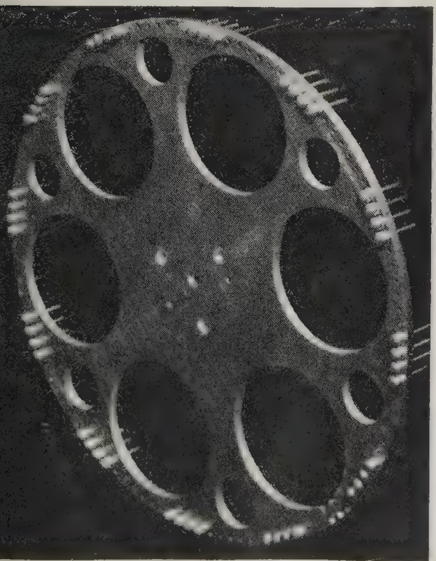


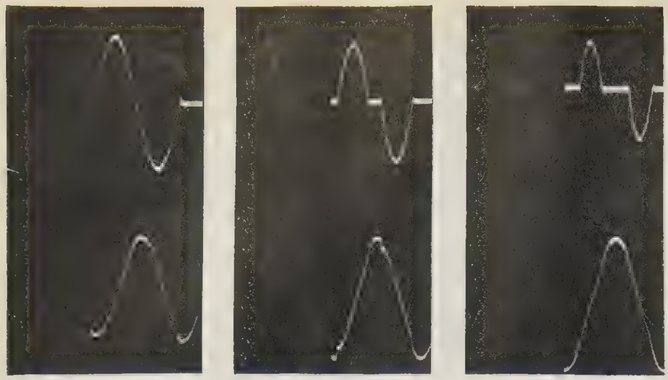
Fig. 3. Timing disk of the new inductive timer

in rating have not yet been established for the largest of these 3 sizes. However, in recent laboratory tests, this unit indicated capability of handling well in excess of the largest present seam-welding loads.

Welding-Heat Control

Further developments in electronic resistance-welding control have led to means for the accurate control, by one pair of ignitrons, of both current magnitude and duration of current flow delivered to the welding machine. This method of controlling current, or "heat," offers several advantages as will be enumerated and discussed later in this paper.

Heat control, by this new electronic method, is accomplished by interposing a variable zero-current space or gap between the several individual current loops generating heat for one weld. Owing to the



(A) (B) (C)

Fig. 5. Oscillograms of tube-controlled 2-cycle spot welding showing reduction in welding current or "heat" by ignition delay

Current scale in *B* and *C* is reduced to show more clearly the wave shapes

Top traces—Primary current of welding machine

Bottom traces—Line voltage

A—Effective current during weld of 990 amperes; relative current scale 100

B—Effective current during weld of 520 amperes; relative current scale 75

C—Effective current during weld of 174 amperes; relative current scale 46

inductive nature of welding loads, relatively small gaps result in appreciable crest-current reduction. Current gaps of this nature are obtained by delaying or retarding the ignition point of each tube beyond the natural power angle of the welding load. Heat reduction by this method suggests, and in some respects may be likened to, the lessened power output of an automobile motor with retarded spark.

The additional apparatus required for electro heat control consists of variable grid-control circ means associated with the weld-timing circuit a co-operating with it in such manner that, althou the number of current loops obtained for one w remain as present, the loop magnitude may be var at will.

Oscillograms *A*, *B*, and *C* of figure 5 show sp welding with 2 cycles of welding current applied each case to a machine having a power factor of 0. The root-mean-square or effective value of weldi current is reduced in *B* and *C* by delayed ignitic In *A* the current during the weld is 990 ampe effective value, in *B* 520 amperes, and in *C* 1 amperes. The scales of the 3 oscillograms w made successively smaller to show more clearly t shapes of the current waves. The scale for *B* 73 per cent that of *A*, and for *C* it is 46 per cent th of *A*.

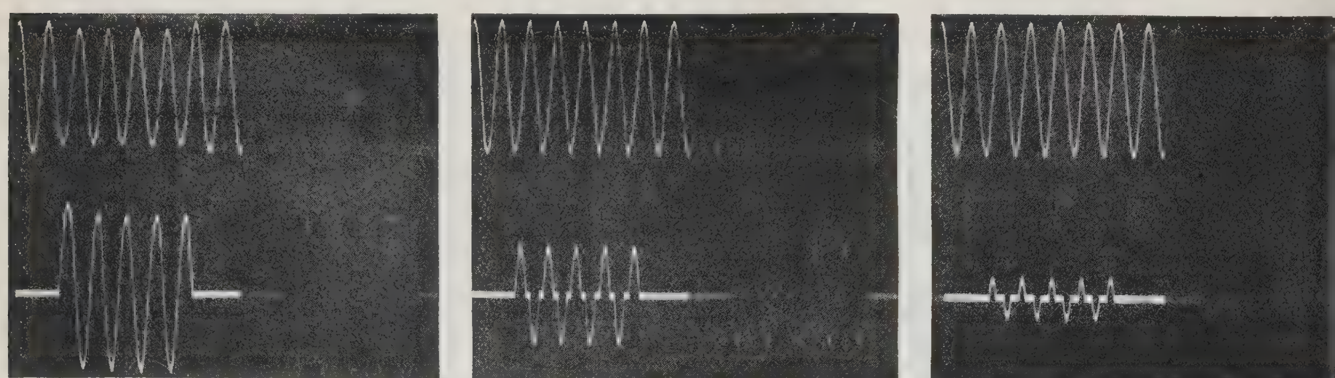
In similar manner, oscillograms *W*, *X*, and *Y* figure 6 show reduction in welding current in sea welding with a load having a power factor of 25 p cent and with current "on" $4^1/_2$ cycles and "o 3 cycles. In *W*, the effective current during each w is 1,100 amperes, in *X* 540 amperes, and in *Y* 1 amperes. In figure 6, however, the current sc has been left constant to show successive reductio in crest value more clearly.

The current loop obtained with delayed igniti contains both a sine and a transient term and m be expressed, neglecting tube arc drop, as

$$i = I\left[\sin(\theta + \omega t) - (\sin\theta)\,\epsilon^{-\frac{\omega R}{X}t}\right]$$

where

i = instantaneous value of primary welding current
I = crest value of full sine-wave current with no delay in ignit of the tube



(W) (X) (Y)

Fig. 6. Oscillograms of tube-controlled seam welding with $4^1/_2$ cycles "on" and 3 cycles "off," showing reductio welding current by ignition delay

The current scale is the same for *W*, *X*, and *Y* to show successive reductions in crest current values more clearly. In *W* ignition has b shifted, for first half-cycle, to a point in advance of natural current "zero," to show the effect of "early" firing

Top traces—Line voltage

Bottom traces—Primary current of welding machine

W—Effective current during weld of 1,100 amperes

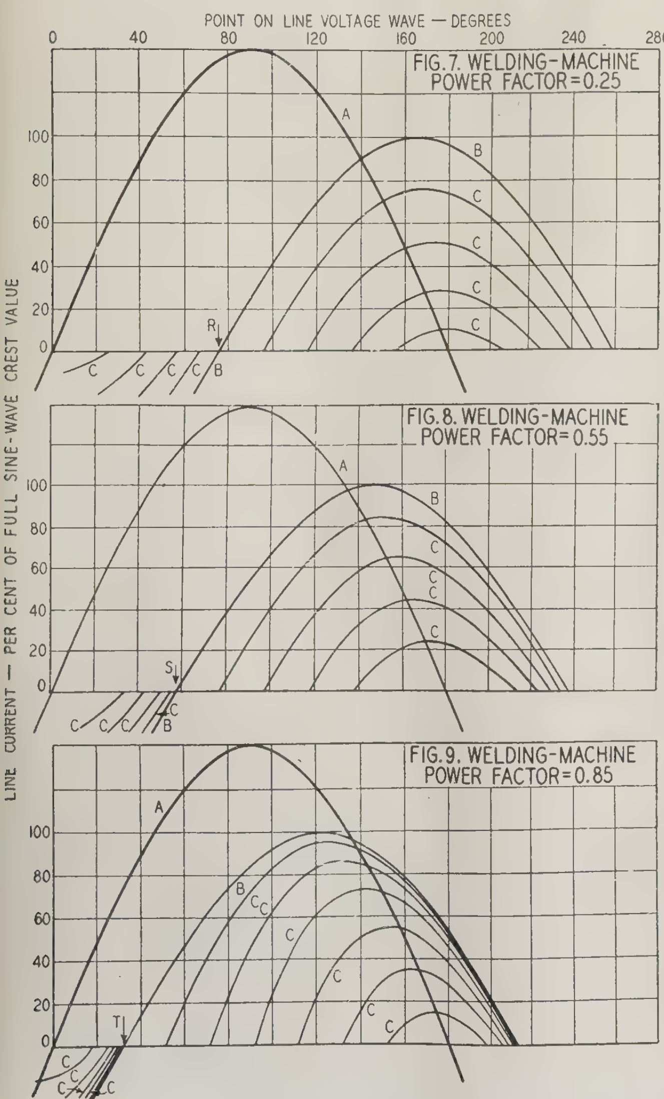
X—Effective current during weld of 540 amperes

Y—Effective current during weld of 170 amperes

θ = angle of delay of ignition beyond the natural power angle of the welding load in radians
ω = 2π times the line frequency
t = time after ignition in seconds
R and X = resistance and reactance in ohms, respectively, of the welding load in terms of the welding transformer primary

This expression applies for one loop only, or to where i falls to zero when the rectifying action of the tube prevents further current flow. A similar loop but of reverse polarity is obtained when the second tube is ignited 180 degrees following ignition of the first. The transformer secondary current will be of similar form.

In figure 7 are shown plots of the successively smaller current loops, all in one view, as caused by successively delaying the ignition point in 20-degree steps beyond the natural power angle of the load. Since "load" power factor influences the current-wave shape, and also its effective value, figures 7, 8, and 9 are shown, representing loads having power factors of 0.25, 0.55, and 0.85 respectively. In operating a purely resistive load the current loop, would, of course, assume the shape of the voltage wave following ignition. Such tendency may be noted on comparing figure 7 to figure 9. The power factor of resistance-welding loads is usually low, and is governed far more by the welding-machine throat opening necessary to receive the particular work than by the work itself. The power factor of a large percentage of welding machines now in use is in the neighborhood of 0.25. However, machines for special work in which small throat opening is possible may involve power factors of 0.50 or even 0.85.



Figs. 7–9. Curves of delayed-ignition welding current showing successive reduction in size of the current loop as the ignition point is successively delayed

A—Line voltage wave
B—Welding current loop with no ignition delay
C—Current loops resulting from delayed ignition
R, S, and *T* are power angles for welding loads with power factors of 0.25, 0.55, and 0.85 respectively

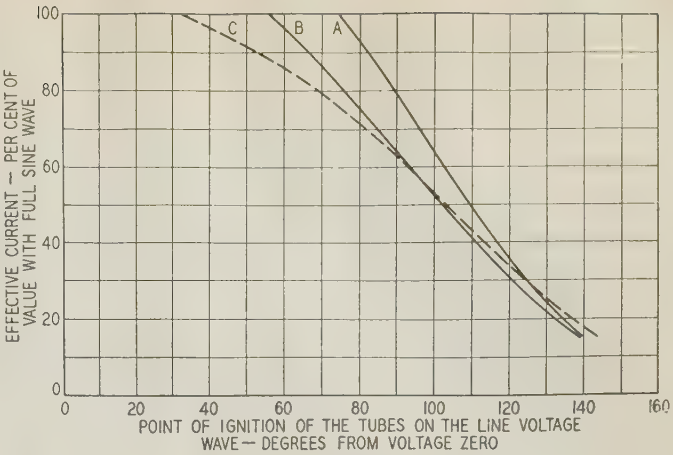


Fig. 10. Curves showing resulting effective value of current during weld as caused by point of ignition on the voltage wave

Curves *A, B,* and *C* are for welding-machine power factors of 0.25, 0.55, and 0.85 respectively

In figure 10 the effective value of delayed ignition current is shown as a percentage of maximum full-wave value as the point of ignition on the voltage wave is varied. Zero degrees represent zero line voltage.

In similar manner, figure 11 shows the effective value of delayed ignition current, plotted, however, against the size of the current gap between current loops in degrees.

Figure 12 indicates the comparative values of crest welding current as obtained by the ordinary tap-changing method and by the delayed-ignition method. Obviously, when the current loops are of shorter duration than 180 degrees their crest magnitude must be somewhat higher than that of a full sine wave delivering the same heat. It may be noted that at 25 per cent reduction in welding current (representing a 44 per cent reduction in actual welding heat), the current crests are only 8 per cent higher with delayed ignition on a load with a power factor of 0.55 than if the current were reduced by tap changers. When the current is reduced to 50 per cent (representing a reduction of 75 per cent in actual welding heat) the crest current value is corre-

spondingly 21 per cent higher than when the current is reduced by tap changers.

Advantages offered by the new form of heat control for resistance welding may be enumerated as follows:

1. Welding heat adjustment is made continuous as contrasted to step-by-step adjustment by tap changers. Continuous adjustment is becoming more and more desired in the newer precision welding, particularly of alloys and nonferrous metals, as is evidenced by the increased number of taps found on welding machines used for such work.

2. Heat adjustments are easily made by a dial-type knob as contrasted to the laborious shifting of heavy tap switches. The welding heat may, in fact, be "tuned in" during a series of spot welds or during the course of a seam weld.

3. The means for heat adjustment may be located to suit the operator's convenience, or, if desired, may be padlocked against tampering.

4. Electronic control of welding heat is readily adaptable to welding at high production speed on variable-gauge structures. A large number of spot welds may be made in rapid succession, and yet certain spots, as desired, may be welded at increased or decreased welding heat. Either manual or cam-operated means may be employed to alter heat settings between welds. In seam welding, heat may be changed to suit the structure during travel of the welding-electrode wheels.

5. Automatic heat regulation, eliminating the effects of line-voltage variation, is made entirely feasible by the electronic ignition-delay means. Although preliminary work has been done on such regulating means, commercial apparatus has not yet been made available.

6. Electronic heat control offers economies in apparatus cost, not yet fully evaluated, in the partial or complete elimination of tap-changing equipment. On the basis of complete elimination of tap changers by the addition of this ignition delay feature to electronic timing control, the cost of a separate tapped autotransformer for welding machines in the higher kilovolt-ampere ratings is saved, and the cost of added tapped primary turns (requiring a larger transformer) for the smaller machines is saved, in addition to elimination of the multipoint tap-changing switches necessary in each case. Experience only will indicate the degree to which it will be advisable to extend this economy.

Additional advantages, and possibly some disadvantages, will be disclosed in the accumulation of experience with electronic delayed ignition. Although predictions might be made regarding certain possibilities, these are considered as yet too intangible to justify enumeration as advantages or disadvantages. Two points of difference in the 2 methods of heat control deserve consideration at this time, however. One of these is the higher crest input of welding heat attendant to ignition-delay current as compared to sine-wave current. While experience, to date, in welding with each of the 2 heat-adjustment methods has shown no marked difference in results obtained, further experience on a variety of gauges and metals may possibly show differences between, and corresponding relative merits of, the 2 methods. A second point of difference, and a possible disadvantage of ignition-delay heat control, is that reduced heat by this method does not result in as great a reduction in effective line current as when the heat is reduced in like amount by the tap-changing method. When the effective value of secondary current is reduced from 100 per cent to 50 per cent by ignition delay (resulting in a welding heat reduction to 25 per cent) the line or primary current will also be 50 per cent of maximum effective value. On the contrary, a secondary-current reduction of 50 per cent by tap changers will result in a line or primary current of only 25 per cent of the maximum effective value. In the former case, however, current is not flowing during the entire voltage cycle, the current gap occurring at or near the peak value of the voltage wave for loads of typical power factors. The net effects on line voltage have not been evaluated. A harmonic analysis of line-voltage effects should readily yield information on this point.

It seems a reasonable conclusion, therefore, that both the time duration and the magnitude of current flow as factors in resistance welding may be brought to a degree of precision and flexibility in their control that was impossible before the advent of electronic devices. One of the most difficult of all factors in resistance welding, that is, timing, has already been subjected to precise control, as described in an earlier paper introducing ignitron timers.[7] These same tubes, through the method described in the present paper, now may serve to control accurately another difficult factor, that is

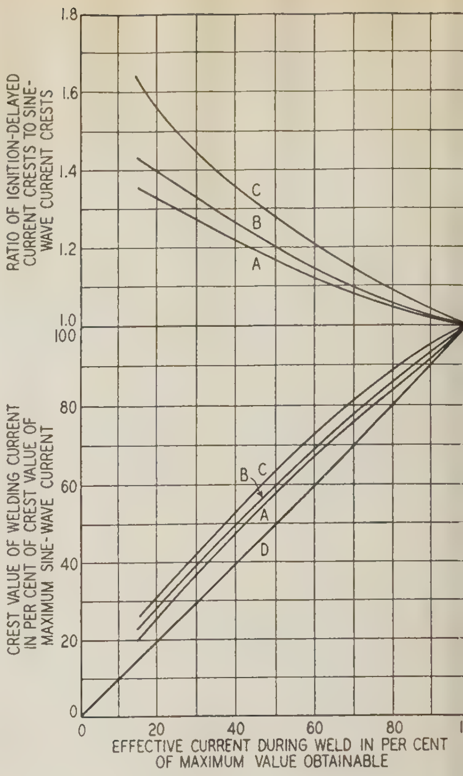


Fig. 12. Curves showing the comparative values of crest welding currents as obtained by the tap-changing method and by the ignition-delay method

Curves *A*, *B*, and *C* are for ignition-delay method with welding-machine power factors of 0.25, 0.55, and 0.85 respectively

Curve *D* is for tap-changing method with welding machine of any power factor

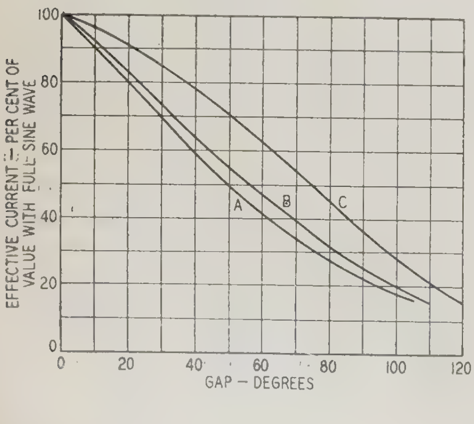


Fig. 11. Curves showing the influence of size of gap between current loops on the effective value of the loops

Curves *A*, *B*, and *C* are for welding-machine power factors of 0.25, 0.55, and 0.85 respectively

current magnitude, or heat. It is only logical to suppose that this additional refinement will result in still further increase in the fields to which resistance welding will be applied.

Load-Center Operation

Delayed ignition in electronic control offers an additional feature, not associated primarily with heat control, but with improved performance in load-center seam welding. The proposition of load-center operation is considered most practicable where production requirements demand 2 or more similar seam-welding machines operating on similar work. In this case, one high-capacity ignitron timing-control unit may be employed energizing a bus, common to all machines, with interrupted power. Each machine may then be connected to this bus at will by means of an ordinary magnetic contactor. If adjustments in welding heat are desired at the individual welding machines, such adjustment may be accomplished by tap changers. The welding heat of all machines in common may be adjusted by suitable ignition delay at the common control unit.

However, for practical reasons, a slight amount of ignition delay is desirable in load-center operation, at all times, as a preventative against possible magnetic saturation of the welding-machine transformers. This follows from the fact that should the connected seam-welding load be energized by the tubes at each weld prior to the natural power angle of the load, that is, prior to the point of natural zero current on the voltage wave, abnormally large initial-current loops will be delivered at each weld. These transients or d-c components are cumulative and may result in transformer saturation on any welding duty in which each individual weld begins at the same polarity of line voltage. Slight ignition delay will, therefore, insure the absence of such transients in spite of variations in power factor of the connected welding load as caused by connecting and disconnecting machines of slight variance in power factor.

The safety factor thus provided by a small amount of ignition delay coupled with the seam-welding control units which are now available for handling line currents of several thousand amperes justifies very serious consideration of large load-center installations whenever production requirements demand batteries of seam-welding machines producing a given article.

Appendix

Operation of the ignitron seam-welding control circuit shown in figure 4 is, in detail, as follows:

The control principle of igniter-type tubes *A* has been described in the previous text. Control of the hot-cathode gaseous-discharge tubes *B* is effected by holding the grids negative with respect to the cathode for nonconduction, and positive for conduction. Provided, then, that the anode is at positive potential and the grid, also, is made positive, the tube will almost instantly ionize and conduct current at 15 volts arc drop.

Two tubes *B* are series connected in each ignitron ignition circuit and both, therefore, must be energized before the igniter-type tube can "fire." The 2 tubes *B* are respectively controlled by the inductive timer previously described and designated as components *C*, *D*, *E*, and *F* in figure 4, and the welding-heat control circuit containing elements *G*, *H*, *I*, *J*, *K*, and *R* in figure 4.

The function of the inductive timer is to control the welding current in cycles "on" and cycles "off." This it does by delivering a positive voltage pulse to the grids of the lower of tubes *B* as each timing pin (plugged into disk *D*) passes through the gap of magnet *C*. This occurs for each half-cycle, whether positive or negative for which welding current is desired.

The function of the welding-heat control circuit is to control accurately the point of fire of the igniter-type tube on the voltage wave and thereby the magnitude of the welding-current loop. To accomplish this an alternating voltage of controllable lagging phase is delivered to the grids of upper tubes *B*. It is apparent that the grids of these tubes will first assume positive potential at that angle on the line-voltage wave to which the control voltage is out of phase. This form of grid control applied to gaseous tubes is known as "phase-shift control" and is commonly applied to rectifiers for variation of d-c output. A second circuit function of the heat-control circuit is to provide a slight, although also controllable, phase differential in the voltage delivered to the 2 grid transformers *J* and *K*. The purpose of this feature is to provide compensation for perfect "balance" (absence of any d-c component in the welding current) between the 2 ignitrons which may differ slightly in arc drop.

The phase-lagged voltage of the heat control circuit is obtained from midtap of the supply transformer and slider of heat-control potentiometer *H*. Current flowing through reactor *G* and resistor *R*, series connected across the supply transformer, furnishes quadrature voltages to the respective halves of midtapped potentiometer *H*. It is apparent that the phase-lagged voltage of this circuit may be varied from 0 to 180 degrees lag depending at which end of potentiometer *H* the slider is set. With the slider at mid position on *H*, the control voltage will lag 90 degrees since the reactance ohms of *G* equal the resistance ohms of *R*.

When the slider of "balance" potentiometer *I* is at mid position, grid transformers *J* and *K* deliver to the grids of respectively controlled tubes *B*, voltages exactly 180 degrees apart in phase as also are the anode voltages of these tubes. When, however, the slider of *I* is at the *J* end of the potentiometer, transformer *J* will be supplied directly while the entire resistance of *I* will be contained in the supply of primary voltage to *K*. Transformer *K*, as well as *J*, is designed to demand sinusoidal magnetizing current which current, in flowing through *I*, must somewhat lag the voltage actually supplied to *K* with respect to that of *J*. In this manner, the output voltage of transformer *K* can be made either to lag or lead that of *J* and hence either igniter-type tube may be delayed with respect to its companion tube, compensating for possible slightly lower arc drop or a few degrees more prompt ignition.

In practice the d-c type zero-center galvanometer *M* is used to denote presence of any d-c component in the alternating load current flowing in shunt *L*. Through "balance" potentiometer *I* this d-c component is then "tuned" out.

Conditions necessary to firing of an igniter-type tube are therefore, first, that the anode is momentarily of positive potential with respect to the mercury pool cathode; second, that the inductive timer has supplied positive potential to the grid of the lower tube *B;* third, that the heat-control circuit has applied positive potential to the grid of upper tube *B;* and fourth, that the control relay is closed. With these 4 conditions satisfied, ignition current will flow from igniter to pool, in the particular tube, whereupon it will fire and pass line current to the welding machine. In so doing, the low arc drop of the tube will be insufficient to force further unneeded ignition current through tubes *B* and the igniter. Once fired, the tube will carry current to the end of the current loop.

In practice, the angular position of magnet *C* is so set in relation to the timing disk as to deliver positive voltage to the lower tube *B* well in advance of the electrical angle at which the heat-control circuit, at full output, will energize the upper tubes *B*. Lower tube *B*, at passage of a timing pin, is ionized, and so remains until the series tube is ionized, being supplied meanwhile by subignition current through the resistor connected in shunt to the upper tube *B*. This latter tube is eventually also energized by the heat-control circuit when ignition of the particular ignitron actually occurs.

Control relay *N* establishes the 2 ignition circuits which thereafter respond to the timing in cycles "on" and "off" as preset by the pin arrangement of the timing disk and to the "heat" setting as preset by potentiometer *H*.

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Negative-Sequence Reactance of Synchronous Machines

A theoretical analysis of the method proposed by the AIEE committee on electrical machinery for measuring the negative-sequence reactance of synchronous machines is presented in this paper. The analysis proves that the method leads to a correct measurement of the negative-sequence reactance for the operating condition most frequently met in applying the method of symmetrical components, but that the method of measurement does not determine the correct negative-sequence reactance of the machine for the operating conditions used in the proposed AIEE test. The results of experimental tests are given to verify the theory presented.

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ONE of the characteristic coefficients employed in the theory of symmetrical components that has proved very useful in calculating unbalanced currents arising from short circuits is the negative-sequence reactance, usually represented by x_2. The AIEE committee on electrical machinery has proposed a test procedure[1] for measuring the negative-sequence reactance of synchronous machines. This method of measurement is open to question, however, since it is valid only for sine-wave voltages and currents, and the voltage and current in the machine under the conditions of measurement are not sinusoidal.

Doherty and Nickle[2] have shown that the short-circuit current and the open-phase voltage of a synchronous machine operating with a single-phase short circuit in steady state may contain all odd harmonics, and that the magnitudes of these harmonics are in a known geometric ratio in ascending order. Since a-c instruments indicate the effect of all harmonics present in the measured quantities, the measurements made under the condition referred to, which is employed in the AIEE method, might be incorrect and thus introduce an error into the determination of negative-sequence reactance. The purpose of this paper is to analyze the proposed AIEE method, show precisely what is measured by means of such a method, and to suggest a rational procedure in determining the quantity x_2.

Summary and Conclusions

The experimental tests are shown to verify in detail the theory presented. The theory can be accepted as correct then, and the following conclusions drawn:

1. The AIEE method actually measures a reactance $^1/_2(x_d'' + x_q'')$, the arithmetic mean of the subtransient reactances. This is the correct value of x_2 for the machine when it is operating with a relatively large external reactance—the condition most commonly met in the practical application of the method of symmetrical components.

2. The AIEE method does not, however, give the correct x_2 for the operating condition used, that is, a single-phase line-to-line short circuit, for which $x_2 = \sqrt{x_d''x_q''}$

3. The correct x_2 for the single-phase line-to-line short circuit may be obtained from the value of x_2 measured in accordance with the AIEE method,

$$\sqrt{x_d''x_q''} = \frac{1 - b^2}{1 + b^2} \cdot x_2\,(\text{AIEE})$$

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1. For all numbered references, see list at end of paper.

where

$$b = \frac{\sqrt{x_q''} - \sqrt{x_d''}}{\sqrt{x_q''} + \sqrt{x_d''}}$$

4. If the fundamental components of the quantities measured by means of the AIEE method are used, a correct value of x_2, namely $\sqrt{x_d''x_q''}$, is determined for that operating condition.

5. The AIEE method is strictly correct when the subtransient reactance is the same for any position of the poles. This condition may be approached in a salient-pole machine having a complete squirrel-cage winding of low resistance, for which x_d'' is approximately equal to x_q'' and b is approximately equal to zero.

6. It is recommended that x_d'' and x_q'' be measured and x_2 be calculated for the case at hand, from the equations given by Park and Robertson.[5] If x_d'' and the AIEE x_2 are known, x_q'' is determined, since x_2 (AIEE) $= \frac{1}{2}(x_d'' + x_q'')$.

The Nature of Negative-Sequence Reactance

For circuits with constant parameters, such as ordinary coils and transmission lines, a linear current-voltage relation that defines the negative-sequence reactance of the circuit exists; that is, negative-sequence reactance consumes negative-sequence reactive voltage when negative-sequence current flows. In synchronous machinery, certain circuit parameters, namely, reactances that vary with the pole position, are not constant and the definition of negative-sequence reactance is much more complex.

The AIEE standards,[3] which agree with the Wagner and Evans text,[4] define the negative-sequence reactance of a synchronous machine as "the ratio of the fundamental component of reactive voltage, due to the fundamental negative-sequence component of armature current, to this component of armature current at rated frequency."

Park and Robertson[5] have shown that the value of the negative-sequence reactance of a synchronous machine, in terms of the direct- and quadrature-axis reactances, depends on the external circuit. They give values of x_2 as follows:

When a short circuit occurs at the machine terminals

$$x_2 = \sqrt{x_d''x_q''} \tag{1}$$

When sinusoidal negative-sequence voltage is maintained at the machine terminals

$$x_2 = \frac{2x_d''x_q''}{x_d'' + x_q''} \tag{2}$$

When the machine is operated with a relatively large external reactance in series with the armature

$$x_2 = \tfrac{1}{2}(x_d'' + x_q'') \tag{3}$$

where

x_d'' is the 3-phase subtransient reactance, direct axis.
x_q'' is the 3-phase subtransient reactance, quadrature axis.

The AIEE definition of x_2 implies that fundamental negative-sequence current is circulated in the machine, which would represent a special operating condition obtained by connecting a large external reactance in series with the armature circuit. For this case $x_2 = \frac{1}{2}(x_d'' + x_q'')$. On the contrary, the AIEE method of measuring x_2 is based on the theory of symmetrical components, not on the AIEE definition of x_2. It so happens, as demonstrated, that the AIEE method actually measures the correct x_2 for the condition (large external reactance) specified by the definition, that is, $\frac{1}{2}(x_d'' + x_q'')$, although it does not measure the correct x_2 for the operating condition actually used in the AIEE test; for this, $x_2 = \sqrt{x_d''x_q''}$.

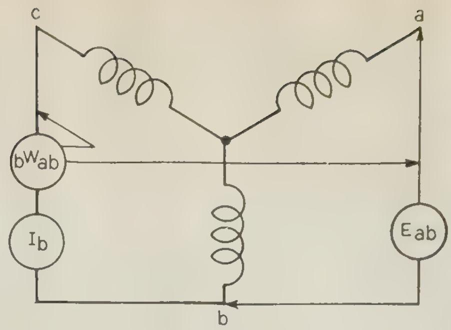

Fig. 1. Diagram of circuit used for measuring sustained armature current and short-circuit line voltages as required by the AIEE method of determining negative-sequence reactance

It must be recognized that no single value of x_2 will suffice for all operating conditions. It is shown later that if either x_d'' and x_q'', or x_d'' and the AIEE x_2 are known, the correct x_2 can be calculated by means of the Park and Robertson expressions for the case at hand.

AIEE Method of Measuring Negative-Sequence Reactance of a Synchronous Machine

The AIEE committee on electrical machinery has proposed the following method[1] to measure the negative-sequence reactance of a synchronous machine. With the machine driven at rated speed and a single-phase line-to-line short circuit applied, the sustained armature current and the potential between the open line and either short-circuited line are measured (see figure 1). At the same time, the reading of a single-phase wattmeter, of which the current coil is connected in series with the short circuit and the potential coil is excited by the open-to-short-circuit line voltage, is obtained.

From these measurements x_2 can be calculated from the expression

$$x_2 = \frac{E_{ab}}{\sqrt{3}I_b} \cdot \frac{{}_bW_{ab}}{E_{ab}I_b} \tag{4}$$

In proposing this method the committee has recognized that harmonics might introduce errors into the measurement. Since the real effect of the harmonics was not known, the committee simply suggested that oscillograms be taken in special cases, although it does not indicate how the oscillograms are to be used.

Analysis of the AIEE Method by Means of Synchronous-Machine Theory

Expressions can be derived from fundamental synchronous machine theory for all the quantities measured in the AIEE method. If these equations, which account for all harmonics, are substituted in equation 4, the quantity actually measured by this method may be found. In this way the effect of the harmonics upon the measurement is determined.

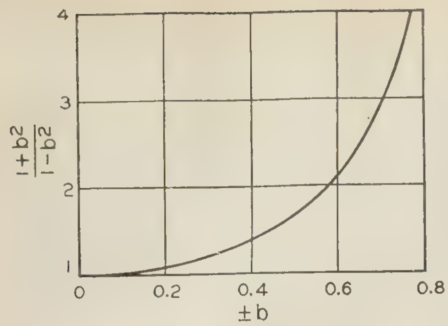

Fig. 2. Curve showing the variation with b of the function $\dfrac{1 + b^2}{1 - b^2}$

For a single-phase line-to-line short circuit the sustained armature current is given by Doherty and Nickle[2] as

$$i = \frac{\sqrt{3}e_0}{x_d + \sqrt{x_d''x_q''}}(\cos\theta + b\cos 3\theta + b^2\cos 5\theta + \ldots) \quad (5)$$

where

$$b = \frac{\sqrt{x_q''} - \sqrt{x_d''}}{\sqrt{x_q''} + \sqrt{x_d''}} \quad \text{(see appendix II)}$$

The effective value of this current is given as

$$I = \frac{\sqrt{3}E_0}{x_d + \sqrt{x_d''x_q''}} \cdot \frac{1}{\sqrt{1 - b^2}} \quad (6)$$

An expression for the voltage between the open- and short-circuited lines can be found by applying the constant linkage theorem[6] to the generalized armature linkage expressions[7] for this short circuit. From equation $A13$ of appendix I,

$$e_{ab} = \frac{3e_0\sqrt{x_d''x_q''}}{x_d + \sqrt{x_d''x_q''}}(\cos\theta + 3b\cos 3\theta + 5b^2\cos 5\theta + \ldots) \quad (7)$$

The effective value of the voltage represented by equation 7 is (from appendix I)

$$E_{ab} = \frac{3E_0\sqrt{x_d''x_q''}}{x_d + \sqrt{x_d''x_q''}}\sqrt{\frac{1 + 6b^2 + b^4}{(1 - b^2)^3}} \quad (8)$$

The wattmeter reading ${}_bW_{ab}$ will indicate the average product of the current (equation 5) and the voltage (equation 7). From appendix I,

$${}_bW_{ab} = \frac{3\sqrt{3}E_0^2\sqrt{x_d''x_q''}}{(x_d + \sqrt{x_d''x_q''})^2} \cdot \frac{1 + b^2}{(1 - b^2)^2} \quad (9)$$

Equations 5, 7, and 9, derived in appendix I, are subject to certain assumptions, namely:

1. Negligible saturation. This is, of course, not true practically; however, reactances can be defined precisely only for the unsaturated case. Since this paper deals only with reactances the assumption is a valid one.

2. The open-circuit voltage is a sine wave. This is practically true in many present-day alternators as there are numerous schemes used to effect it, such as pitching, distributing, and connecting the armature phases in star.

3. The machine is constructed symmetrically, that is, the armature phases are identical in all respects; each pole possesses symmetry about its axis and all poles are identical in shape and winding.

4. The machine has no electrical resistance. This refers to the armature and field winding resistance and to the resistance of any additional rotor circuits such as amortisseur windings, field collars, and eddy-current paths.

In order to simplify the problem, a symmetrical machine with unsaturated reactances and zero resistance is considered.

The AIEE expression for x_2 (equation 4) must be valid regardless of the resistance of the machine and therefore is valid for the special case of zero resistance. The fundamental equations 6, 8, and 9 can be substituted in the AIEE expression (equation 4):

$$x_2 = \frac{\dfrac{3\sqrt{3}E_0^2\sqrt{x_d''x_q''}}{(x_d + \sqrt{x_d''x_q''})^2} \cdot \dfrac{1 + b^2}{(1 - b^2)^2}}{\sqrt{3}\left[\dfrac{\sqrt{3}E_0}{x_d + \sqrt{x_d''x_q''}}\right]^2 \cdot \dfrac{1}{1 - b^2}}$$

which reduces to

$$x_2 = \frac{1 + b^2}{1 - b^2}\sqrt{x_d''x_q''} \quad (10)$$

It has been shown by equation 1 that the correct x_2 for the AIEE operating case is $\sqrt{x_d''x_q''}$. Accordingly, the term $(1 + b^2)/(1 - b^2)$ in equation 10 is a measure of the error introduced into the AIEE method by the harmonics in the measured quantities. The AIEE method, then, does not measure the correct x_2 for the operating condition actually employed by the method, that is, $\sqrt{x_d''x_q''}$.

The reactance actually measured in the laboratory can be predicted from the values of subtransient reactance, for

$$x_2\,(\text{AIEE}) = \sqrt{x_d''x_q''} \cdot \frac{1 + b^2}{1 - b^2} \quad (11)$$

A curve of $(1 + b^2)/(1 - b^2)$ can be plotted for all values of b, and from this curve $(1 + b^2)/(1 - b^2)$ can be found once b is known for any machine (see figure 2).

If the expression for b in terms of the subtransient reactances

$$b = \frac{\sqrt{x_q''} - \sqrt{x_d''}}{\sqrt{x_q''} + \sqrt{x_d''}}$$

is substituted in equation 10,

$$x_2 = {}^1/_2(x_d'' + x_q'') \quad (12)$$

Thus the AIEE method actually measures the arithmetic mean of the subtransient reactances, which, as already indicated, is the correct value of x_2 for operation with a arge external reactance in the armature circuit.

Short circuits occurring at the machine terminals can be calculated directly by the Doherty and Nickle equations. On the contrary, the method of symmetrical components is most commonly used in practice for the solution of power-system short circuits. In these cases, the short circuits would occur with a large external reactance in series with the machine, and the correct x_2 would be ${}^1/_2(x_d'' + x_q'')$. The AIEE method (see equation 12) measures the arithmetic mean of the subtransient reactances, and thus would give the correct x_2 for the most common use of the method of symmetrical components.

If it is desired to use the method of symmetrical components for other cases than for which $x_2 = {}^1/_2(x_d'' + x_q'')$, the proper x_2 must be calculated from the equations given by Park and Robertson.[5]

In suggesting that oscillograms be taken in connection with the measurements by the AIEE

Table I—Reactance Measurements and Calculations

1 Machine	2 b	3 x_d'' in Ohms	4 x_q'' in Ohms	5 $\frac{x_d''+x_q''}{2}$ in Ohms	6 x_2 (AIEE) in Ohms	7 $\sqrt{x_d''x_q''}$ in Ohms	8 $\frac{1+b^2}{1-b^2}$	9 x_2 (AIEE) Corrected in Ohms	10 x_2 Fundamental Components in Ohms
1	0.50	1.38	12.9	7.14	7.16	4.22	1.69	4.24	4.33
	0.47	1.36	10.4	5.88	6.06	3.76	1.57	3.86	4.18
	0.28	1.34	4.24	2.79	2.71	2.38	1.18	2.30	2.37
	0.06	10.26	13.06	11.66	11.42	11.58	1.005	11.42	11.9
	0.01	12.68	13.25	12.96	13.05	12.98	1.00	13.05	12.8
	−0.004	1.35	1.37	1.36	1.35	1.36	1.00	1.35	14.2
	−0.149	2.45	1.34	1.90	1.90	1.81	1.05	1.81	1.86
	−0.468	10.23	1.35	5.79	5.82	3.72	1.57	3.71	3.94
	−0.503	12.34	1.35	6.85	6.70	4.08	1.68	3.99	4.28
2	0.234	1.08	2.81	1.95	1.92	1.74	1.12	1.71	1.75
3	0.28	0.592	1.87	1.23	1.20	1.05	1.18	1.02	1.14
4 A	0	0.272	0.260	0.266	0.259	0.266	1.00	0.259	0.292
4 B	0.252	0.437	1.226	0.832	0.817	0.731	1.14	0.716	0.74

4*A*—With amortisseur winding 4*B*—Without amortisseur winding

method, the committee has implied that the oscillograms be analyzed for the fundamental components of the measured voltage and current and that these components be used in the reactance calculations.

If the fundamental terms are taken from equations 5, 7, and 9 and substituted in equation 4, x_2 becomes

$$x_2 = \frac{\dfrac{3E_0\sqrt{x_d''x_q''}}{x_d + \sqrt{x_d''x_q''}}}{\dfrac{\sqrt{3}\cdot\sqrt{3}E_0}{x_d + \sqrt{x_d''x_q''}}} \cdot \frac{\dfrac{3\sqrt{3}E_0^2\sqrt{x_d''x_q''}}{(x_d + \sqrt{x_d''x_q''})^2}}{\dfrac{\sqrt{3}E_0}{x_d + \sqrt{x_d''x_q''}} \cdot \dfrac{3E_0\sqrt{x_d''x_q''}}{x_d + \sqrt{x_d''x_q''}}} = \sqrt{x_d''x_q''} \tag{13}$$

Since the correct value of x_2 for the single-phase line-to-line short circuit is $\sqrt{x_d''x_q''}$, this method would determine the correct x_2 for the conditions of the test but not the same x_2 as would be found from the meter readings.

In a machine having $x_d'' = x_q''$, b would be zero and the armature current and the open-line voltage with a sustained single-phase short circuit applied would be sinusoidal. In such a case the measurement of x_2 by the AIEE method would be strictly correct and the equations of Park and Robertson would reduce to

$x_2 = x_d'' = x_q''$

Experimental Confirmation of the Theoretical Results

Experimental tests were made to verify the theory presented. These tests show by actual measurement that:

1. The AIEE method determines the arithmetic mean of x_d'' and x_q''.
2. As measured by the AIEE method, x_2 can be corrected to $\sqrt{x_d''x_q''}$ with the factor $\frac{1+b^2}{1-b^2}$.
3. $\sqrt{x_d''x_q''}$ is the correct value of x_2 for the single-phase line-to-line short circuit on the machine.

In satisfying these conditions, 2 types of tests were made: first, measurements of machine reactances; second, measurements of short-circuit currents and open-line voltages for a single-phase line-to-line short circuit. In order to gain a fair representation of practical conditions, the tests were made on a variety of machines. These were:

Machine 1. Westinghouse wound rotor induction motor rated at 10 horsepower, 220 volts, 60 cycles, 1,200 rpm, 3 phase.

Machine 2. General Electric synchronous motor, rated at 15 kva, 220 volts, 60 cycles, 3 phase, 1,800 rpm, with a pole face amortisseur winding.

Machine 3. General Electric synchronous motor rated at 9 kva, 220 volts, 60 cycles, 3 phase, 1,800 rpm, with a complete amortisseur winding.

Machine 4. General Electric synchronous motor rated at 125 horsepower, 440 volts, 60 cycles, 3 phase, 900 rpm, used with and without an amortisseur winding.

Machine 1 had a 3-phase winding on the rotor and a 2-phase winding on the stator. The 3-phase winding was used as armature and the 2-phase winding as field, giving in effect a field winding on both the direct and quadrature axes. When the direct-axis field was excited with the quadrature-axis field open-circuited, x_q'' was greater than x_d'' and b was about 0.5. With the direct-axis field excited and the quadrature-axis field short-circuited x_d'' was equal to x_q'' and b was equal to zero. When the direct-axis field was excited through a large reactance and the quadrature-axis field was short-circuited, x_d'' was greater than x_q'' and b was about −0.5. The value of b could be adjusted to any value from +0.5 to −0.5 by properly connecting the field windings and the external reactance. This machine could thus be given electrical characteristics equivalent to many machines; in this case 9 sets were used.

Reactance Measurements. The subtransient reactances were measured in each case by the static method.[5] The negative-sequence reactances were measured and calculated in each case by the AIEE method. The measured values of reactance for each case are listed in table I.

From the measured data certain combinations of the subtransient reactances were calculated: first, the arithmetic mean of x_d'' and x_q''; second, the geometric mean $\sqrt{x_d''x_q''}$; and third, the value of $b = \dfrac{\sqrt{x_q''} - \sqrt{x_d''}}{\sqrt{x_q''} + \sqrt{x_d''}}$. The correction factor $\dfrac{1+b^2}{1-b^2}$

Table II—Armature Current and Open-Line Voltage for Sustained Single-Phase Line-to-Line Short Circuits

1	2	3	4	5	6	7	8	9	10	11	12	13
					I_b Total Current	I_1 in Amperes			V Total	V_1 in Volts		
						Calculated		Measured	Open Line Voltage	Calculated		Measured
Machine	x_1 in Ohms	$\sqrt{x_d''x_q''}$ in Ohms	x_2 (AIEE) in Ohms	$\sqrt{3}E_g$ in Volts	Measured in Amperes	$\sqrt{x_d''x_q''}$	x_2 (AIEE)			$\sqrt{x_d''x_q''}$	x_2 (AIEE)	
	14.45	4.22	7.16	62	3.98	3.32	2.85	3.37	58.8	24.3	35.6	25.3
	14.45	3.76	6.06	63	4.02	3.46	3.07	3.55	48.8	22.5	32.2	25.7
	14.43	2.38	2.71	72	4.55	4.28	4.20	4.21	25.1	17.7	19.7	17.3
	14.42	11.58	11.42	96	3.70	3.69	3.71	3.66	77.0	74.1	73.5	75.4
1	14.42	12.98	13.05	112	4.10	4.08	4.00	4.06	96.0	91.8	92.1	90.0
	14.42	1.36	1.35	74	4.75	4.69	4.70	4.75	12.6	11.1	11.0	12.0
	14.43	1.81	1.90	57	3.65	3.50	3.49	3.47	13.5	11.0	11.5	11.2
	14.45	3.72	5.82	75	4.85	4.13	3.70	4.06	60.5	26.6	37.3	27.7
	14.45	4.08	6.7	67	4.45	3.62	3.17	3.66	64.8	25.6	36.8	27.1
2	7.62	1.74	1.92	120	13.8	12.7	12.2	13.4	48.5	40.1	46.7	40.7
3	5.08	1.05	1.20	74.5	12.25	12.15	11.85	12.0	29.8	22.1	24.7	23.0
4 A	1.64	0.266	0.259	98.5	51.2	51.8	51.8	51.2	25.9	23.3	23.9	25.9
4 B	1.64	0.731	0.817	60	26.5	25.3	24.4	25.6	42.8	32.1	34.6	32.8

Values of b can be found in table I   4*A*—With amortisseur winding   4*B*—Without amortisseur winding

found from figure 2 for the proper value of b, was then applied to the value of x_2, obtained by the AIEE measurements, to give the expression $\sqrt{x_d''x_q''}$; that is, calculations were made according to the equation $\sqrt{x_d''x_q''} = x_2 \text{ (AIEE) } (1 - b^2)/(1 + b^2)$.

The measured and calculated values of reactance for the various machines are listed in table I. In column 10 of table I are listed the values of x_2 determined from the fundamental components of the quantities measured in the AIEE method; the data for this calculation may be found in the appropriate columns of table II.

It can readily be noted that the results listed in column 5 of table I, values of $\frac{1}{2}(x_d'' + x_q'')$, check closely with those in column 6, values of x_2 (AIEE). This comparison shows that the AIEE method actually measures a reactance $\frac{1}{2}(x_d'' + x_q'')$, as was indicated in the theoretical analysis, equation 12. The average discrepancy in this comparison is less than 2 per cent.

Column 9 of table I which represents each measurement of x_2 made by the AIEE method divided by $(1 + b^2)/(1 - b^2)$, compares closely with column 7, calculations of $\sqrt{x_d''x_q''}$. This comparison verifies again the correctness of the theory presented (see equation 10) and presents a simple means of correcting the AIEE x_2 to values of $\sqrt{x_d''x_q''}$, should they be desired.

Column 10 of table I shows values of x_2 calculated from the fundamental components of the voltage and current measured in the AIEE method. The comparison of column 10 to column 7 verifies the theory presented in equation 13 and indicates a means of determining directly the correct value of x_2 for the operating case used in the AIEE method, that is, $x_2 = \sqrt{x_d''x_q''}$. The average discrepancy in this comparison is 4 per cent; this larger discrepancy is partly explained by the fact that the values in column 10 were calculated from oscillograms that could be analyzed within only 5 per cent.

Short Circuit Currents and Open Line Voltages. The second phase of the experimental tests was the actual measurement of short-circuit current and open-line voltages. These tests were made to verify the theoretical analysis in a slightly different manner.

Measurements were made of the sustained short-circuit current and the open-line voltage, with the machine operating with a single-phase line-to-line short circuit (see figure 1). Oscillograms, as well as meter readings, were taken of the voltage and current. The oscillograms were analyzed for their fundamental components by Fourier's method. All these measurements are listed in table II.

The fundamental component of armature current and the fundamental component of the open-line voltage were calculated for each case by means of the theory of symmetrical components. The expressions used[4] for the special case of zero resistance were

$$I_1 = \frac{\sqrt{3}\,E_0}{x_1 + x_2}$$

and

$$E_1 = \frac{3x_2E_0}{x_1 + x_2}$$

The calculations were made using 2 values of x_2, as calculated from $\sqrt{x_d''x_q''}$, and as measured by the AIEE method. In order to simplify the analysis all these calculations were made for the case of zero resistance.

Table II lists for the various tests the measured and calculated values of short-circuit current and open-line voltage and their fundamental components.

Referring to table II, it will be noted that column 7, calculated values using $x_2 = \sqrt{x_d''x_q''}$, checks column 9, the measured values; column 8, calculated using the AIEE x_2, does not, except for the case when b is equal to zero. When b is equal to zero, the 2 values of x_2 become equal and all 3 columns agree. In a similar manner, the measured and calculated values of open-line voltage agree: column 11, values of open-line voltage calculated using $x_2 = \sqrt{x_d''x_q''}$, agrees with the measured values in column 13; column 10, values calculated

using x_2(AIEE), does not, except when b equals zero.

The comparison of these test and calculated values (table II) indicates that the correct x_2 for a single-phase line-to-line short circuit occurring directly at the machine terminals is $\sqrt{x_d''x_q''}$. Thus, for the operating condition used by the AIEE method, experiment shows that $x_2 = \sqrt{x_d''x_q''}$, as given by Park and Robertson.[5] From this it can be deduced that the AIEE method does not measure the correct x_2 for the operating case used.

Assumptions and Errors in Measurements. All the test measurements were made at reduced voltages in order to eliminate the effect of saturation. The voltage and current readings were plotted and the impedance corresponding to the straight portion of the saturation curve was calculated. In this way, the assumption of no saturation was fulfilled.

On the machines tested, the no-load voltage wave, except for tooth harmonics, was close to a sine wave.

Naturally the machine circuits contained resistance, but by employing a wattmeter with each reactance measurement the true reactive component of the impedance was obtained. Thus, in checking the theory presented, the assumption of zero resistance was duly recognized. The resistance was neglected in the calculation of the actual single-phase short circuits (table II). In the machines tested, the resistance was never more than 20 per cent of the corresponding reactance and thus neglecting the resistance would introduce only a 2 per cent error in the calculation of impedance and current.

In some cases duplications of the reactance measurements were made after an interval. The measured data could be reproduced to within 2 or 3 per cent, which is slightly greater than the discrepancy noted in the comparison of results. In other words, the comparisons are well within the accuracy to be expected in laboratory measurements of these quantities.

Appendix I—Derivation of the Equations From Fundamental Theory of Synchronous Machines

Generalized Armature-Linkage Expressions

The generalized armature linkage expressions for an ideal 3-phase synchronous machine as given by Park,[7] are:

$$\left.\begin{aligned}
\psi_a &= \frac{l_d+l_q}{3}\left[i_a - \frac{i_b+i_c}{2}\right] + \frac{l_d-l_q}{3}[i_a\cos 2\theta' + \\
&\qquad i_b\cos(2\theta'-120°) + i_c\cos(2\theta'+120°)] + \\
&\quad \frac{l_0}{3}[i_a+i_b+i_c] + M_dI_d\cos\theta' - M_qI_q\sin\theta' \\
\psi_b &= \frac{l_d+l_q}{3}\left[i_b - \frac{i_a+i_c}{2}\right] + \frac{l_d-l_q}{3}[i_a \times \\
&\quad \cos(2\theta'-120°) + i_b\cos(2\theta'+120°) + i_c\cos 2\theta'] + \\
&\quad \frac{l_0}{3}[i_a+i_b+i_c] + M_dI_d\cos(\theta'-120°) - M_qI_q\sin(\theta'-120°) \\
\psi_c &= \frac{l_d+l_q}{3}\left[i_c - \frac{i_a+i_b}{2}\right] + \frac{l_d-l_q}{3}[i_a \times \\
&\quad \cos(2\theta'+120°) + i_b\cos 2\theta' + i_c\cos(2\theta'-120°)] + \\
&\quad \frac{l_0}{3}[i_a+i_b+i_c] + M_dI_d\cos(\theta'+120°) - M_qI_q\sin(\theta'+120°)
\end{aligned}\right\} \quad \text{(A1)}$$

where θ' is the angular position of the direct axis referred to the axis of phase a.

These equations represent the instantaneous flux linkages in the respective phase windings with any currents flowing in the armature. The ideal synchronous machine, to which these equations apply, is characterized by the assumptions discussed in the body of the paper.

Single-Phase Line-to-Line Short Circuit

The AIEE method of measuring x_2 specifies a sustained single-phase line-to-line short circuit on the machine. For this condition of operation (see figure 3):

$$\left.\begin{aligned}
i_a &= 0 \\
i_b &= -i_c = i \\
I_0 &= \text{field current before short circuit, corresponding to} \\
&\quad \text{nominal voltage } e_0
\end{aligned}\right\} \quad \text{(A2)}$$

At the instant after the short circuit occurs, the armature current is given by Doherty and Nickle as

$$i = \frac{-\sqrt{3}\,e_0}{x_d'' + \sqrt{x_d''x_q''}}[\cos\theta + b\cos 3\theta + b^2\cos 5\theta + \ldots] \quad \text{(A3)}$$

where

$$b = \frac{\sqrt{x_q''} - \sqrt{x_d''}}{\sqrt{x_q''} + \sqrt{x_d''}}$$

and θ is the angular position of the rotor with respect to the axis of the 2 short-circuited phases. This expression does not include the

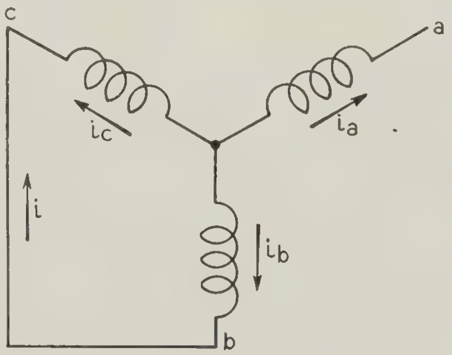

Fig. 3. Diagrammatic representation of a synchronous machine undergoing a single-phase line-to-line short circuit

effect of any initially trapped linkages which would eventually die out and in which there is no interest in this analysis.

Doherty and Nickle have shown that as steady state is reached the short-circuit armature current decreases in the same ratio as the field current, which increases at short circuit in the ratio

$$\frac{x_d + \sqrt{x_d''x_q''}}{x_d'' + \sqrt{x_d''x_q''}} \quad \text{(A4)}$$

The steady-state armature current becomes

$$i_s = \frac{-\sqrt{3}\,e_0}{x_d + \sqrt{x_d''x_q''}}[\cos\theta + b\cos 3\theta + b^2\cos 5\theta + \ldots] \quad \text{(A5)}$$

The effective value of i_s in equation $A5$ can be calculated, for it is known that for a harmonic series of currents[8]

$$I_{eff} = \frac{1}{\sqrt{2}}\sqrt{I^2_m + I^2_{2m} + I^2_{3m}}$$

Therefore

$$I = \frac{\sqrt{3}\,e_0}{\sqrt{2}(x_d + \sqrt{x_d''x_q''})}\sqrt{1 + b^2 + b^4 + b^6 + \ldots} = \frac{\sqrt{3}\,E_0}{x_d + \sqrt{x_d''x_q''}}\cdot\frac{1}{\sqrt{1-b^2}} \quad \text{(A6)}$$

Open Line Voltage

The open line voltage e_{ba} can be found from the linkages ψ_{ba} by Faraday's law

$$e_{ba} = -\frac{d\psi_{ba}}{dt} \tag{A7}$$

in which ψ_{ba} represents the linkages included in phases a and b and can be expressed as

$$\psi_{ba} = \psi_a - \psi_b \tag{A8}$$

Substituting the current relations of equation $A2$ in the linkage expressions $A1$,

$$\psi_a = \frac{l_d'' - l_q''}{3}[i\cos(2\theta' - 120°) - i\cos(2\theta' + 120°)] + M_dI_0\cos\theta'$$

and

$$\psi_b = \frac{l_d'' + l_q''}{3}\left[\frac{3i}{2}\right] + \frac{l_d'' - l_q''}{3}[i\cos(2\theta' + 120°) - i\cos\theta'] + M_dI_0\cos(\theta' - 120°)$$

The subtransient inductances are used here to account for the induced currents in all the rotor circuits.

Subtracting the expression for ψ_b from that for ψ_a,

$$\psi_{ba} = -i\frac{(l_d'' + l_q'')}{2} - i(l_d'' - l_q'')\cos(2\theta' + 120°) + \sqrt{3}M_dI_0\sin(\theta' + 120°) \tag{A9}$$

This equation must be transformed to the same reference axis used in equation $A3$; that is, the axis of phases b and c in series carrying the same current. The transformation is:

$$\theta' = \theta + 90°$$

where θ is the pole position referred to the axis of b and c in series and θ' is the pole position referred to the axis of phase a. Then the expression for ψ_{ba} becomes

$$\psi_{ba} = -i\frac{(l_d'' + l_q'')}{2} + i(l_d'' - l_q'')\cos(2\theta + 120°) + \sqrt{3}M_dI_0\cos(\theta + 120°) \tag{A10}$$

where i represents the current given in equation $A3$.

If the current given by equation $A3$ is substituted for i in equation $A10$ and account is taken of the change in field current given by $A4$, the linkage expression becomes

$$\psi_{ba_s} = \frac{l_d'' + l_q''}{2}\cdot\frac{\sqrt{3}\,e_0}{x_d + \sqrt{x_d''x_q''}}(\cos\theta + b\cos 3\theta + b^2\cos 5\theta + \ldots) + \sqrt{3}M_dI_0\cdot\frac{x_d'' + \sqrt{x_d''x_q''}}{x_d + \sqrt{x_d''x_q''}}\cos(\theta + 120°) + \frac{l_d'' - l_q''}{4}\cdot\frac{\sqrt{3}\,e_0}{x_d + \sqrt{x_d''x_q''}}\left\{\begin{matrix}[(1+b)\cos\theta + (1+b^2)\cos 3\theta + b(1+b^2)\cos 5\theta + \ldots] \\ [\sqrt{3}(1-b)\sin\theta + \sqrt{3}(1-b^2)\sin 3\theta + \sqrt{3}b(1-b^2)\sin 5\theta + \ldots]\end{matrix}\right\} \tag{A11}$$

The voltage e_{ba} can be found from the linkages ψ_{ba} by differentiating with respect to time, since $\theta = \omega t + \alpha$. Differentiating equation $A11$ and collecting similar terms,

$$e_{ba} = \frac{3e_0}{2}(1+b)\left[\frac{x_d'' + \sqrt{x_d''x_q''}}{x_d + \sqrt{x_d''x_q''}}\right][\cos\theta + 3b\cos 3\theta + 5b^2\cos 5\theta + \ldots] \tag{A12}$$

This equation can be simplified by employing the definition of b in terms of reactance, that is

$$b = \frac{\sqrt{x_q''} - \sqrt{x_d''}}{\sqrt{x_q''} + \sqrt{x_d''}}:$$

$$e_{ba} = \frac{3e_0\sqrt{x_d''x_q''}}{x_d + \sqrt{x_d''x_q''}}(\cos\theta + 3b\cos 3\theta + 5b^2\cos 5\theta + \ldots) \quad \text{(A1}$$

Equation $A13$ is an expression for the voltage between lines a ar b when the machine is operated with a sustained single-phase sho circuit between lines b and c, with the assumption of zero resistan in the machine.

The effective value of e_{ba} is

$$E_{ba} = \frac{3E_0\sqrt{x_d''x_q''}}{x_d + \sqrt{x_d''x_q''}}\sqrt{1 + 9b^2 + 25b^4 + 49b^6 + \ldots} \quad \text{(A1}$$

If the series is summed, equation $A14$ reduces to

$$E_{ba} = \frac{3E_0\sqrt{x_d''x_q''}}{x_d + \sqrt{x_d''x_q''}}\sqrt{\frac{1 + 6b^2 + b^4}{(1-b^2)^3}} \quad \text{(A1}$$

Wattmeter Indication

The wattmeter used in the AIEE method is excited with tl open-line voltage e_{ba} and the short-circuit current i_b. Its indic tion, ${}_bW_{ba}$, is the average product of the instantaneous values e_{ba} and i_b:

$${}_bW_{ba} = \frac{1}{\pi}\int_0^\pi e_{ba}\,i_b\,d\theta$$

From equation $A5$,

$$i_b = \frac{\sqrt{3}\,e_0}{x_d + \sqrt{x_d''x_q''}}[\cos\theta + b\cos 3\theta + b^2\cos 5\theta + \ldots]$$

and from equation $A13$,

$$e_{ba} = \frac{3e_0\sqrt{x_d''x_q''}}{x_d + \sqrt{x_d''x_q''}}[\cos\theta + 3b\cos 3\theta + 5b^2\cos 5\theta + \ldots]$$

Power is produced only by current and voltage of the same fr quency; therefore,

$$p_1 = A\cos^2\theta$$
$$p_2 = 3b^2A\cos^2 3\theta$$
$$p_3 = 5b^4A\cos^2 5\theta$$

where

$$A = \frac{3\sqrt{3}\,e_0\sqrt{x_d''x_q''}}{(x_d + \sqrt{x_d''x_q''})^2}$$

The total average power is

$${}_bW_{ba} = \frac{A}{\pi}\int[\cos^2\theta + 3b^2\cos^2 3\theta + 5b^4\cos^2 5\theta + \ldots]d\theta \quad \text{(A1}$$

but

$$\int_0^\pi \cos^2\theta\,d\theta = \frac{\pi}{2}$$

$$\int_0^\pi \cos^2 3\theta\,d\theta = \frac{\pi}{2}$$

$$\int_0^\pi \cos^2 5\theta\,d\theta = \frac{\pi}{2}$$

then equation $A16$ reduces to

$${}_bW_{ba} = A[1 + 3b^2 + 5b^4 + 7b^6 + \ldots]$$

and if the series is summed

$${}_bW_{ba} = \frac{3\sqrt{3}e_0^2\sqrt{x_d''x_q''}}{2(x_d + \sqrt{x_d''x_q''})^2}\cdot\frac{1 + b^2}{(1-b^2)^2}$$

In terms of the effective nominal voltage

$$ {}_bW_{ba} = \frac{3\sqrt{3}E_0^2\sqrt{x_d''x_q''}}{(x_d + \sqrt{x_d''x_q''})^2} \cdot \frac{1 + b^2}{(1 - b^2)^2} \tag{A17} $$

Appendix II—Symbols and Nomenclature

i = instantaneous short circuit current
i_s = steady-state instantaneous short-circuit current
i_a, i_b, i_c = instantaneous currents in phases a, b, and c, respectively
I = effective value of short-circuit current (square-root-of-mean-square value of i)
I_1 = effective value of fundamental component of current i
i_1 = fundamental component of current i, maximum
I_d = current in main field winding
I_q = current in quadrature field winding
I_0 = average (d-c) field current corresponding to nominal voltage e_0
ψ_a, ψ_b, ψ_c = armature flux linkages in phases a, b, and c, respectively
e_0 = maximum nominal voltage per phase
E_0 = nominal voltage per phase, effective value
e_{ab} = instantaneous voltage, lines a to b
E_{ab} = effective voltage, lines a to b
E_1 = effective value of fundamental component of voltage E_{ab}
e_1 = fundamental component of open-line voltage e_{ab}, maximum
ψ_{ab} = linkages included in phases a and b
ψ_{ab_s} = steady-state value of linkages
x_d = synchronous reactance (3-phase) along direct axis
x_q = synchronous reactance (3-phase) along quadrature axis
x_d'' = subtransient reactance (3-phase) along direct axis
x_q'' = subtransient reactance (3-phase) along quadrature axis
l_d, l_q, l_d'', l_q'' = inductances corresponding to the reactances x_d, x_q, x_d'', and x_q'' at normal frequency
M_d = mutual inductance between main field winding and a single armature phase winding when pole is coaxial with the armature winding
l_0 = zero-sequence inductance
θ = angular position of rotor at any instant, referred to the center line of phases b and c in series ($\theta = \omega t + \alpha$)
α = angular position of rotor at instant short circuit occurs, referred to center line of phases b and c in series
θ' = angular position of rotor at any instant, referred to the center line of phase a($\theta' = \omega t + \alpha'$)
ω = angular velocity corresponding to normal frequency
t = time in seconds
b = $\dfrac{\sqrt{x_q''} - \sqrt{x_d''}}{\sqrt{x_q''} + \sqrt{x_d''}}$ = geometric ratio of harmonics in short-circuit current
I_a, I_b, I_c = total effective current in armature phases a, b, and c, respectively
z_1 = positive-sequence armature impedance per phase
z_2 = negative-sequence armature impedance per phase
x_1 = positive-sequence armature reactance per phase
x_2 = negative-sequence armature reactance per phase
x_2 (AIEE) = negative-sequence armature reactance per phase as measured by the AIEE method
${}_bW_{ab}$ = wattmeter reading with voltage E_{cb} and current I_b
p_1, p_2, p_3 = instantaneous power corresponding to the fundamental, third and fifth harmonics, respectively

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Discussions

Of AIEE Papers—as Recommended for Publication by Technical Committees

Electrical Apparatus for Diesel Cars and Locomotives

Discussion of a paper by G. F. Smith published in the April 1936 issue, pages 335–41, and presented for oral discussion at the general session of the AIEE North Eastern District meeting, New Haven, Conn., May 7, 1936.

S. B. Schenck (nonmember; Bessemer and Lake Erie Railroad Company, Greenville, Pa.): Within 2 years, 78 American railroads and 63 American industrial concerns together have invested over 11,000,000 dollars in electrical equipment for more than 1,000 gasoline and diesel-propelled motive power units. Over 1,200 generators and 2,600 propulsion motors have been installed on these units. Engines range from 60 to 2,000 horsepower each, and collectively operate at speeds ranging between 500 and 2,400 revolutions per minute. While most of these units are propelled by gasoline engines, and some of the fundamental design features mentioned in this paper were worked out for the gasoline propelled power, the greatest refinements in design and application have occurred in the past five years for diesel electric motive power.

One of the most desirable characteristics for electric transmissions centers in continued reduction of apparatus weight, and the paper gives information of present status. Although the weight has been reduced in 10 years from around 60 pounds

Discussions in This Issue

On this and the following 2 pages appear all remaining unpublished discussions of technical papers presented at the general session of the AIEE North Eastern District meeting, New Haven, Conn., May 6–8, 1936.

Members anywhere are encouraged to submit written discussions of any paper published in Electrical Engineering, which discussion will be reviewed by the proper technical committee and considered for possible publication in a subsequent issue.

to near 35 pounds per horsepower, it has a long way to go to meet the 15 pounds per horsepower obtainable in mechanical or hydraulic transmission for engines of 250 horsepower or less.

Weight reduction of electric equipment appears to have practically paralleled the reduction of first costs of diesel power. For example, diesel electric locomotives can be purchased today for 100 dollars per engine horsepower compared to 200 dollars 10 years ago. Even at present cost levels, diesel motive power is priced close to 2.5 times that of equivalent main line steam power. The electrical apparatus usually involves about 30 per cent of the first cost of a locomotive, and the economic advantage to be obtained by further cost reduction is obvious. It also emphasizes the importance of further reduction in weight and cost toward the level of mechanical and hydraulic transmissions. The cost of mechanical transmissions for 250 horsepower is said to be about 25 per cent of the corresponding electric drives. Hydraulic transmissions cost more than mechanical ones, but their weight and cost advantages have been among the chief incentives for European railroads to apply them to cars with dual 600 horsepower diesel engines, and to prepare designs for larger engines to be used on locomotives.

The paper indicates the desirability of high ratio between maximum operating speed and lowest continuous operating speed of propulsion motors. There may be possibilities for obtaining the equivalent of increased ratio by the use of 4-cycle engines operated without supercharge at their normal rating, and then bringing supercharge gradually into action to raise the engine power to its maximum.

Herman Lemp (Ingersoll-Rand Company, New York, N. Y.): The author has covered the subject in an able manner. From the paper one gathers that controlling the generator output automatically by the service demand can be covered by either of 2 fundamentally different methods, both of which are open to innumerable variations and refinements.

1. By having the speed of the generator set determine both inversely the fuel supply and generator excitation (speed control or torque control as modified by engine speed).
2. By inherent control of generator excitation such as by differential field on main generator, differential field on exciter, a special designed exciter, and metadyne exciter.

The first group has the advantage that it takes care of the temperamental behavior of any internal combustion engine and of temperature variations in generator and exciter windings at one time. It is usually more complicated and subject to variation, depending, as it does, upon electric contacts, and pressure valves, but it operates satisfactorily in practice.

The second group is simpler in design, and has its chief drawback in the heating of generator and exciter winding, thereby varying the loading of engine from cold to warm.

For switching locomotives, the weight of equipment is no drawback, the additional first cost of copper and iron is well worth the low maintenance resulting from simplicity of design.

With differential field on main generator, units can be run in multiple on a common busbar, and will divide the load in accordance with engine performance (a 4 or 6 cylinder engine may be run in parallel), an advantage not to be brushed lightly aside. Anyway, switching locomotives so equipped have been in constant service for more than ten years.

The writer appreciates the author's pointing out the essential difference in desirable characteristics between traction motors operated from a constant-potential trolley and those supplied by variable voltage from Diesel operation. This is distinctly a step in the right direction, namely, to design traction motors for Diesel service with low C^2R losses versus low core losses.

In the light of further investigations, particularly for larger units, the writer might suggest a closer study of providing 2 generators, one mounted on each end of engine and coupling them either in series or multiple, while retaining the traction motors always in multiple. Traction motors, when in series, lack the grip during acceleration on slippery rails. When one slips, the other motor, in series therewith, also slips. Motors should always be in multiple for best acceleration. There would seem no greater difficulty in coupling generators from series to parallel or when the engine is at idling speed and excitation removed, than in coupling motors as is the usual practice.

P. A. McGee (General Electric Company, Philadelphia, Pa.): The success of Diesel-power-plant rail motive power in this country to date is due largely to the employment of the flexible electric transmission. Electric transmission permits a perfect regulation between the tractive effort and speed of a locomotive within the capacity of the diesel power plant. It would be difficult, if not impossible, with any other device to develop the graduated increments of tractive effort which are essential to obtain the high values of adhesion which account for the economy and success of the many small diesel power plant locomotives which have so effectively replaced steam locomotives of considerably greater horsepower in rail switching services in this country.

In the case of high-speed light-weight trains, such as the "Zephyr" on the Burlington, and the "Comet" on the New Haven, the success of the relatively small diesel plants employed on these trains is largely due to the efficiency, reliability, and small maintenance resulting with the electric transmission. These advantages will become all the more pronounced as the horsepower of the diesel plants increases.

Electric transmission, however, is inherently heavy and costly, and the designers of rail Diesel equipment are naturally enough endeavoring to develop more direct and possibly cheaper drives along mechanical lines with which mechanical engineers and the automotive industry are more familiar and sympathetic. Although developments of mechanical drives are not very encouraging to date for even the small outputs required with switching locomotives and rail cars employed in this country, their consideration should at least act as an incentive to the electrical engineers to more fully exploit the possibilities of electric power transmission and reduce as far as possible the weight and cost of existing equipment.

With the great interest which is now being taken in Diesel rail motive power, this paper on the electrical transmission of this power is very opportune.

Curves 1 and 2 in the paper explain in a satisfactory manner the question of drive efficiency by both mechanical and electric transmission. There is a popular misunderstanding that the electric transmission of power is considerably less efficient than straight mechanical drives, and although this condition may exist at certain fixed speeds, there is every indication that over a range of speed which would be met on any ordinary rail profile, that there is little, if any, advantage in efficiency with mechanical drives. In the case of switching locomotives over 30 per cent of the diesel fuel consumed is accounted for by idling and engine rotational losses which are independent of the method of drive.

With main line high-speed trains and where switching locomotives are required to perform transfer duty between yards, the question of obtaining the maximum output of the diesel power plant at the rails over the range of operating speed is frequently more important than the question of over-all fuel consumption and in consequence the generator characteristics shown on curve 7 of the paper with simple differential regulation which limits the engine output at medium and relatively high locomotive speeds, is not desirable. It then becomes necessary to employ some type of generator speed regulation to approximate curve 5 explained in the paper.

Curve 5 shows a constant generator output over a range of 80 per cent to 160 per cent ampere rating. For high-speed operation, it would be desirable to extend this range of constant output to lesser values of current rating and for heavy switching duty it would be desirable to extend this range to higher values of current rating. Such extensions of constant output range would, of course, require a larger voltage range with both the generator and traction motors and their cost and weight would in consequence be adversely affected. The writer would like, however, to ask Smith how he arrives at the economic limit in constant output range for a given output and as to whether the extension of the range shown in figure 5 of the paper would be costly.

The description with figures 5, 7, 8, and 10 of the various methods employed to obtain a satisfactory generator characteristic are interesting and must be given close consideration by railway engineers.

The effect of the ratio of maximum speed to continuous speed of a locomotive on the weight of traction motors is perhaps one of the most illuminating points brought out in the paper and very clearly illustrates the price which must be paid when high speed requirements are associated with low speed services, such as yard switching work.

This feature might have been further developed by referring to the possibilities of double reduction gear combined with

limitations in locomotive speed. During the past few years, the writer has given some thought to this possibility and estimates that the weight and cost of the traction motors alone on straight electric and Diesel electric locomotives might be reduced by at least 50 per cent with double reduction gears.

In the case of single-phase switching locomotives a locomotive with single reduction gear and having motors weighing approximately 40,000 pounds could be substituted by a double reduction gear locomotive having motors weighing 18,000 pounds, without affecting the maximum speed and actually giving a greater horsepower output.

In order to develop fully the adhesion values possible with electric traction motors, it is necessary that design engineers give more thought to the connection and arrangement of the traction motors in the electric circuit. The author has at 5 points in his paper referred to the more or less standard arrangement of series-parallel control of traction motors. By such a motor combination the author states that the overload on the generator need not exceed 60 per cent even with 200 per cent overload on the motors. As is pretty well understood, traction motors in series are in the first instance in a most unsatisfactory mechanical arrangement due to weight transfer, and in the second instance they are in a most unsatisfactory electrical circuit due to their instability the moment wheel slippage starts, with a tendency to continue slipping and the imposition of the complete circuit voltage across one armature. Due to this unfortunate combination of circumstances, motors with series-parallel control give an extremely "slippy" locomotive if the service requires the development of high adhesive values and the price paid for the lesser duty on the generator by such a control set-up is a severe limitation in the performance of the locomotive and higher maintenance with the traction motors.

The writer would like to ask Smith what increase in the main generator weight and cost would result if traction motors were at all times connected in parallel.

J. W. Marsh (nonmember; The Engineer Board, Fort Belvoir, Va.): The paper is informative and the only thing the writer can offer is a suggestion relative to mechanical drive.

With reference to figures 1 and 2 of the paper, the comparison of 660-horsepower electrical transmission with 800-horsepower mechanical transmission is apparently without foundation. A 4-step transmission certainly would be ineffective on a speed range of from zero to 80 miles per hour and acceleration to 8 miles per hour by clutch slippage would be excessive. The region of mechanical transmission in powers over 200 horsepower per unit is relatively unexplored.

No weight is given to the comparative weight and cost of electrical and mechanical drive locomotives. There is compared in figure 1 of this discussion a mechanical drive locomotive which could be built in the price and weight range of the gasoline electric locomotives recently built for Picatinny Arsenal (weight 64,775 pounds, price 17,533 dollars, Picatinny Arsenal). The diagram shows what would be obtained with a duplex mechanical transmission, using 2 engines, each delivering 175 horsepower to the transmission ("LeRoi *RX-1S*").

With this type of transmission rail, traction is never lost but drops as shown by the vertical lines at each gear change. Gears will be synchronized and shifted pneumatically. Clutch slippage will occur only on accelerating from zero to 1.5 miles per hour. Clutches are available capable of slipping the maximum torque for 30 seconds without deleterious effects. In the case illustrated the clutch would be required to slip 65 per cent of its capacity for 6 seconds to accelerate a 500-ton train to 1.5 miles per hour. Beyond 1.5 miles per hour there is no time limit for any tractive force or speed.

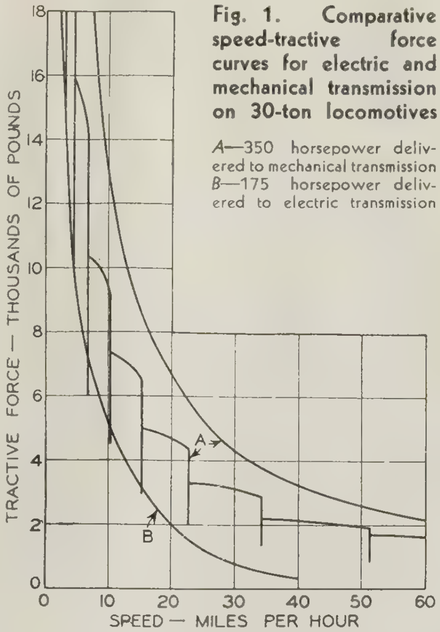


Fig. 1. Comparative speed-tractive force curves for electric and mechanical transmission on 30-ton locomotives

A—350 horsepower delivered to mechanical transmission
B—175 horsepower delivered to electric transmission

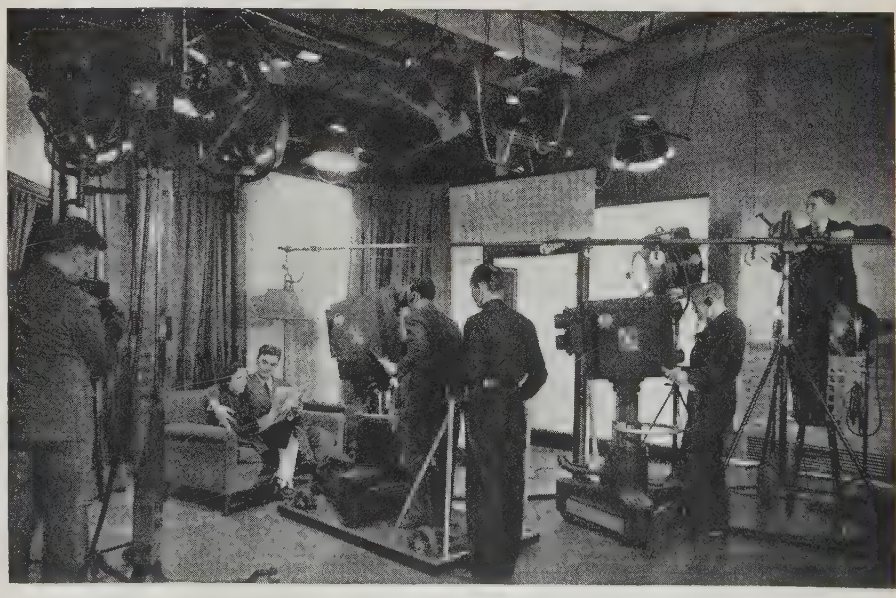
A transmission as just outlined has, of course, not been built but is in the design stage. Its characteristics are shown here merely to illustrate the possibilities of mechanical drive and the competition which will have to be met by electrical transmission.

E. B. Fitzgerald (American Car & Foundry Company, Berwick, Pa.): Figures 1 and 2 of the paper compare an 800 horsepower mechanical drive equipment with an 800 horsepower electric drive. These curves indicate a top speed of 60 miles per hour (governed engine speed) for the mechanical drive, and 80 miles per hour for the electric drive. Having been associated with the development and application of both mechanical and electric-drive equipments for 10 years, the writer feels that in fairness to the mechanical drive, the following points should be brought out:

1. Selection of proper transmission and axle ratio (figures 1 and 2 of the paper) will result in the same top speed for the mechanical drive as is indicated for the electric drive.
2. If the mechanical drive is compared to the electric drive on the basis of horsepower per ton of weight available at the rail, which is really what counts, and first cost, the picture changes because of the increased weight and cost of the electrical transmission.

Under the subheading "Generator With Torque Control" the author states, "With this arrangement, the full torque of the engine, which it can deliver at any engine speed can be used regardless of engine condition, battery condition or temperature for the main generator field." In addition, torque control as in the case of the power plant illustrated in figure 4 automatically increases or decreases power available for traction depending on the output demand of the auxiliary generator which may vary from a few kilowatts to 60 to 80 kilowatts parasite load. This is not possible with differential control.

The NBC Experiments With Television



News

Of Institute and Related Activities

AIEE Winter Convention Features 5-Day Program

IN KEEPING WITH plans to resume a 5-day schedule for the Institute's winter convention, a tentative technical program of many specialties has been arranged for the 1937 convention, which is to be held January 25–29, in New York, N. Y. The convention will begin on Monday with registration in the morning and the opening session in the afternoon. Technical sessions will be held Monday afternoon and during both mornings and afternoons of the next 3 days. Friday, the fifth day, will be devoted exclusively to inspection trips to places of interest. The smoker will be held on Tuesday evening, Edison Medal presentation on Wednesday evening, and the dinner-dance at the Hotel Astor on Thursday evening. More information regarding these features will be announced in the January issue.

Technical Sessions

Thirteen technical sessions have been arranged tentatively, and many timely papers will be presented. Several papers in the field of power transmission will deal with lightning investigations on transmission lines and measures of protection against lightning. In another session, bearing a close relation to the indirect effects of lightning on apparatus, will be several papers which have to do with the latest information on impulse voltage testing technique and measurements. The first report of power-system stability by the Institute's subcommittee on interconnection and stability factors will contribute valuable authentic information to the literature on this important subject. Ultra-high-speed reclosing of circuit breakers is another timely topic, of interest to transmission and protection engineers, which is scheduled in the protective devices session.

Engineers who have to do with the planning and performance of distribution systems will find several papers in the distribution session to be of interest as well as the papers in the sessions on communication and protective devices. The economics of overhead secondary circuits will be analyzed in a paper which presents a cross section of experiences obtained from 5 companies.

For those interested in the application of mathematics to certain electrical engineering problems a session on tensors, including also a paper on dyadic algebra and another on complex vectors, will hold much interest. It is hoped that these papers and the ensuing discussion will do much toward indicating the advantages and limitations of tensor methods. For those interested in the theory and design of electrical machinery 3 sessions will be held on this subject. Sessions on communication, selected subjects, electronics, electrophysics, and electric welding constitute the remainder of the program tentatively established.

Technical Conferences

During the convention technical conferences on the subjects of education, electrical networks, the AIEE d-c machinery test code, and the AIEE a-c machinery test code will be held. The technical conferences are intended to afford opportunity for groups of specialists to meet and discuss subjects informally, no provision being made for publication of the papers or discussions.

Rules on Presenting and Discussing Papers

At some of the technical sessions, a few papers may be presented only by title. This will permit the devotion of more time to discussion. At other sessions, papers will be presented in abstract, 10 minutes being allowed for each paper unless otherwise arranged, or the presiding officer meets with the authors preceding the session to arrange the order of presentation and allotment of time for papers and discussion. Authors will be notified officially in each case about one month in advance.

Any member is free to discuss any paper when the meeting is opened for general discussion. Usually 5 minutes is allowed to each discusser for the discussion of a single paper or of several papers on the same general subject. When a member signifies his desire to discuss several papers not dealing with the same general subject, he may be permitted to have a somewhat longer time.

It is preferable that a member who wishes to discuss a paper give his name in advance to the presiding officer of the session at which the paper is to be presented. Each discusser should step to the front of the room and announce, so that all may hear, his name and professional affiliations. Three typewritten copies of discussion prepared in advance should be left with the presiding officer.

Other discussion to be considered for publication must be submitted, typed double spaced, in triplicate to C. S. Rich, secretary, technical program committee, AIEE headquarters, 33 West 39th St., New York, N. Y., on or before February 12, 1937. Discussion received after this date will not be accepted.

Membership—

Mr. Institute Member:

A substantial gain in membership in the Institute has been realized recently; from May 1 to November 1 the number is 575. This was accomplished through the co-operative efforts of the individual members and the Section and national membership organizations. The increase should continue along with the expansion of technical employment now under way. We urge your continued interest and efforts as you come in contact with qualified nonmembers.

Each member is in a position to acquaint prospective members concerning Institute activities, and by so doing the record of progress shall continue. Give your Section membership committee the names of good member prospects on the blank form recently sent to you.

Vice-Chairman, District No. 2,
National Membership Committee

Tentative Technical Program

In this program, reference to the issue and, in so far as possible, to the page in ELECTRICAL ENGINEERING, is given for all papers published up to and including this issue.

Communication

Calorimetric Measurements of Dielectric Loss, H. H. Race and S. C. Leonard, General Electric Company. December issue, pages 1347–56

Current Propagation and Potentials in Earth Return Circuits, E. D. Sunde, Bell Telephone Laboratories, Inc. December issue, pages 1338–46

The Inductive Co-ordination of Common-Neutral Power Distribution Systems and Telephone Circuits, J. O'R. Coleman, Edison Electric Institute, and R. F. Davis, American Telephone and Telegraph Company.

Selected Subjects

Studies of Stability of Cable Insulation, Herman Halperin and C. E. Betzer, Commonwealth Edison Company. October issue, pages 1074–82

*Life Testing of A-C Dry Electrolytic Condensers, F. W. Godsey and Preston Robinson, Sprague Specialties Company.

*Characteristics of Laminated Silver-Steel Contacts Tips for Small Arcing Contacts, O. R. Schurig and J. B. Ford, General Electric Company.

Electronic Transient Visualizers, H. J. Reich, University of Illinois. December issue, pages 1314–18

Power Distribution

*Overhead Secondary Distribution—A Review of 5 Economic Studies, W. P. Holben, Duquesne Light Company.

Pole Flexibility as a Factor in Line Design, H. P. Seelye and Myron Zucker, The Detroit Edison Company.

Trends in Distribution Overcurrent Protection, G. F. Lincks and P. E. Benner, General Electric Company.

Automatic Boosters on Distribution Circuits, L. M. Olmsted, Duquesne Light Co., October issue, pages 1083–96.

Tensor Analysis

The Tensor—A New Engineering Tool, A. Boyajian, General Electric Company. August issue, pages 856–62

Dyadic Algebra Applied to 3-Phase Circuits, A. Pen-Tung Sah, National Tsing Hua University. August issue, pages 876–82

Tensor Algebra in Transformer Circuits, L. V. Bewley, General Electric Company. November issue, pages 1214–19

Tensor Analysis of Multielectrode Tube Circuits, Gabriel Kron, General Electric Company. November issue, pages 1220–42

Complex Vectors in 3-Phase Circuits, A. Pen-Tung Sah, National Tsing Hua University. December issue, pages 1356–64

Power Transmission

*Lightning Investigation on Transmission Lines—VI, W. W. Lewis and C. M. Foust, General Electric Company.

Lightning Investigation on 220-Kv Lines of the Pennsylvania Power and Light Co., Edgar Bell, Pennsylvania Power & Light Company. December issue, pages 1306–13

*First Report of Power-System Stability, AIEE subcommittee on interconnection and stability factors.

Lightning Protection for Low-Voltage Transmission Lines, A. W. Gothberg and A. S. Brookes, Public Service Electric and Gas Company. Scheduled for January issue

Characteristics of Suspension Insulator, C. L. Dawes and Reuben Reiter, Harvard University. Scheduled for January issue

Induction Machinery

Induction Motor Resistance Ring Width, P. H. Trickey, Diehl Manufacturing Company. February issue, pages 144–50

Induction Motors Under Unbalanced Conditions, E. O. Lunn, Peterson and Cowan Elevator Company, Ltd. April issue, pages 387–393

Capacitor Motors With Windings Not in Quadrature, A. F. Puchstein and T. C. Lloyd, Robbins and Myers, Inc. November issue, pages 1235–39

Induction Motors on Unbalanced Voltages, H. R. Reed and R. J. W. Koopman, Michigan College of Mining and Technology. November issue, pages 1206–13

Protective Devices

The Control Gap for Lightning Protection, Ralph Higgins and H. L. Rorden, Ohio Brass Company. September issue, pages 1029–34

A New Thermal Fuse for Network Protectors, L. A. Nettleton, Brooklyn Edison Company. October issue, pages 1096–9

Ultra-High-Speed Reclosing of High-Voltage Transmission Lines, Philip Sporn, American Gas and Electric Company, and D. C. Prince, General Electric Company.

*A New Service Restorer, W. R. Nodder, Pacific Electric Manufacturing Corporation.

A Single-Element Polyphase Directional Relay, A. J. McConnell, General Electric Company.

Synchronous Machinery

Proposed Transformer Standards, J. E. Clem, chairman, 1934–36 Transformer Subcommittee. Scheduled for January issue

*Contributions to the Theory of Synchronous Machines, A. S. Langsdorf, Washington University.

Negative-Sequence Reactance of Synchronous Machines, W. A. Thomas, Antioch College. December issue, pages 1378–84

Two-Reaction Theory of Synchronous Machines, S. B. Crary, General Electric Company. Scheduled for January issue

Operational Solution of A-C Machines, A. R. Miller and W. S. Weil, Jr., Lehigh University. November issue, pages 1191–1200

* These papers may be presented, but they have not been accepted for publication at the time of going to press. Of these papers, those which are accepted will be published, in so far as is possible, in the January 1937 issue.

Electronics

Heater-Cathode Insulation Performance, Hans Klemperer, Westinghouse Electric & Manufacturing Company. September issue, pages 981–5

Amplification Loci of Resistance-Capacitance Coupled Amplifiers, A. C. Seletzky, Case School of Applied Science. December issue, pages 1364–71

An Electronic Voltage Regulator for D-C Generators, F. H. Gulliksen, Westinghouse Electric & Manufacturing Company. August issue, pages 873–6

New Developments in Electronic Welding Control, J. W. Dawson, Westinghouse Electric & Manufacturing Company. December issue, pages 1371–4

Sealed-Off Tubes for Welding Control, David Packard and J. H. Hutchings, General Electric Company.

Electrical Machinery

Self-Regulated Rectifiers, W. M. Goodhue and R. B. Power, Harvard University. November issue, pages 1200–06

An Analysis of the Shaded-Pole Motor, P. H. Trickey, Diehl Manufacturing Company. September issue, pages 1007–14

*Abrasion—A Factor in Electrical Brush Wear, V. P. Hessler, Iowa State College.

*Arc Characteristics Applying to Flashing on Commutators, R. E. Hellmund, Westinghouse Electric & Manufacturing Company.

Electrophysics

A-C Characteristics of Dielectrics—II, Alfredo Baños, Jr., Massachusetts Institute of Technology. December issue, pages 1329–37

*Dielectric Strength of Insulation to Impulse Switching and 60-Cycle Voltages, P. L. Bellaschi and W. L. Teague, Westinghouse Electric & Manufacturing Company.

Parallel Inverter With Inductive Load, C. F. Wagner, Westinghouse Electric & Manufacturing Company. September issue, pages 970–80

Calculation of Resistances to Ground, H. B. Dwight, Massachusetts Institute of Technology. December issue, pages 1319–28

*Expansion Theorems for Ladder Networks, M. G. Malti and S. E. Warschawski, Cornell University.

Instruments and Measurements

Impulse Voltages Chopped on Rising Front, P. L. Bellaschi, Westinghouse Electric & Manufacturing Company. September issue, pages 985–90

*Impulse Generator Circuit Charts for Selecting Circuit Constants, J. L. Thomason, General Electric Company.

*Short Time Breakdown of Gaps, J. H. Hagenguth, General Electric Company.

*Development of a Modern Watt-Hour Meter, I. F. Kinnard and H. E. Trekell, General Electric Company.

*Watt-Hour-Meter Bearings, I. F. Kinnard and J. H. Goss, General Electric Company.

Electric Welding

The Resistance Welding Circuit, C. L. Pfeiffer, Western Electric Company. August issue, pages 868–73

Demonstrations.

Report of ECPD Committee on Professional Recognition Makes Far-Reaching Proposals

Supplementing the report of the annual meeting of the Engineers' Council for Professional Development and the excerpts from the annual reports of 3 ECPD committees published in the November issue (pages 1280–5), full text of the annual report of the ECPD committee on professional recognition is presented here. The report was presented to ECPD at its annual meeting held on October 6, 1936, but official action on the recommendations contained therein was deferred pending further consideration at a future meeting of ECPD. These recommendations are of far-reaching importance to all engineers, and particularly to members of national engineering societies. Comments on these recommendations are invited.

TO THE committee on professional recognition is assigned the fourth phase of the ECPD program, namely, to develop procedure and recommendations for "bringing some correlation into the various methods of formal recognition of the development of an engineer" (1935 Annual Report of ECPD, page 3).

The present avenues or stages of formal recognition of the development of an engineer are 3 in number. They are all represented in ECPD as a co-ordinating agency. Listed in order of progressive and chronological sequence, they are:

1. *Professional Education*, as evidenced generally by graduation from an approved college of engineering.

2, *Registration as a Professional Engineer*, representing legal recognition and admission into the engineering profession.

3. *Membership in a professional grade of a recognized engineering society*, representing recognition of the attainments of the individual by his fellow engineers.

These 3 stages of formal recognition of the development of an engineer are now established. *Our problem is to improve their correlation.*

The proposal of any additional procedure of certification or recognition would only be adding a fourth method to the 3 methods of progressive recognition already established. It would introduce new competition or conflict and new difficulties of correlation, and would therefore not be a solution of the problem of harmonious coordination.

It is true that the 3 methods of progressive recognition already established are not yet, within themselves, sufficiently uniform. With equal interest in all 3, ECPD should address itself to achieving results of more uniform significance. Nothing in this report is to be construed as recommending any lowering of present standards or requirements.

1. Under the heading of "Professional Education," ECPD (through its committee on engineering schools) is seeking to established recognized national standards of quality and attainment for engineering schools through its program of accrediting. "Graduation from an approved course in engineering" should eventually have a more definite and more uniform significance.

2. Under the heading of "Engineers' Registration," it is recognized that this method of formal recognition is not yet universal nor sufficiently uniform. Only 35 of the 48 states have engineers' registration laws; and in 3 of these states the laws are incomplete, covering only a fraction of the profession. Moreover, with such laws enacted at different dates (since 1907) and under varying circumstances, there are naturally some variations in their qualification requirements. "ECPD should therefore address itself to rendering all possible assistance to effect uniform registration laws in states which do not have them, to improving the registration laws that now exist, and to effecting among these present laws a higher degree of uniformity as to requirements and as to form of recognition" (1935 report of committee on professional recognition, adopted by ECPD October 8, 1935).

3. Under the heading of "Membership in Engineering Societies," it is recognized that there is considerable variation in qualification requirements for admission to the same or corresponding grades of membership in the various national organizations. Significance and recognition will be advanced if these requirements are brought to a more uniform level, both in constitutional prescription and in application. ECPD has adopted the recommendations of this committee (1933 and 1934) for "Standard Grades of Membership," namely, Student Member, Junior Member, Member, and Fellow; and support should be given to the establishment of this uniform system of grades upon the basis of the advantage to the entire engineering profession resulting from more uniform formal recognition. ECPD has also adopted the "Minimum Definition of an Engineer" formulated by this committee (1933); and under the approved "Standard Grades of Membership," the grade of Member is defined as "the full-fledged engineer, that is, *the engineer who has passed the requirements in the minimum definition of an engineer.*" All of the interested engineering societies should therefore be urged to make the "Minimum Definition of an Engineer" their goal as a minimum requirement for admission to the "Member" grade. The "Minimum Definition" prescribes professional education, specified experience, and the *passing of written examinations*. Instead of duplicating such examinations, the engineering societies may accept the results of corresponding examinations, passed in securing professional registration under the state laws.

Correlation

If the 3 avenues or stages of recognition of development of an engineer (1—education, 2—registration, 3—membership) are appreciated as logically progressive and successive, much apparent conflict is resolved, and consistent correlated relationship is made manifest. The [ECPD] committee on student selection and guidance has for its province the problems preceding and anticipatory to phase 1; the committee on engineering schools covers phase 1; the committee on professional training is chiefly concerned with the problems covering the period of individual development between phase 1 and phase 2, and beyond; and the committee on professional recognition is concerned with all as they are related to the recognition of the engineer.

Under the concept of the progressive sequence of the 3 stages of recognition (1—education, 2—registration, 3—membership) each successive stage should be predicated, so far as practicable, upon the prior attainment or completion of the stage preceding it. Thus will any remaining conflict be minimized, and correlation improved.

Accordingly, evidence of completion of (1) professional education (by graduation and/or examinations) should be made universally a prerequisite for (2) registration under the state laws and for (3) admission to "Member" grade in the national engineering societies. ECPD should therefore give its assistance in amending any state registration law in which evidence of professional education (by graduation and/or examinations) is not yet clearly specified as an essential prerequisite. Likewise, ECPD should urge all interested engineering societies to adjust their requirements so as to specify evidence of professional educa-

Future AIEE Meetings

Winter Convention
New York, N. Y., Jan. 25–29, 1937

North Eastern District Meeting
Buffalo, N. Y., May 5–7, 1937

Summer Convention
Milwaukee, Wis., June 21–25, 1937

Pacific Coast Convention
Spokane, Wash., Date to be determined

Middle Eastern District Meeting
Akron, Ohio, Fall 1937

tion (by graduation and/or examinations) as an essential prerequisite for membership.

Similarly, to improve correlation, registration should be made so far as practicable, a minimum prerequisite for admission to the professional grades* of membership of the national engineering societies. Each organization can easily determine for itself which of its membership grades shall be regarded as professional. For admission to the professional grades of membership, with such temporary exceptions as may be practically indicated, state registration should be established as a minimum requirement. This does not mean that any candidate is to be accepted for membership merely because he is registered or licensed. The engineering society may not yet be satisfied with the qualification requirements for registration in some of the states, and may desire therefore to impose such additional requirements as it deems proper. State registration is here recommended as a desirable basic requirement, not as an all-sufficient requirement.

Unless any engineering society takes the position that the states maintain qualification requirements higher than should be expected of professional grades of membership of the society, there would appear to be no valid objection to requiring registration as a prerequisite. If, on the other hand, any engineering society takes the position that the registration requirements in any state are inferior, there would appear to be no objection to challenging any candidate who fails even to meet such inferior requirements.

Exceptions can of course be made for applicants from the 13 states which do not yet have registration laws, or from the 3 states in which the registration laws are incomplete. Exceptions can also be made for applicants who can show specific exemptions in their state registration laws, permitting their continued responsible practice of engineering without registration.

It is therefore recommended that the interested engineering societies consider the eventual adoption of the following requirement:

"Before admission or transfer to professional grades of membership in this society, an applicant shall show that he is or has been a legally registered professional engineer, unless he resides in a state in which an engineers' registration law has not been enacted, or unless he shows specific legal exemption under the engineers' registration law of the state in which he resides, permitting him to engage in the responsible practice of professional engineering without registration."

The adoption of this recommendation will prove correlation between membership grades and professional registration. It will also improve correlation between recognized professional status and corresponding grades of membership in the different engineering societies. It will, incidentally, facilitate the full establishment and application of the "Minimum Definition of an Engineer" as a requirement for admission to membership. Such co-ordination of (2) registration and (3) membership will improve the standards and status of both and will be a constructive contribution toward harmonious and consistent relationship of the "various methods of formal recognition of the development of an engineer."

A further contribution to the correlation and significance of formal recognition would be to simplify, toward greater uniformity, the wide variation of degrees conferred upon graduation from engineering schools and, in particular, to eliminate the professional degree (C.E., M.E., E.E., etc.) as a degree in course. For perfect correlation, ECPD should recommend that these professional degrees if conferred at all, be not conferred by the engineering schools upon any graduate before he has passed the stage of professional registration. Again, registration should be regarded as a necessary, though not as a sufficient, prerequisite. Each individual school may impose such additional requirements and tests as it deems appropriate.

Survey of Present Correlation

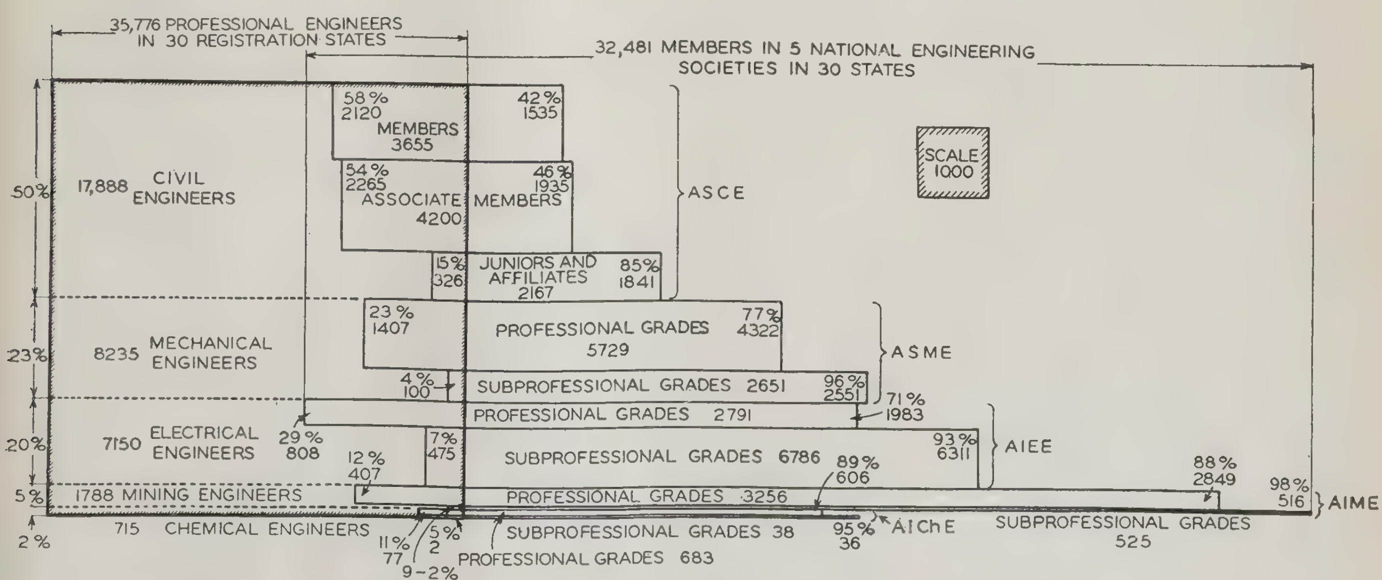
In order to establish an index of the present degree of correlation between (2) state registration and (3) society membership, a survey has been prepared under the auspices of ECPD at the request of the committee on professional recognition. This survey, in summary form, yields the percentages of correlation shown in table I.

The complete report of this count, by

* Professional grades of membership include: Honorary Member, Member, and Associate Member of the American Society of Civil Engineers; Honorary Member and Member of the American Institute of Mining and Metallurgical Engineers; Honorary Member, Fellow, and Member of The American Society of Mechanical Engineers; Honorary Member, Fellow, and Member of the AIEE; Active Member of the American Institute of Chemical Engineers.

Table I—Percentage of Members of National Engineering Societies Registered as Professional Engineers in 30 Registration States

ASCE		ASME		AIEE		AIME		AIChE	
Grade	%	Grade	%	Grade	%	Grade	%	Grade	%
Assoc. M. . 54 Members . . 56 Hon. M . . 60	56	Members . . . Fellows . . . . Hon. M.	25	Members . . Fellows . . . . Hon. M. . .	29	Members . . Hon. M	13	Active M. . 11	
Juniors. . . . 15 Affiliates. . . 16	15	Juniors. . . . . Associates . .	04	Associates . .	07	Juniors. . . . . Associates . .	02	Juniors . . . . Associates . .	05



Correlation of Registration and Engineering Society Membership in 30 States Having Registration Laws

Square area at left indicates total number of engineers registered in 30 states. Rectangular areas indicate membership in national engineering societies in the same states. The portions of the rectangular areas extending inside the square indicate the numbers of members of the various societies that have become registered engineers

states, of the relative numbers of members registered under the state laws in each grade of membership in each of the 5 participating engineering societies, is submitted as an appendix to this report. These data are also shown in graphic form for New York, for Iowa, and for 30 states for which registration figures were available. [Because of space limitations, only the chart showing registration data for 30 states is reproduced here.]

The percentages of correlation found in this survey are higher than was generally anticipated. They are higher for some of the societies than for others. By this survey, a clear line of differentiation is indicated between the professional grades and the other grades.

An improvement in the indicated percentages of correlation is to be desired for advancing the recognition of the profession. The development of recommendations for the improvement of such correlation is a phase of the ECPD program assigned to the committee on professional recognition.

Summary of Recommendations

1. ECPD should urge upon its participating engineering societies the actual adoption and application of the "Minimum Definition of an Engineer" as a minimum requirement for admission to Membership. Evidence of professional education (by graduation and/or examinations) should be specifically included in the prescribed membership qualifications.
2. ECPD should urge and encourage the early adjustment of membership grades in the participating engineering societies to conform to the "Standard Grades of Membership" previously formulated and ratified.
3. ECPD and its participating bodies should render all possible assistance to effect uniform registration laws in states which do not have them, to improve and to strengthen the registration laws where they now exist, and to effect among these present laws a higher degree of uniformity as to requirements and as to form of recognition.
4. ECPD should give its assistance and support in amending any state registration law in which evidence of professional education (by graduation and/or examinations) is not yet clearly specified as an essential prerequisite.
5. ECPD should recommend to all participating and interested engineering societies that state registration of a candidate be established as a *minimum* prerequisite for admission to professional grades of membership, with such provisional exceptions as present circumstances may justify.
6. ECPD should urge and support the simplification, toward greater uniformity, of the wide variations of degrees conferred upon graduation from engineering schools.
7. ECPD should recommend to all engineering schools that the professional degree be eliminated as a degree in course; and that (when it is awarded for post-collegiate professional recognition) it be not conferred until after the candidate has secured professional registration as a *minimum* prerequisite.

Personnel of ECPD Committee on Professional Recognition

The ECPD committee on professional recognition is composed of representatives of the 7 constituents of ECPD, as follows:

C. N. Lauer, *chairman*, American Society of Mechanical Engineers.

J. W. Barker (M'26, F'30), AIEE.

H. C. Parmelee, American Institute of Chemical Engineers.

D. B. Steinman, National Council of State Boards of Engineering Examiners.

F. M. Becket, American Institute of Mining and Metallurgical Engineers.

F. L. Bishop (H. S. Rogers, alternate), Society for the Promotion of Engineering Education.

J. P. H. Perry, American Society of Civil Engineers.

Dallas District Meeting Program Enjoyed by 533

Taking its place among the largest and most effective of the Institute's District meetings, the sixth South West District meeting, held in Dallas, Texas, October 26–28, had a total registration of 533. Attendance at sessions was unusually large in comparison with the registration, and keen interest was maintained until the end of the closing session. All 6 Sections and 14 Student Branches located in the District were represented, and considerable numbers of students from certain schools attended, the total student registration being 179.

Opening Session

With Vice-President L. T. Blaisdell presiding, the meeting was opened on Monday morning, October 26. Mr. Blaisdell expressed his pride in the work of the South West District which in the past has held a number of successful and interesting meetings. He also expressed his sincere appreciation of the work of the local committees which made preparations for the 1936 meeting. He then introduced Mayor Sergeant, of Dallas, who gave a brief address of welcome extending cordial greetings to all members and guests present and mentioning some of the attractive features of Dallas and of the Texas Centennial Central Exposition. He stated that the celebration of the 100th anniversary of Texas coincides with the celebration of the 95th anniversary of Dallas.

Vice-President Blaisdell then introduced President MacCutcheon, who presented a brief address on the subject: "Texas—The October Headquarters of the AIEE." After paying tribute to the friendliness, enthusiasm, and loyalty of the Institute's leaders in the South West District, he said: "All of you have not realized that your Institute has 15,000 members, 26 national officers, 62 Sections, and 118 Student Branches; that last year was held a summer convention with 39 technical papers, a winter convention with 59 such papers, and 2 regional conventions such as this where 30 papers were presented; and that in addition 20 technical conferences were held."

President MacCutcheon next called attention to some of the salient features of the meeting program, and then continued: "We are gathered here as engineers. What is an engineer? The dictionary states 'one versed in or practicing any branch of engineering.' This is not particularly helpful. From the same source: 'a person who manages or carries through any enterprise,' which seems too broad. I was led to ask several fellow engineers as to their understanding of 'What is an engineer?' A chief draftsman states 'an engineer is one who is trained to apply the theories of science to practical every-day problems.' An engineer of long experience and an executive of a construction company believes: 'an engineer is a man who can do with one dollar what any man can do with two.' An assistant chief engineer of a manufacturing company feels that an engineer is 'a man who through technical training and experience is able to consider a given set of facts and conditions relating to materials, structures, or machines, and arrive at a logical, practical, and economical conclusion.' An engineer's wife defines an engineer as 'one whom it is difficult to live with, and impossible to live without.' A chief engineer gave this conception: 'An engineer is one who through training, study, and practice adapts and controls the materials and forces of nature to the benefit of himself, his fellow engineers, and the rest of the human family.' Possibly each one of you can still better define an engineer. In a good engineer is required integrity, dependability, resourcefulness, forcefulness, adaptability, diplomacy, friendliness, broadmindedness, and knowledge. We are at this convention to acquire in a still greater degree all of these qualifications.

"To me the most outstanding characteristic of the Institute members is their free-will offering of service in the interest of their profession. As a notable example I point to the time so willingly given by your general chairman, Mr. Blaisdell, by his staff of 7 chairmen, and the 50 members of their committees, in arranging for this convention and all its details, providing this opportunity for the public presentation of a large amount of engineering information. I wish to pay tribute to the authors who have so gladly given of their time, knowledge, and experience in the preparation of these papers and to thank them for their contribution.

"In my mind the Institute may be likened to a great electrical network with many generating and distributing stations, tied together throughout Section, District, and national organization. Our output is engineering knowledge. Today the Dallas Section and the District 7 are carrying a peak load, feeding out through the network to every other District and Section. They have raised their voltage. The load is heavy, but the power factor is high. . . ."

At the conclusion of this address, Vice-President Blaisdell introduced several National, District, and Section officers and then turned the meeting over for the conduct of the technical part of the session.

Technical Sessions

The technical program for the entire meeting was carried out substantially as published in the September issue of Electrical Engineering, page 1041. The paper by J. B. Hodtum, scheduled for the Tuesday morning session, was omitted because the author was unable to be present. The paper by C. H. Frier, scheduled for Monday morning, was presented on Tuesday morning.

The papers were selected to be of particular interest to members in the South West District. The excellent attendance at all technical sessions showed how well the committees had attained this objective. The presiding officers for the several sessions were: Monday morning, N. F. Rode, Agricultural and Mechanical College of Texas, College Station, and F. J. Meyer,

Oklahoma City, Okla.; Tuesday morning, D. H. Levy, Dallas, and L. C. Starbird, Dallas; Tuesday afternoon, O. S. Hockaday, Fort Worth, Texas, and C. W. Mier, Dallas; Wednesday morning, D. D. Clarke, Kansas City, Mo., and M. H. Lovelady, San Antonio, Texas; Wednesday afternoon, J. B. Thomas, Fort Worth, and B. D. Hull, St. Louis, Mo.

The varied program on Wednesday afternoon attracted an especially large audience and the presentation of the 2 technical papers and the illustrated lecture by Professor R. G. Kloeffler of Kansas State College, Manhattan, was followed with keen interest by the audience of nearly 300.

Luncheon Meeting With Dallas Electric Club

One of the events of outstanding interest was the luncheon held jointly with the Dallas Electric Club on Monday, at which Doctor Matthew Luckiesh (A'11, M'15), director of the lighting research laboratory of the General Electric Company, Nela Park, Cleveland, Ohio, presented an address on the subject "Let's See" in a manner intensely interesting to all of the more than 200 persons present, including a considerable number of women. He discussed the development of artificial lighting and emphasized the need for greatly improved illumination in the industries, on the highways, and in the home, in order to provide adequately for the higher speeds, more intensive work, and other conditions prevailing in modern civilization. He showed that less work is involved in seeing as the intensity of light is increased and that no one at present knows what is enough light. A strong plea for wider application of science in lighting was made. Lee Cook, chairman of the Dallas Section of the Institute, presided at the luncheon.

Student Technical Sessions

In parallel with the regular sessions of the meeting, student technical sessions were held on Tuesday morning and afternoon, with a total of 9 papers presented and an average attendance of more than 100. R. S. Hestand, chairman, Southern Methodist University Branch, Dallas, presided.

The 9 papers presented at the 2 sessions were:

1. Short-Wave Directional-Antenna Performance, written by F. Q. Gemmill and E. Eberhard, and presented by J. W. Howard, all of the University of Kansas, Lawrence.
2. A High-Voltage Vacuum-Tube Voltmeter, M. H. Brown, Texas Agricultural and Mechanical College, College Station.
3. Possibilities and Limitations of Air Motors for Small Isolated Electric Plants, E. B. Ankenman, Kansas State College, Manhattan.
4. Rebuilding Fractional-Horsepower Motors for Capacitor Start, E. W. Logan, Missouri School of Mines and Metallurgy, Rolla.
5. Electrical Methods of Acoustical Improvements of Buildings, H. O. Campbell, Oklahoma Agricultural and Mechanical College, Stillwater.
6. The Effect of Copper-Sulphate Treatment on Ground Resistance, A. H. Duncan, Kansas State College, Manhattan.
7. An Electrical Wind Direction Recorder, R. B. Pearce, Texas Agricultural and Mechanical College, College Station.
8. A Rolling Ball Slipmeter, J. H. Treadwell, Rice Institute, Houston, Texas.
9. Air Conditioning, J. L. McKinley, University of Arkansas, Fayetteville.

Analysis of Registration at Dallas Meeting

Classification	Location: Dallas	District 7*	Other Districts	Totals
Members	50	80	22	152
Students	20	158	1	179
Men guests	100	52	6	158
Women guests	15	26	3	44
Totals	185	316	32	533

* Outside of Dallas Section.

The content and presentation of the papers by the students inspired many favorable comments among members of the Institute who attended these sessions. The students exhibited keen interest throughout the day as was shown by the attendance, the attention to the presentations, and participation in the discussions.

Conference on Student Activities

The District conference on student activities was held as a luncheon meeting on Tuesday with Professor J. S. Waters, of Rice Institute, Houston, Texas, chairman of the District committee on student activities, presiding. Each of the 14 Branches in the District was represented by its counselor, or chairman, or both, only one counselor and one chairman being absent. Branch counselors present were:

W. B. Stelzner, University of Arkansas, Fayetteville
L. M. Jorgenson, Kansas State College, Manhattan
E. W. Hamlin, University of Kansas, Lawrence
I. H. Lovett, Missouri School of Mines and Metallurgy, Rolla
M. P. Weinbach, University of Missouri, Columbia
Chester Russell, Jr., University of New Mexico, Albuquerque
A. Naeter, Oklahoma Agricultural and Mechanical College, Stillwater
C. T. Almquist, University of Oklahoma, Norman
J. S. Waters, Rice Institute, Houston, Texas
H. F. Huffman, Southern Methodist University, Dallas, Texas
H. C. Dillingham, Texas Agricultural and Mechanical College, College Station
C. V. Bullen, Texas Technological College, Lubbock
J. A. Correll, University of Texas, Austin

Branch chairmen present were:

R. C. Eldridge, Jr., University of Arkansas, Fayetteville
H. H. Harris, Kansas State College, Manhattan
J. W. Howard, University of Kansas, Lawrence
E. W. Logan, Missouri School of Mines and Metallurgy, Rolla
C. E. Owings, University of Missouri, Columbia
Robert Bonney, University of New Mexico, Albuquerque
Wm. C. Russell, Oklahoma Agricultural and Mechanical College, Stillwater
W. F. Hildebrand, University of Oklahoma, Norman
W. E. Brice, Rice Institute, Houston, Texas
R. S. Hestand, Southern Methodist University, Dallas, Texas
J. J. Caldwell, Jr., Texas Technological College, Lubbock
I. E. Caldwell, University of Texas, Austin
W. W. Otis, Washington University, St. Louis, Mo.

The conference was attended also by F. Ellis Johnson, chairman of the Institute's committee on Student Branches.

After a discussion of the problems involved in awarding prizes for the student technical papers presented at this meeting, it was decided that, because of late receipt by the judges of copies of the papers, the awards would be deferred for about 3 weeks. In a general discussion regarding prizes, it was suggested that the publication in Electrical Engineering of outstanding student papers would be the most desirable type of prize that could be offered and would be more effective than the publication of papers written especially for students.

Each Branch chairman reported briefly regarding the principal activities of his Branch. These reports showed a keen interest in the conduct of Branch activities and extensive knowledge of the many features involved. Considerable emphasis was placed upon the importance of refreshments as an incentive toward increased attendance. Methods of increasing interest, discussed also, included provisions for speakers of recognized ability several times a year as well as talks by members of the Branches.

Professor Chester Russell, Jr., of the University of New Mexico, Albuquerque, was elected chairman of the District committee on student activities to take office on May 1, 1937.

President MacCutcheon complimented the counselors and students upon the excellent type of the conference.

District Executive Committee Meeting

The executive committee of the South West District met at luncheon on Wednesday with the chairmen of all 6 Sections in the District present; in addition, the secretaries of 2 Sections and the vice-chairman of 1 were present. Those present were:

L. T. Blaisdell, vice-president, South West District
L. C. Starbird, secretary, South West District
H. R. Fritz, chairman, St. Louis Section
A. L. Maillard, chairman, Kansas City Section
E. D. Freeman, chairman, Oklahoma City Section
C. E. Bathe, secretary, Oklahoma City Section
L. E. Cook, chairman, Dallas Section
O. S. Hockaday, secretary, Dallas Section
E. G. Conroy, chairman, San Antonio Section
M. C. Hughes, chairman, Houston Section
A. M. MacCutcheon, national president
H. H. Henline, national secretary
M. H. Lovelady, vice-chairman, San Antonio Section

Vice-President Blaisdell presided and District Secretary Starbird recorded the proceedings.

Professor Joseph W. Ramsay, of the University of Texas, Austin, was selected to represent the District on the national nominating committee. The chairman of each Section discussed briefly the methods used by his Section to increase interest in the meetings and membership in the Section. These reports were followed by a discussion of local membership in Sections, technical committees in District and Sections, and the desirability of publishing more articles of broad interest in Electrical Engineering. President MacCutcheon urged that the Sections try to interest their members in the activities of the national technical committees to such an extent that they will submit suggestions regarding committee personnel.

Entertainment and Inspection Trips

The committees offered a well-balanced program of inspection trips and special entertainment features, with the Texas Centennial Central Exposition serving as the principal attraction. Nearly 80 registered for the inspection of the electrical

features of the Centennial on Monday afternoon and for the inspection of the exterior illumination on Monday evening. Also on Monday evening about 225 saw the 400th showing of the Cavalcade of Texas, a pageant depicting on a mammoth outdoor stage the principal events in the history of Texas under 6 flags. This performance was dedicated to the members of the Institute and was so announced.

In addition to the visit to the exposition, there were several inspection trips on Monday afternoon which were of especial interest to power and telephone engineers, including the 85,000-kw generating station and the underground network system of the Dallas Power & Light Company, a low-cost farm-electrification project of the Texas Power & Light Company, and the toll office of the South Western Bell Telephone Company.

The dinner dance on Tuesday evening was attended by 268 and was a most enjoyable occasion. On Wednesday afternoon a group went to Fort Worth by special bus to attend the Frontier Centennial Exposition, including a showing of the spectacular Casa Mañana musical revue. The program of additional events for the women was comprehensive and attractive, including several showings at the Centennial, tea on the Chrysler Roof, and others.

In commenting upon the program of entertainment features, the entertainment committee said: "The most outstanding point in connection with the entertainment program was the friendly and enthusiastic spirit shown by the members and guests. . . ." This response from the participants only reflected the excellence of the whole program and the effective manner in which it was executed.

Electrical Insulation Is Subject of Continued Study by National Research Council

FOR the ninth annual meeting of the National Research Council's committee on electrical insulation, some 110 or more chemists, physicists, electrical engineers, and others interested in the ramified subject of dielectric research gathered in Cambridge at the Massachusetts Institute of Technology, November 5–7, 1936. Bound together by a common interest in the improvement of electrical insulating materials and methods, this loosely organized group has grown by wholly natural processes from the meager beginning initiated 8 years or so ago by a small group of farsighted leaders who saw the subject as one of fundamental importance to the electrical industry. These annual meetings, in addition to serving as a medium for the exchange of new information and for the frank discussion of new problems and new phases of old problems, also have served effectively as something of a proving ground for almost everything within the scope of the subject from new theories to new synthetic products.

Of interest and significance in connection with the evolution of this work is the increasing participation of, and the important contributions of the chemist and the physicist to a problem commonly regarded not so long ago as being largely mechanical. Through the committee important advances have been made toward a co-ordination of efforts in and a correlation of results from many related fields. These trends have been noticeable for some time, but seemed quite pronounced this year.

Representing MIT as official host, Dr. Vannevar Bush (A'15, F'24) dean of engineering and vice president of MIT, and this year's chairman of the division of engineering and industrial research of the National Research Council, delivered the address of welcome, encouraging the committee in the conduct of its work. Cofounder and continuing chairman, Dr. J. B. Whitehead, dean of the school of engineering at Johns Hopkins University and past-president of the AIEE (1933–34), presided at the technical sessions.

Fluorescent Microscopy

Other features of the program included several subcommittee and informal group meetings for the discussion of special problems, the annual dinner meeting held Thursday evening at the Copley Plaza Hotel, Boston; a luncheon Friday noon tendered by MIT at its Walker Memorial, and a personally conducted inspection of the MIT laboratories. At the Thursday dinner meeting, MIT Professor Ernst A. Hauser, presented a most amusing and illuminating demonstration lecture on the subject of "colloids" as they do or may have a bearing upon the further development of electrical insulating materials. He also revealed something of the possibilities of the art of "fluorescent microscopy," by means of which the researcher "is enabled to make direct observational measurements of many phenomena otherwise mathematically difficult or impossible to determine or predict . . . can observe and directly measure the character and extent of emulsification by observation of the fluorescence of the interfaces. . . ." Also, a showing of Prof. H. E. Edgerton's famous high speed motion pictures was made, demonstrating their adaptability to research problems.

Full Technical Program

A total of 26 titles representing the efforts of 35 authors appeared on the program for the 3 technical sessions. For those who may be interested in following some phases of this work further, the complete list of titles and authors is printed herewith. Of these, Doctor Whitehead's annual report has been published in ELECTRICAL ENGINEERING (November 1936, pages 1180–5) and papers by Doctor Baños and by Professor Dawes and Mr. Reiter covering the scope of the work reported on by them are now on the docket for publication in forthcoming issues of ELECTRICAL ENGINEERING; Dr. Race's paper may be found on pages 1347–56 of this issue. Many other subjects represent extensions of work in fields previously reported in AIEE papers, or work that is likely to be covered by future papers if and when the information has been developed to a point warranting such treatment.

Theory of Dielectrics

RECENT PROGRESS IN RESEARCH (annual report of chairman), J. B. Whitehead (A'00, F'12, past-president) Johns Hopkins University, Baltimore, Md.

THEORY OF DIELECTRIC CONSTANT OF SOLIDS, J. H. VanVleck, Harvard University, Cambridge, Mass.

Author stated that the interaction of dipoles is in fact a kinetic problem that should be handled by methods of statistical mechanics. He has attempted to do this, and produced some evidence tending to explain apparent inconsistencies between theory and experiment.

THE DIELECTRIC PROPERTIES AND THE CONSTITUTION OF LIQUIDS, Hans Mueller, Massachusetts Institute of Technology, Cambridge.

A discussion of the same [above item] difficulty in the case of liquids and stated that if a translation as well as a rotation of the polar molecule be considered, some of the discrepancies of experiment are accounted for.

RELATION BETWEEN DIELECTRIC CONSTANT AND DENSITY IN COMPRESSED GASES, F. G. Keyes and J. L. Oncley, Massachusetts Institute of Technology, Cambridge.

A third method of approach to the problem of determining quantities for use in Lorentz's formula.

DIPOLE ROTATION IN CERTAIN ORGANIC SOLIDS, A. H. White, Bell Telephone Laboratories, New York, N. Y.

Data tending to show that in some types of molecules there is free orientation of dipoles, while in others the structure is such as to preclude such motion.

LIMITATIONS OF THE VON SCHWEIDLER METHOD OF A-C COMPUTATION, A. Baños, Jr. (A'31) Massachusetts Institute of Technology, Cambridge.

A discussion of the use of von Schweidler's theorem in correlating a-c and d-c data, as used by Dr. Whitehead in several researches. He showed mathematically that while 3 terms of the series are sufficient for loss correlation, as used by Dr. Whitehead in predicting a-c losses from d-c data, 3 terms would not be at all sufficient for correlation of capacitance. [*See pages 1329–37*]

PROPERTIES OF VACUUM INSULATION, J. G. Trump (A'31) Massachusetts Institute of Technology, Cambridge.

A description of some experiments on the use of vacuum as an insulator, his work being with static electricity. With a pressure of 0.00004 millimeter of mercury the breakdown strength between stainless steel electrodes is about 5 million volts per centimeter at 0.02 millimeter separation, dropping down to 100,000 volts per centimeter at 0.70 millimeter separation. Pressure of 0.0005 millimeters of mercury is as good an insulator as better vacuum. With pressures as low as this the limitations have to do with the materials of the surfaces of the electrode.

DIELECTRIC LOSS IN AIR CONDENSER, A. V. Astin, Bureau of Standards, Washington, D. C.

A report on work being done at the Bureau of Standards on the power factor of 3-electrode air capacitors. The values of power factor found were in the range of 0.0001 to 0.000001. The loss is not due to the gas itself, but to a surface condition of the electrodes. Measurements made on the same plates with variable separation indicate that the film may be represented as a series resistance. Lowest power-factor values were obtained with gold and nickel, highest values with aluminum. No data were given on monel metal or stainless steel, which are being used by other people on the assumption that they have very low losses.

Oxidation in Insulating Liquids

STABILITY OF SATURANTS FOR SOLID-TYPE PAPER-INSULATED CABLES, G. M. L. Sommerman (A'31) American Steel & Wire Company, Worcester, Mass.

Oils containing rosin are generally more stable than oils without rosin. Metal decreases the stability of oil, the effect being less marked, how-

ever, in oils of low viscosity index (naphthenic base). Gassing and wax formation under stress is less pronounced in oils rich in naphthenic or aromatic compounds.

Oxidation of Hydrocarbons as Influenced by Electrodeless Discharge, N. A. Milas, Massachusetts Institute of Technology, Cambridge.

A chemical explanation of oxidation.

Chemical and Electrical Studies of Oil Oxidation, J. C. Balsbaugh (A'23, M'35), R. G. Larsen, and D. A. Lyon (A'29), Massachusetts Institute of Technology, Cambridge.

A report of results of oxidation under direct contact with oxygen. No strict correlation found between power factor and any definite oxidation product, the best correlation being with total oxygen absorbed by the oil. One important difference between the method used in this investigation and those of most previous investigations is that the oxidation products were maintained in contact with the oil. Authors mentioned that the degassing of a sample that had a power factor of 4 per cent resulted in reduction of the power factor to 0.5 per cent.

Progress Report on Some Chemical Changes Associated with Oxidation of Insulating Oil, R. N. Evans, Brooklyn Edison Company, Brooklyn, N. Y.

A description of tests in which oil was deteriorated by ultra-violet radiation, the measure of deterioration being the amount of acid formed. Reported that where no oxygen is present, no acid is formed. Copper in the oil accentuates the effect. Paper in the oil reduces the effect.

Gas Evolution in Mineral Oils When Subjected to Electrical Discharge, L. J. Berberich (A'30) Socony-Vacuum Oil Company, Paulsboro, N. J.

Data from tests on gassing of oil under stress, the electrical discharges being chiefly in a gas space above the oil. In general it was found that the more refined the oil, the more rapid the gassing. Addition of 0.2 gram of paper in the form of a powder to 70 cubic centimeters of oil greatly increased the gassing. A mixture of highly refined with lightly refined oil gave results closer to those for the lightly refined oil. Comparison of oils of naphthene base with those of paraffine base showed little difference in respect to gas formation. The gas evolved consists of from 80 to 90 per cent hydrogen, less than 1 per cent carbon dioxide and the rest chiefly combustible gas.

Oil Oxidation, R. W. Dornte, General Electric Company, Schenectady, N. Y.

Author stated that oils fall into 3 classes with regard to oxidation: (1) those in which the rate of oxidation increases with time, indicating presence of an inhibitor; (2) those in which the rate is constant; and (3) those in which the rate decreases with time, indicating using up of reagent. Addition of copper enormously increases the rate of oxidation, whereas addition of iron, tin, or lead increases the rate only slightly. Principal products of oxidation of oil are water and carbon dioxide.

Oxidation and Power Factor in Insulating Oil, J. B. Whitehead.

Authors oxidized at 125°C. oil of the type used in oil-filled cables and measured a number of physical, chemical, and electrical characteristics. Among others, they found linear relations between oxygen absorption and time, acid number and time, carbon dioxide formed and time, power factor and acid number, final d-c conductivity and acid number. Dielectric constant increased 5 per cent when acid number reached 0.2, but started down again as acid number increased. Ratio of d-c to a-c resistivity ranged from 250 for fresh oil to about 7 for the most oxidized oil, the power factor of which was 1.3 per cent at 45°C., at which temperature all electrical measurements were made.

Sorption of Some Organic Acids from Oil by Paper, J. D. Piper, N. A. Kerstein, and A. G. Fleiger, Detroit Edison Company, Detroit, Mich.

Authors stated that anhydrous oxidation products produce no increase in power factor, provided they are soluble in the oil. Acid increases the power factor markedly. Presence of paper in the oil makes the increase with acid much more marked because of sorption of acid by the paper.

The Influence of Moisture on the D-C Conductivity in Impregnated Paper, D. A. McLean and G. T. Kohman, Bell Telephone Laboratories, New York, N. Y.

Concerns the effect of moisture on the conductivity of unsaturated paper. The moisture content of the paper was determined by heating to 150°C., evacuating and trapping the moisture. The paper used was standard capacitor paper and voltages up to 40 kilovolts per millimeter (1,000 volts per mil) were used. Exposure to various degrees of humidity showed enormous increase of conductivity with per cent relative humidity. Increase of conductivity with time for high moisture content was noted, but not explained.

Insulating Materials and Practice

Research in Rubber Insulation, C. R. Boggs, Simplex Wire and Cable Company, Cambridge, Mass.

Reports that, after much research, selenium can now be used satisfactorily as a vulcanizing agent. If rubber is de-proteinized, its water absorption characteristics are greatly improved. Author stated that the best rubber insulation known is deproteinized vulcanized latex, as it has high dielectric strength, low power factor, low specific inductive capacity, high insulation resistance, and good water-absorption characteristics.

Corona Characteristics of Rubber Insulated Wires, E. B. Paine (A'04, M'12) and H. A. Brown (A'16, M'26) University of Illinois, Urbana.

Effect of corona on rubber discussed, especially in relation to the high-frequency transient voltages resulting from corona discharges.

Power Factor Changes of Some Commercial Rubber Insulations with Aging, W. J. Warren (A'34) University of Illinois, Urbana.

See preceding item.

Koroseal—A New Synthetic, Elastic Material, H. E. Fritz, B. F. Goodrich Company, Akron, Ohio.

Data on a synthetic "rubber-like" material, for which is claimed good mechanical and fair electrical characteristics; also good heat-resistance. This material has been used for insulated cable up to 600 volts.

DuPrene, J. B. Miles, Jr., E. I. duPont de Nemours & Company, Wilmington, Del.

Miles (duPont) gave electrical data on a synthetic "rubber-like" material "DuPrene," He said the electrical properties of which are said to be much inferior to rubber, though it is far superior in a great many other ways.

The Dielectric Characteristics of Suspension Insulator Disks, C. L. Dawes (A'09, M'28) and Reuben Reiter, Harvard University, Cambridge, Mass.

Data concerning the effects of humidity, corona, and mechanical stress on the electrical characteristics of porcelain and glass units.

The Relation of Surface Electrical Resistance to Chemical Durability, E. M. Guyer, Corning Glass Works, Corning, N. Y.

Calorimetric Measurements of Dielectric Loss at High Frequency and High Voltage, H. H. Race (A'24, M'31) General Electric Company, Schenectady, N. Y.

A description of a method of measuring dielectric losses in insulators at 1,000,000 cycles and 10-kilovolts by means of a calorimeter. Instead of allowing the calorimeter to come to a steady condition, which would require several hours, makes use of differential temperature between 2 thermocouples as an indication of heat generated. This permits a measurement to be made in a few minutes. Calibration is by direct current. Results obtained are far more consistent than any one could have expected. [*See pages 1347–56*]

Cable Research in Europe Versus Cable Research in America, K. S. Wyatt (A'32) Detroit Edison Company, Detroit, Mich.

Some of the high lights of a recent trip to Europe, stressing the importance of more laboratory control of processes of manufacture.

A. C. Walker of the Bell Telephone Laboratories suggested a theory as to how moisture is held in fibrous insulation. Author drew a picture of what he thought a cotton fibre must look like and on this basis computed moisture content under different conditions. The check with measured values was very good.

Through the courtesy of Dr. R. W. Atkinson (A'09, F'28) director of high-voltage research for the General Cable Corp., Perth Amboy, N. J., the foregoing summary and comments prepared by Dr. Louis Meyerhoff are made available.

Columbia University Scholarship Not Awarded

The special tuition scholarship at Columbia University placed at the disposal of the AIEE each year, which has been awarded to some student for many years past, was peculiarly affected by the rising barometer in business conditions which resulted in not one definite application for the scholarship for the academic year 1936–37, according to Committee Chairman W. I. Slichter (A'00, F'12, National treasurer). There were several inquiries early in the spring, but each of these inquirers failed to send in the definite information required of contenders, that is, academic record and supporting letters; this fact was interpreted by the committee to mean that the individuals had found employment that was more attractive to them than advanced study.

An analysis of this condition shows that it is quite logical but that it does not mean that the interest in this scholarship will not revive later. Many students have carried their education to completion at a severe financial sacrifice to their families, and even to the extent of incurring indebtedness; these students naturally feel that they should begin to earn and repay these obligations as soon as possible. It is expected that with still further improvement in economic conditions many students will be able to complete their regular college work and take an additional year of graduate work without serious financial burden to their families.

ECPD Receives Grant From Carnegie Foundation

On October 22, 1936, the trustees of the Carnegie Corporation of New York resolved that "the sum of $16,000 be, and it hereby is appropriated to the Carnegie Foundation for the Advancement of Teaching, toward support of the program of the Engineers' Council for Professional Development."

Continued Carnegie interest in engineering activities thus is evidenced once again. It began in 1903 with a donation of $1,050,000 by Andrew Carnegie for erection of the Engineering Societies Building at 33 West 39th Street, New York City. In later grants the Carnegie Corporation has generously supported the Society for the Promotion of Engineering Education in its investigation of engineering schools extending over a period of 5 years, as well as the summer schools for engineering teachers instituted and maintained by that organization.

The present $16,000 grant is made in support of the work of ECPD for the year October 1, 1936, to September 30, 1937. Additional appropriations in the amount of $3,450 have been made to Council by The Engineering Foundation. Rental cost for Council's office recently opened in the Engineering Societies Building is being met by 3 of the participating societies, the American Society of Civil Engineers, The American Society of Mechanical Engineers, and

the AIEE. Including this rental value a total fund of more than $21,000 is available, of which $19,450 will be applied directly to the advancement of Council's program for enhancing the professional status of the engineer through the co-operative support of the national organizations directly representing the professional, technical, and legislative phases of the engineer's life.

In connection with the newly opened office of ECPD, Brigadier-General R. I. Rees,* retiring assistant vice-president of the American Telephone and Telegraph Company, had planned to devote full time to his duties as vice-chairman of the Council. General Rees was also chairman of the ECPD committees on professional training and ways and means.

* Deceased November 23, 1936; see news item elsewhere in this issue.

The Institute Budget for 1936–37

INFORMATION regarding the finances of the Institute is now furnished to the membership on 2 occasions: first, in the report of the board of directors at the close of the fiscal year when complete financial statements are presented by certified public accountants, and, second, with the adoption by the board of a budget of income and expenditures for the appropriation year beginning October 1, as recommended by the finance committee.

The financial reports for the fiscal year, which ended April 30, 1936, were published in due course in the July 1936 issue of ELECTRICAL ENGINEERING. The following data are now presented to the membership for its information regarding the scope of the appropriation budget for 1936–37 as adopted by the board of directors at its meeting on October 20, 1936.

PUBLICATIONS

Electrical Engineering-Transactions. In the tabulation that forms a part of this article it may be noted that approximately $68,600 was expended in the preceding appropriation year for text matter; of this sum, $3,750 was required for the TRANSACTIONS. The budget for that year originally contemplated the publication of 1,400 pages of such material. Subsequent developments made it necessary to revise the appropriation to take care of increased technical material, the total text pages reaching 1,472 with the publication of the September issue.

The budget for 1936–37 provides for the publication of approximately 1,500 pages of technical papers, discussions, special articles, and news of Institute and related activities, and contemplates an increase in the number of such pages devoted to special articles considered to be of broad interest to the membership. The publication committee is planning a study of the present publication policy with a view to submitting recommendations at a later date as to what changes, if any, should be made in order to improve the service which ELECTRICAL ENGINEERING is rendering to all members of the Institute.

The expenditures for Institute TRANSACTIONS reflect the cost of publishing in bound form the same text pages which have appeared in ELECTRICAL ENGINEERING for the corresponding period, the publication being distributed only to members and others who maintain a subscription at the prescribed rates. The 1936 volume of the TRANSACTIONS should be ready for shipment to such subscribers early in January 1937.

Yearbook. There having been sufficient indication that a revised edition of the YEARBOOK is again desired by the membership each year, the present budget provides for the publication of a 1937 edition, containing about the same material as heretofore and corrected to give business and mailing addresses as of March 1, 1937. A return card was recently circulated to the entire membership for the purpose of verifying the address records at Institute headquarters, and all members are urgently requested to return this card with information regarding present business affiliations and any necessary changes in mailing addresses.

Copies of the Institute YEARBOOK are available to members of the Institute upon receipt of request, an announcement appearing in the news column of ELECTRICAL ENGINEERING when the book is ready for distribution.

INSTITUTE MEETINGS

The appropriations for national and District meetings absorb the expenses for publicity (announcements to the membership other than those issued in ELECTRICAL ENGINEERING), meeting supplies, such as convention badges, printed forms, meeting programs, and other such items, and the traveling expenses of any members of headquarters staff who may attend the meeting. The new budget provides for such expenses for the winter convention to be held in New York City in January 1937 and the summer convention in Milwaukee in June 1937 and also provides for the South West District Meeting held in Dallas, October 26–28, 1936, and the North Eastern District Meeting to be held in Buffalo, May 5–7, 1937. In addition, provision is made for items of expense incidental to the 1937 Pacific Coast Convention, which will become payable during this appropriation year, as well as routine expenses incurred at Institute headquarters and chargeable directly to meetings' activities.

INSTITUTE SECTIONS

In the appropriation for Institute Sections, each Section is assured of financial support to the maximum extent provided for in the Institute by-laws, that is, a flat appropriation of $175, plus an allowance of $1 for each active member located within the territory of the Section on August 1. Traveling expense allowances for Section representatives are referred to elsewhere in this article; other expenses, as indicated in the budget tabulation, comprise the cost of miscellaneous supplies furnished to Sections by Institute headquarters, as well as that proportion of the staff payroll which is allocated to Section activities.

INSTITUTE BRANCHES

Routine expenditures incidental to the meetings of Institute Branches have been provided for to the same extent as last year; a slight increase has been allowed in the appropriation for miscellaneous printing and supplies furnished to Branches by Institute headquarters, to which appropriation is also charged that portion of headquarters salary expense incurred because of services performed for the Branch organizations.

ADMINISTRATION

As may be noted from the budget tabulation, more than $^3/_4$ of the appropriation for administrative purposes is allocated to salaries—41 per cent of the total salaries paid to 20 members of headquarters staff, in addition to the full salaries of the national secretary and office manager, being charged to the appropriation. The Institute still maintains the smallest staff per 1,000 members of any of the Founder Societies.

Other items in this appropriation are provided for to the same extent as last year, and cover such expenditures as postage, printing and mailing of general announcements to the membership, telephone and telegraph services, insurance, office equipment, stationery and printing, miscellaneous supplies, and services not chargeable to specific appropriations.

TRAVELING EXPENSES

The budget again provides for reimbursement of substantially all the expense incurred for railroad fare, Pullman accommodations, and meals en route, in the traveling allowance to District, Section, and Branch representatives, to members of the board of directors, and to members of the national nominating committee. The traveling allowances are reimbursed on a uniform rate of $7^1/_2$ cents per mile, one way, mileage being obtained from the *Official Table of Distances* compiled by the United States War Department. The authorized traveling expense appropriations are as follows:

1. For each vice-president of the Institute to one meeting each year of each Section and each Student Branch within his geographical District, it being understood that joint meetings of Sections and Branches will be arranged as far as may be expedient.

2. For the vice-president, the District secretary, and chairman of the District committee on Student activities, and either the chairman or the secretary of each Section within a District (or, if neither can attend, an alternate chosen by the executive committee of the Section) to one meeting each year of the District executive committee held within the District.

3. For the vice-president and secretary of each District, the counselor and the incoming Student chairman of each Branch within the District, and the appointed member of the committee on Student Branches located in the District, to one conference on student activities within the District each year under the auspices of the committee on student activities of the District. *Alternates for counselors*

Table I—Institute Income and Expenses for Year Ending September 30, 1936, and Budget for Year Ending September 30, 1937

	Actual Income and Expenses Year Ending 9/30/36	Budget for Year Ending 9/30/36
ncome		
Dues	$187,562.84	$195,125.00
Students' fees	9,604.50	9,500.00
Entrance fees	6,573.85	6,000.00
Transfer fees	1,314.00	1,250.00
Advertising	25,653.10	26,500.00
Elec. Engg.—non-mem. subscriptions	13,045.68	13,000.00
Trans. subscriptions	7,111.54	7,000.00
Miscellaneous sales	6,707.19	6,000.00
Badge sales	1,676.50	1,600.00
nterest on securities	8,483.63	8,500.00
Total	$267,732.83	$274,475.00
Expenses		
Publications		
Text matter	68,599.19	71,180.00
Advertising section	11,077.38	11,500.00
Yearbook	6,351.28	6,560.00
nstitute Meetings	10,837.68	12,250.00
nstitute Sections		
Appropriations	20,818.94	23,000.00
Trav. exp. convention delegates	10,207.37	4,300.00
Other expenses	5,630.32	5,950.00
nstitute Branches		
Meetings expense	924.03	1,000.00
Trav. exp. District conferences on student activities	4,887.11	6,200.00
Trav. exp. counselor delegates to conv.	1,635.56	800.00
Other expenses	1,752.62	1,950.00
Administration		
Salary expense	32,139.50	32,500.00
Postage	3,217.21	3,400.00
Stationery and printing	3,199.87	3,300.00
Office equipment	606.64	500.00
Telephone, telegraph, traveling, insurance, supplies, and misc. service expenses	3,272.26	2,900.00
Membership Committee	6,946.72	7,800.00
Standards committee	5,238.95	8,245.00
Finance committee	1,600.00	1,600.00
Amer. Standards Assn.	1,000.00	1,375.00
Amer. Engg. Council	11,500.00	12,000.00
Engineering Societies Employment Service	2,756.17	1,973.00
Traveling exp. general		
Board of directors	6,695.81	7,785.00
Geographical Dists.	1,616.72	3,150.00
National Nominating committee	652.06	900.00
President's appropriation	1,245.35	2,000.00
AIEE representatives	24.00	100.00
United Engg. Trustees		
Building assessment	5,882.28	5,882.00
E.C.P.D. space assessment	....	630.00
Deprec. and renewal fund assessment	....	5,000.00
Library assessment	8,759.64	8,790.00
Library stacks	2,111.38	....
Other committees and misc. expenses	10,891.31	11,495.00
Reserve Capital Fund investment	....	7,500.00
Total	$252,077.35	
Excess income, of which board of directors have authorized investment of $7,500.00 for Reserve Capital Fund	15,655.48	
Total	$267,732.83	$273,515.00

not authorized. The allowance is available to alternates for Branch chairmen only upon advance approval by the vice-president of the District in each case.

4. For one delegate from each Section to the annual summer convention.

5. For all District secretaries to the annual summer convention.

6. For one Student Branch counselor from each District, to represent the committee on student activities of the District, to the annual summer convention.

7. For all members of the national nominating committee to the annual meeting of the committee, held during the winter convention.

8. For members of the board of directors and the executive committee to meetings of the respective bodies.

It will be noted that the traveling expense budget again provides for the attendance of the District secretaries at the conference of officers, delegates, and members held during the summer convention, an item omitted from the budgets of the past few years because of economic reasons; furthermore, the board of directors also authorized an allowance of traveling expenses to each District conference on Student activities for the appointed member of the committee on Student Branches located in that District.

Other Activities

The remainder of the budget comprises those items for which detailed explanations—beyond a statement of the appropriation or activity, the amount expended last year, and the anticipated expense for the present year—probably are unnecessary. Nevertheless, brief comments regarding a few items may be of interest. For example, the Institute is prepared to meet its share of an annual increase of $20,000 in the Depreciation and Renewal Fund of United Engineering Trustees, Inc., payments to this reserve from the operating revenues of U.E.T. having necessarily ceased in 1931.

Incidental to the recent grant received by the Engineers' Council for Professional Development from the Carnegie Foundation for the Advancement of Teaching, to be applied directly to the development of Council's program for enhancing the professional status of the engineer, three of the national engineering societies have agreed to underwrite the expense for office rent of this undertaking; the budget of the Institute provides accordingly for its share of this expense.

At its June 1936 meeting, the board of directors, upon recommendation of the finance committee, voted to increase the appropriation for American Engineering Council from $10,000 to $12,000 annually, and the 1936–37 budget assumes the continuance of an annual contribution of the latter sum.

Beginning on January 1, 1937, the number of Institute representatives on the American Standards Association will be restored to three, unfavorable financial conditions having caused the withdrawal of one representative a few years ago. The budget provides for a proportionate increase in the assessment of American Standards Association on that basis.

Several projects of the AIEE standards committee for which funds were available in the 1935–36 budget did not materialize, with the result that there is a corresponding increase in funds for this activity provided for in the present budget. Two projects in particular, the revision of Standard 45 (Recommended Practice for Electrical Installations on Shipboard) and the proposed new Standard on Electrical Definitions, will involve unusual expenditures this year. Other cases in which revised editions or new standards are contemplated are as follows:

Measurement of Test Voltages in Dielectric Tests
Railway Motors
Transformers, Induction Regulators and Reactors
Industrial Control Apparatus
Graphical Symbols—17g1, 17g2, 17g3, 17g5, and 17g6 under revision. Several will be reprinted during the year.
Automatic Stations
Electrical Measuring Instruments
Insulators
Indicating Instruments
Electrical Recording Instruments
Test Code for Transformers & Test Code for Synchronous Machines
Test Code for D-C Machines
Test Code for Induction Machines

Upon the recommendation of the finance committee, the board of directors provided in last year's budget for the engagement of investment counsel, on a yearly basis, to supervise and make recommendations relating to the management of the investments of the Reserve Capital Fund and the special funds for which the Institute acts as trustee. It was felt that in view of the existing factors affecting the investment situation, which make it difficult for a layman to keep informed of changing conditions, such assistance would prove helpful in protecting the security of these investments. The 1936–37 budget provides for the renewal of our contract for another year, and also covers the customary annual audit of Institute accounts by certified public accountants.

It is possible to dwell upon but briefly, in so short a presentation, the various factors affecting the nature and relative importance of the items comprising the annual budget, and additional information regarding particular activities will be furnished upon request. The board of directors and finance committee endeavor each year to adopt a budget that places the proper emphasis on the different phases of Institute activities, and which limits the annual expenditures to the total of anticipated income for the corresponding period. Thus, the prompt collection of membership dues plays a most important part in carrying out the full program of Institute activities provided for in the budget, a situation which is fully appreciated by the membership, as evidenced by the unusually high percentage of active members at this time.

North Central District Executive Committee Meets

The executive committee of the Institute's North Central District (6) held its annual meeting in Denver, Colo., November 2, 1936. Those in attendance were: R. H. Fair, vice-president, District 6; N. R. Love, vice-chairman, Denver Section; R. H.

Owen, secretary, Denver Section; H. M. Craig, chairman, Nebraska Section; W. H. Gamble, chairman, District committee on Student activities; and T. H. Granfield, District secretary. President A. M. MacCutcheon also attended the meeting and contributed materially to the discussions relating to Institute policies.

L. N. McClellan, chief electrical engineer, United States Bureau of Reclamation, Denver, Colo., was unanimously chosen as the nominee for vice-president. A. L. Turner, chief engineer, Northwestern Bell Telephone Company, Omaha, Neb., was chosen to represent the District on the national nominating committee. The vice-president was authorized to appoint an alternate to represent the District in the event that Mr. Turner is not able to attend the meeting of that committee.

W. D. Hardaway, superintendent, hydroelectrical production and transmission, Public Service Company of Colorado, Denver, was chosen to succeed himself as chairman of the District committee on prize awards for papers presented during the calendar year 1937. Other members of this committee are to be appointed by the chairmen of the Denver and Nebraska Sections, each chairman to appoint one member to represent his Section.

There was some discussion concerning activities of the student Branches within the District, particularly with reference to the proposed student conference to be held at the South Dakota State College, Brookings, in April 1937. It was considered desirable to hold the conference at Brookings in spite of the fact that inspection trips could not be combined with the conference.

Membership activities also were discussed, and representatives of the Denver Section stated that the practice of having members bring prospective members to Section meetings as guests had been found to be helpful in obtaining new members.

The executive committee attended a luncheon with members of the Denver Section, given at the Denver Club in honor of President MacCutcheon, after which a brief review of District activities in general was held by the committee.

Review Courses Offered by N. Y. Section

If there is sufficient demand, the power group of the Institute's New York Section will repeat the review courses in structural planning and design and in electrical engineering, with the first sessions to be held during the second week of February 1937. Applicants for New York State professional engineers license who intend to enroll for either or both of these courses should put their names and addresses on file with Otto W. Manz, Jr., chairman of the Section's related activities committee, 55 Johnson Street, Brooklyn, N. Y.

Announcements of the final arrangements of these and any other courses to be sponsored by the power group are scheduled to be mailed some time in December. Enrollment for all educational activities of the Section is open to nonmembers as well as members of the Institute.

C. F. Kettering Honored by General Motors Corp.

Charles Franklin ("Boss") Kettering (A'04, F'14) was the guest of honor at a luncheon given Tuesday, November 10, 1936, at the Waldorf Astoria Hotel in New York City by Alfred P. Sloan, Jr., president of General Motors Corporation in commemoration of the 25th anniversary of Kettering's development and successful application of the electric starter for automobiles. The 188 other guests present constituted an unusual and impressive concentration of business, industrial, and educational leaders of national and international repute. Co-featured with Honor Guest Kettering was the evidence of his handiwork, the first commercially successful electric starter, pedestal-mounted and unveiled at the psychological moment, supplemented by an historic model of a Cadillac motor car,

C. F. Kettering

Regarded by qualified persons as the Number 1 man of contemporary industrial research

representing the model of 1911, to which the starter was first applied.

Introduced by Host Sloan; radio commentator Edwin C. Hill, who outlined Guest Kettering's colorful and self-made career; Board Chairman W. A. Harriman of the Union Pacific Railroad, who commented on the extent and significance of current improvements in surface transportation and urged "that among other problems studied, the engineers should apply their efforts toward rewinning public opinion." Flier "Eddie" V. Rickenbacker, who traced briefly the high lights of the startling development of modern air transportation, and pointed to a truly Jules Vernian future; Shipbuilder Daniel H. Cox, who pointed out that water transportation, too, was keeping apace of modern trends, with the help of internal-combustion engines.

Responding in characteristically modest fashion, Doctor Kettering observed that he had "no desire to philosophize on the past, only to direct attention toward an infinite future." Other comments: "Unintelligent motion in research is far more productive for human good than intelligent standing-still . . . . . faith and patience are 2 fundamentals upon which research is based. . . . . . research is a form of insurance—a looking forward to see in what direction industry may or may not go . . . . . if the young people of today will look forward instead of backward there will be no need to worry about the future. . . . . You can't stop human progress unless human mentality stops . . . . ."

Other honors that have come to Docto[r] Kettering this year include the Washingto[n] Award (*EE Feb. '36, p. 218*) and a Frankli[n] Medal (*EE May '36, p. 564*).

John Fritz Medal for 1937 Awarded to A. N. Talbo[t]

Arthur Newell Talbot, professor emeritu[s] of engineering in the University of Illinoi[s] has been awarded the 1937 John Fritz Gol[d] Medal, highest of American engineerin[g] honors. Professor Talbot, who is 79, wa[s] cited as "moulder of men, eminent consul[t]ant on engineering projects, leader of research, and outstanding educator in civil engineering." The award is made annuall[y] for notable scientific or industrial achievement by a board composed of 16 past presidents of the 4 national societies o[f] civil, mining and metallurgical, mechanica[l] and electrical engineers.

Professor Talbot was born in Cortland, Ill. October 21, 1857. He received the degre[e] of bachelor of science from the Universit[y] of Illinois in 1881, and the degree of civi[l] engineer in 1885. Honorary degrees hav[e] been conferred on him by 3 institutions He has been engaged in engineering wor[k] since 1881, his activities embracing railroads, roads, bridges, buildings, and municipal public works. In 1885 he joined th[e] faculty of the University of Illinois as assistant professor of engineering and mathematics. In 1890 he was appointed professor of municipal and sanitary engineerin[g] in charge of theoretical and applied mechanics. He became professor emeritus i[n] 1926.

Professor Talbot aided in the upbuildin[g] of the University's testing laboratories an[d] the college of engineering, and he has bee[n] active in the formation and development o[f] the Illinois Engineering Experiment Station. He also has directed studies sponsored by the American Society of Civil Engineers and the American Railway Engineering Association. For outstanding research in railroad track stresses he wa[s] awarded the plaque of the AREA. He stil[l] continues his research at the University o[f] Illinois.

Professor Talbot has been president of th[e] American Society of Civil Engineers, th[e] Society for the Promotion of Engineerin[g] Education, and the American Society fo[r] Testing Materials, and is a member o[f] many other professional organizations. H[e] received the Washington Award in 1924 the Turner Medal (ASCE) in 1928, th[e] Henderson Medal of the Franklin Institut[e] in 1931, and the Lamme Medal of the Society for the Promotion of Engineering Education in 1932. He is the author of man[y] technical articles, and has contributed t[o] professional journals.

Members of the Institute who have received this medal include: Elihu Thomso[n] (A'84, F'13, HM'28, member for life, past president); Guglielmo Marconi (HM'18) Ambrose Swasey (HM'28); and Herber[t] Hoover (HM'29). Other recipients, n[o] longer living, include: Lord Kelvin (William Thomson) (HM'92); George Westing

house (A'02); Alexander Graham Bell (A'84, M'84, past-president); Thomas A. Edison (A'84, M'84, HM'28); Edward D. Adams (A'10); Elmer A. Sperry (A'84, M'93); John J. Carty (A'90, F'13, HM'28, past-president); M. I. Pupin (A'90, F'15, HM'28, past-president); and Frank J. Sprague (A'87, F'12, HM'32, past-president).

General R. I. Rees, ECPD Vice-Chairman, Dies

On November 23, Brigadier-General R. I. Rees, assistant vice president, American Telephone and Telegraph Company died in Detroit, Mich., while on a field trip for Engineers' Council for Professional Development, of which organization he was vice-chairman. General Rees was one of the organizers of ECPD and one of its most active proponents. He was chairman of the ECPD committee on professional training continuously since council was organized, and was also chairman of its committee on ways and means. He was scheduled to retire from the telephone company on December 1, 1936, and thereafter to devote full time to ECPD.

General Rees was born in Houghton, Mich., in 1871; he received his technical training at the Michigan College of Mining and Technology in that city, from which he was graduated in 1897. From 1897 to 1924, he served the United States Army, becoming second lieutenant in 1899 and advancing to brigadier-general in 1918. During the World War he was a member of the General Staff Corps, serving in the war plans, executive, and operations division, in Washington, D. C., and in 1918 he was sent to France in charge of all educational work of the American Expeditionary Forces. In 1919 he was awarded the Distinguished Service Medal "for exceptional meritorious and conspicuous service," and was made an Officer of the Legion d'Honneur.

In 1924, General Rees became assistant vice-president of the American Telephone and Telegraph Company, New York, N. Y., where he was in charge of the division for employment and training of college graduates for the Bell System, and of other educational activities, which position he held until his death. He was the author of "Personnel Management," published by the Alexander Hamilton Institute, and was a member of the following societies: Society for the Promotion of Engineering Education (president 1929–30), American Society of Mechanical Engineers, American Association for the Advancement of Science, American Association for Adult Education (member of executive committee), Personnel Research Federation, American Management Association, Army and Navy Club (Washington), Machinery Club (New York), and Western Universities Club (New York); he was also a member of the corporation of Brooklyn Polytechnic Institute.

ASCE Nominates 1937 Officers. The following candidates for society offices for the year 1937 have been nominated by the American Society of Civil Engineers: for president—Louis C. Hill, Los Angeles, Calif.; for vice-president—L. F. Bellinger, Atlanta, Ga., and R. C. Gowdy, Denver, Colo.; for directors—William J. Shea and Enoch R. Needles, New York, N. Y., Arthur W. Dean, Boston, Mass., R. P. Davis, Morgantown, W. Va., T. Keith Legaré, Columbia, S. C., and Thomas E. Stanton, Jr., Sacramento, Calif. These nominees will be voted on by ballot, and the elected officers will be inducted into office at the January 1937 annual meeting of the Society.

Graduate Courses for Chicago Engineers. The Western Society of Engineers, in cooperation with the leading educational institutions in Chicago, Ill., is making available part-time graduate courses in engineering and related fields to engineers employed in the Chicago area. The courses offered are outlined in a bulletin recently issued by that society. According to the bulletin, the program has been developed by the education committee of the society working with the educational institutions, and is intended to present an opportunity for engineering graduates to secure additional training in specific fields and to work toward an advanced degree.

"Handbook of Chemistry and Physics" (flexible fabrikoid, $4^1/_2$ by $6^3/_4$ inches, 2,024 pages, \$6), revised and brought up to date, has recently been published by the Chemical Rubber Publishing Company, Cleveland, Ohio. For more than 20 years this handbook has served those having need of a variety of accurate tables, formulas, and scientific data in a single volume. New and revised material in this 21st edition includes several new features in the mathematical section, an enlarged collection of laboratory arts and recipes, an enlarged photographic section, a new section on commercial plastics, a revised table of isotopes, and various new tables. The book contains 302 pages of mathematical tables and formulas, 741 of chemical tables, 197 on properties of matter, 154 on heat, 20 of hygrometric and barometric tables, 7 on sound, 108 on electricity and magnetism, 141 on light, and 195 pages of miscellaneous tables.

Steam-Electric Locomotive for Union Pacific. According to a recent announcement, a steam-electric locomotive is now being built for the Union Pacific Railroad in the Erie, Pa., shops of the General Electric Company. It is being designed for passenger service, and will carry a condensing-steam-turbine generating plant using oil for fuel and feeding electric power to traction motors. The new locomotive will be a double-cab unit, rated at 5,000 hp, and built for a speed of 110 miles per hour on level track. Streamlined, practically smokeless, and provided with equipment for air conditioning, it will be modern in every respect.

United Engineering Trustees, Inc.

The Joint Engineering Organizations

United Engineering Trustees, Inc., was organized in 1904 as an instrumentality of the Founder Societies, the 4 national societies of civil, mining and metallurgical, mechanical, and electrical engineers. Its purpose is the managing of property and funds in which these societies have joint interests, and it is governed by trustees duly appointed by the societies as their representatives. It maintains 2 departments: (1) the Engineering Societies Library, and (2) The Engineering Foundation.

The corporation (UET, Inc.) manages the Engineering Societies Building and all trust funds placed in the hands of the United Engineering Trustees, Inc.

The Engineering Foundation, founded by Ambrose Swasey (HM'28) in 1914, is entrusted with the expenditure of income from endowment and other funds. The ultimate objective of Foundation is stated to be: "the advancement of engineering as a most important instrumentality for bringing men of all nations into intimate contact or communication, for supplying them the requisites of a constantly developing enjoyment of life and, consequently, for maintaining peace and progress." Foundation's immediate objective is: "the furtherance of researches by its Founder Societies and other engineering organizations directed toward solutions of problems of benefit to the profession or the public, of technological or human interest, in which engineering methods and knowledge may be utilized."

The Engineering Societies Library is a free public engineering library, which, with its numerous activities, is operated for users at a distance, as well as for those who visit its rooms in the Engineering Societies Building.

In the accompanying article may be found announcement of the election recently held by UET, and abstracts of the annual reports of this organization and of The Engineering Foundation and the Engineering Societies Library. The annual election of The Engineering Foundation, held October 8, 1936, as part of the annual meeting of Foundation, was reported in the November issue, page 1286.

Election of Officers of United Engineering Trustees, Inc.

Officers to serve the United Engineering Trustees, Inc., for the year 1936–37, were

elected at the annual meeting of UET held in the Engineering Societies Building, New York, N. Y., October 22, 1936. G. L. Knight (A'11, F'17), vice-president in charge of mechanical operations, Brooklyn (N. Y.) Edison Company, was re-elected president to serve for the period ending at the annual meeting in October 1937. Otis E. Hovey was elected first vice-president and D. Robert Yarnall, second vice president. Albert Roberts was re-elected to serve as treasurer, and H. R. Woodrow (A'12, F'23) was named assistant treasurer. John Arms, general manager, was re-elected secretary.

The names of all members of the board of trustees of UET for the year 1936–37, including both new and hold-over members, are as follows:

Terms expiring October 1937

Otis E. Hovey	ASCE representative
Walter Rautenstrauch	ASME representative

Terms expiring October 1938

J. P. H. Perry	ASCE representative
A. L. J. Queneau	AIME representative
Henry A. Lardner	ASME representative
G. L. Knight (A'11, F'17)	AIEE representative

Terms expiring October 1939

John P. Hogan	ASCE representative
Albert Roberts	AIME representative
D. Robert Yarnall	ASME representative
H. P. Charlesworth (M'22, F'28, past-president)	AIEE representative

Terms expiring October 1940

H. G. Moulton	AIME representative
H. R. Woodrow (A'12, F'23)	AIEE representative

Of these, John P. Hogan, H. G. Moulton, D. Robert Yarnall, and H. R. Woodrow became reappointed members at the annual meeting, upon presentation of credentials from the Founder Societies. Henry A. Lardner (A'94, F'13, member for life) was appointed to fill the unexpired term of Harold V. Coes. All others are hold-over members. Mr. Rautenstrauch is completing the unexpired term of W. L. Batt.

Committees Appointed

The following committees were appointed by the president:

Finance: Otis E. Hovey, *chairman;* H. G. Moulton, Albert Roberts, Henry A. Lardner, G. L. Knight, *ex-officio.*

Real Estate: H. R. Woodrow, *chairman;* J. P. H. Perry, A. L. J. Queneau, D. Robert Yarnall, G. L. Knight, *ex-officio.*

Pension: Walter Rautenstrauch, *chairman;* John P. Hogan, A. L. J. Queneau, H. P. Charlesworth, G. L. Knight, *ex-officio.*

Memorial to Calvin W. Rice: W. L. Batt, *chairman;* Otis E. Hovey, George D. Barron, G. L. Knight, *ex-officio.*

Annual Report Issued by United Engineering Trustees, Inc.

The annual report of United Engineering Trustees, Inc., for the year ending September 30, 1936, has been submitted by G. L. Knight (A'11, F'17), president. During the year the trustees have endeavored to maintain close relationship with the societies through the president's visits to the governing boards or executive committees of the societies and through the society publications.

Some much needed improvements have been made in the Engineering Societies Building within the past year, the most important of which was the redecoration of the rooms of the Engineering Societies Library. Certain improvements also were made in the elevator mechanisms and service.

Thoughtful study has been given to the matter of resuming payments into th depreciation and renewal fund of th Engineering Societies Building. The build ing is now 30 years old and eventually mus be replaced. To meet such a situation the depreciation and renewal fund wa started in 1909 and received additions eacl year until 1932 when, because of the de creased income of the societies, payment were suspended. A program for takin care of this matter is in progress.

One of the duties of UET is the manage ment of funds entrusted to it by generou

Table I—Summary of Annual Report of Finance Committee of UET

	Legal	Nonlegal	Book Value	Market Value
Operation of Building				
Operating revenue				$ 104,715.8
Less operating expenditures				99,483.6
Operating gain 1936				5,232.1
Operating credit previous years				5,419.7
Credit balance September 30, 1936				10,651.8
Transferred to general reserve fund				2,500.0
Net balance September 30, 1936				$ 8,151.8
Operation of Library				
Maintenance revenue			$43,464.41	
Maintenance expenditures			41,820.08	
Credit balance for year 1936			1,644.33	
Credit balance from previous year			4,534.36	
Credit balance September 30, 1936				$ 6,178.6
Service Bureau revenue			8,416.23	
Service Bureau expenditures and adjustments			5,588.41	
Credit balance for year 1936			2,827.82	
Credit balance from previous year			1,744.14	
Credit balance September 30, 1936				4,571.9
Total net operating credit balance cumulated to September 30, 1936				$ 10,750.6
Funds and Property				
***Combined Fund: Summary of Investments, September 30, 1936**				
Funds included:				
Engineering Foundation fund	$492,897.66	$281,938.83	$ 774,836.49	
Edward Dean Adams fund		90,776.60	90,776.60	
Library endowment fund	92,462.93	75,130.33	167,593.26	
Depreciation and renewal fund		318,526.80	318,526.80	
General reserve fund		2,500.00	2,500.00	
Total	$585,360.59	$768,872.56	$1,354,233.15	
Investments: Legal			$ 705,337.75	
Nonlegal			640,900.70	
Total investment, September 30, 1936			1,346,238.45	$1,365,120.2
Loans from depreciation and renewal fund			2,531.65	
Cash uninvested			5,463.05	
			$1,354,233.15	
Real estate, cost of—September 30, 1936			$1,987,793.92	
Operating cash			20,780.85	
Investments, operating cash			5,000.00	$ 5,139.0
Prepaid fire insurance			2,170.28	
Accounts receivable, gross			731.75	
The Engineering Foundation—unexpended income			11,818.93	
The Engineering Foundation—temporary investments			6,180.00	$ 7,132.5
Alloys of iron research—unexpended income			3,409.82	
The Engineering Foundation custodian fund assets			5,300.00	
Henry R. Towne engineering fund			50,000.43	$ 42,202.7
Total			$3,447,419.13	
Moneys Held for Special Purposes				
Endowment committee (Adams expense fund)			$702.57	
Special Library binding fund (W. S. Barstow)			253.28	
UET Inc. Custodian of John Fritz Medal Fund				
Securities held September 30, 1936			$3,500.00	$ 3,745.0
Cash on hand September 30, 1936			68.33	

* A group of funds managed as one for convenience and economy in investment transactions.

donors for the advancement of the arts and sciences of engineering. To this end maintenance of principal is not only mandatory, but maximum income consistent with safety of principal is essential to carry on the research and other projects of Engineering Foundation in co-operation with and for the Founder Societies. During the past year when interest rates have been forced down and bonds called, leaving only lower-income-producing securities for reinvestment, the trustees were faced with the difficult task of assuring both principal and income. In spite of such conditions, the market value of the UET funds at the end of the year is greater than the book value, and the income increased slightly over 1935.

The total book value of UET funds is $1,404,233.58; the book value of Engineering Societies Building and land is $1,987,793.92; and the unexpended balances on hand, accounts receivable, and temporary investments for the United Engineering Trustees, Inc., and its departments amount to $55,391.63, making a total of $3,447,419.13 managed by United Engineering Trustees, Inc., for the Founder Societies. A summary of the report of the UET finance committee is given in Table I.

Because of the continued downward trend of interest rates and consequent reduction in the average rate on return on bonds held legal for savings banks in New York State, to which the investment policy of UET restricted all new investments of The Engineering Foundation fund, the investment policy of UET has been somewhat liberalized. This was done with the general consent of Dr. Ambrose Swasey (HM'28), founder and principal contributor to The Engineering Foundation fund.

The complete revision of the UET by-laws, adopted January 24, 1935, after having been approved by the societies, has resulted in clarifying and simplifying a great deal of the work of UET, especially that relating to financial matters and to real estate. The real estate committee, appointed as a result of a by-law creating it, has greatly helped in the management of the building through the sifting of recommendations received for building improvements and in co-ordinating these with the work of the finance committee.

Annual Report Issued by Engineering Foundation

The annual report of The Engineering Foundation for the year ending September 30, 1936, has been submitted by H. P. Charlesworth (M'22, F'28, past-president) chairman of the Foundation board, and Doctor Alfred D. Flinn, director.

A summary of the capital fund of The Engineering Foundation and a condensed financial statement follows:

Capital Funds

Endowment, total book value on September 30, 1936	$870,000
E. H. McHenry bequest, in hands of executors until decease of 2 life beneficiaries, appraised at probate of will in 1931, approximately	400,000

The capital funds are held and administered by United Engineering Trustees, Inc. The net income from endowment was $34,126 for the fiscal year ending September 30, 1936. The Foundation board has discretion in use of income. For many of the enterprises which Foundation has aided, large contributions of money, services, and materials have been obtained from others.

Current Resources

Balance on October 1, 1935		$ 31,352
Receipts		
Income from endowment and temporary investment of income balance	$34,531	
Income from minor items	764	35,295
Total resources		$ 66,647

Expenditures

Research projects	$36,571	
Promotion of research and administrative expenses	12,277	
Total for furtherance and support of research		$ 48,848
Balance on October 1, 1936		$ 17,799

Money "contributions" from organizations and individuals, for specific activities, passed through the Foundation's accounts from its organization to September 30, 1936, totaled $249,315.

Activities aided by the Foundation during its fiscal year ending September 30, 1936, comprised:

1. **Earths and Foundations Research,** continued by the American Society of Civil Engineers, including participation in the International Conference on Soil Mechanics and Foundation Engineering at Harvard Graduate Engineering School, in June 1936, which indicated great expansion since the committee began work in 1929. Doctor Terzaghi, of the committee, presided. Field and laboratory research in America and Europe progressed under encouragement of the committee. The committee's support helped in establishing an advanced soil mechanics laboratory at Harvard University. At Yale University support was given to investigation of lateral supporting power of soils to individual piles, anchors, and bulkheads. At Columbia University the equipment and methods of the barodynamic research (see item 3) were utilized for a variety of problems. At the University of Minnesota extensive researches were made in the subjects of earth dams and cofferdams, with the aid of engineering models.

2. **Alloys of Iron Research,** sponsored by American Institute of Mining and Metallurgical Engineers. Iron alloys committee continued preparation and publication of books containing critical digests and bibliographies of the world literature on alloy steels and alloy cast irons: seventh book printed; eighth book on the press; manuscripts for 5 books well advanced. Efforts were continued to obtain additional financial contributions. An exhibit of the committee's work was prepared for the symposium on structural applications of steel and lightweight alloys at the October meeting at Pittsburgh, Pa., of the American Society of Civil Engineers.

3. **Barodynamic Research** (study of weighty masses by means of special centrifuges) with application to mining and civil engineering problems, sponsored by American Institute of Mining and Metallurgical Engineers, continued at Columbia University School of Mines. Professor in charge made an extended journey in Europe and Africa studying mining problems by surface and underground observations. Confirmation of laboratory results was obtained. In the laboratory were developed a device for determining side pressures of loose materials and a new type of artificial support in mines; data on stress distribution in mine pillars and roofs were obtained, and the time effect in rock structures stressed beyond elastic limits was studied.

4. **Cottonseed Processing Research,** by a committee of The American Society of Mechanical Engineers with headquarters at University of Tennessee, was continued in laboratories and field, and a manual on "Mechanical Processing of Cottonseed" was prepared by the committee and published by the University. This book contains a classified bibliography of the literature.

5. **Cutting Fluids** (for lubricating and cooling metal-cutting tools). This research committee of The American Society of Mechanical Engineers was inactive until July 1936, but for the last quarter of the fiscal year pushed its work vigorously at University of Michigan, doing experimental cutting, according to a program, on a large piece of steel known as a "test log."

6. **Critical Pressure Steam Boilers:** a basic investigation by a research committee of The American Society of Mechanical Engineers on which designs for a new type of boiler may be based, at Purdue University. Determinations of viscosity of water and steam, and reactions between steam and metals at elevated temperatures, have particularly been under study.

7. **Fluid Meters.** Long-radius flow nozzles, one of the primary elements used in fluid meters, are being studied experimentally by a committee of The American Society of Mechanical Engineers in several laboratories, with the co-operation of National Bureau of Standards, Massachusetts Institute of Technology, Cornell University, Universities of California, Ohio, and Oklahoma, and several companies, using steam in some instances and water in others, through nozzles ranging from 3 to 16 inches in diameter; also 2-inch pipe orifices. Purpose: to provide more economical and convenient means for precise measurements of large quantities of liquids or gases, as in efficiency tests of steam and hydraulic power installations.

8. **Boiler-Feed-Water Research** was continued by a committee of The American Society of Mechanical Engineers at University of Michigan, on methods of determining oxygen in the waters, and at the nonmetallic minerals experiment station of the United States Bureau of Mines, New Brunswick, N. J., on fundamental studies of the cause and prevention of embrittlement in boiler steel.

9. **Strength of Gear Teeth:** This experimental study has been in progress 12 years under supervision of a committee of The American Society of Mechanical Engineers, using a specially built testing machine, with co-operation of industries. Recent operations have been at Massachusetts Institute of Technology and during the year herein reported were devoted to tests on surface fatigue of cast iron.

10. **"Cutting of Metals" handbook.** Manuscript for this book of approximately 250 typewritten pages has been completed by a special editor engaged by The American

Society of Mechanical Engineers with the aid of an underwriting of $1,500 by Foundation. Collection of the materials was the work of a committee for 4 years. Publication of the handbook is predicted for early 1937.

11. Welding Research: (a) Pure Iron Electrodes, sponsored by American Institute of Electrical Engineers. This basic research, begun in June 1930, was terminated by expiration, on September 30, 1936, of the fifth extension of the 2-year agreement with Lehigh University. It is the intention hereafter to concentrate Foundation's aid in this field upon the comprehensive program of the welding research committee. Interesting fundamental facts have been brought to light at Lehigh University and reported through several channels, including publications of the AIEE, American Institute of Mining and Metallurgical Engineers, and American Physical Society. **(b) Welding Research Committee,** sponsored by American Institute of Electrical Engineers and American Welding Society. Created in 1934 with the co-operation of industries, this committee has completed its plan of organization, outlined the main features of its program, obtained assurances of financial support and other co-operation, and made progress on its activities. These activities include critical reviews, translations, and digests of literature in America and other countries; some 60 fundamental researches mostly in college laboratories; development of extensive co-operation in industrial researches, through 10 subcommittees, and publication of a monthly bulletin (in connection with Journal of American Welding Society) to inform research workers, technical men and executives of industries.

12. Engineers' Council for Professional Development: composed of representatives of the Founder Societies, Society for Promotion of Engineering Education, American Institute of Chemical Engineers, and National Council of State Boards of Engineering Examiners. Reports on the work of ECPD have appeared month by month in ELECTRICAL ENGINEERING.

13. Personnel Research Federation: Forms of employer-employee co-operation: By visits to industrial plants, by correspondence with governmental departments and labor organizations, and by conferences, materials were collected for a report of practical usefulness showing the development of employer-employee co-operation to date and the ways in which this method of industrial management may develop further. This report is to be published in the fall of 1936. At a large and successful general meeting in New York in January, employer-employee interviews and other subjects were discussed. *Personnel Journal,* published monthly, has been notably improved; number of subscribers is increasing encouragingly. The Federation progressed also in its general development.

14. Plastic Flow of Concrete: University of California, engineering laboratories. Long-time tests started in 1926 were continued of various factors in the plastic behavior of concrete, and reports were presented. Thermal stress studies were completed. Three new series were begun: study of moisture loss accompanying plastic flow under sustained load study of the validity of the assumption of plane bending in beams under sustained load; effect of compound composition and fineness of cement upon plastic flow. In September 1936, there were 433 specimens under observation in the laboratory. Results of earlier work have been reported in publications of The American Concrete Institute and through other channels; the later results (including a bibliography) are being prepared for publication next year.

15. Concrete Research. The planning committee on cement and concrete research reported in August 1936 the opinion "that there is a real need for a broad and comprehensive program of cement and concrete research and that The Engineering Foundation might well be the organization to promote such a program." The committee recommended "the appointment of a standing committee on cement and concrete research, the membership of which shall include representatives of national organizations having interests in this field." The director of Foundation is collecting data and opinions from a number of other sources to aid in the further preliminary examination of this proposal during the coming year.

16. Plasticity of Metals: creep and relaxation, under agreement with University of Pittsburgh; research facilities provided in Westinghouse research laboratories. Work begun January 1, 1935, was completed in September 1936. Final report is being written. Special apparatuses were designed and constructed. The results obtained are proving to be of practical value.

17. Defects and Failures of Metals. An informal proposal for a book was examined. Inquiries addressed to a number of persons informed on the subject drew widely divergent opinions. Therefore, the matter was dropped, but with willingness to receive additional suggestions or opinions.

The general plan and policy of Foundation is outlined in its "Platform" adopted by the Foundation board on June 14, 1935, and approved by the Founder Societies in the Fall of 1935. (See April 1936 issue, page 423.) Further information on Foundation and its activities may be obtained from the Foundation office, 29 West 39th Street, New York, N. Y.

Annual Report Issued by Engineering Societies Library

The annual report of the Engineering Societies Library for the year ending September 30, 1936, has been submitted by Doctor Harrison W. Craver, director. It contains the usual information on the use of the library, its finances, and acquisitions.

While attendance has not reached the height of 4 or 5 years ago, the reading done has been more serious and less casual, as evidenced by more calls for books and longer stays by readers. Users of the library numbered 37,586, of whom 26,784 visited it and made direct use of the collection. The remaining 10,802 were assisted in various ways: 124 by loans of 141 books, 109 by special bibliographies, 121 by translations, 2,363 by supplying 20,020 photoprints, 3,013 by letters, and 5,072 by telephone.

Corresponding figures for 1934, the last 12-month period recorded, were 40,789 users, of whom 29,928 were readers. It is interesting to note that the decreased use is a decrease in visitors, the requests from nonvisitors having remained at the same level. During the past year the library has served 2 nonvisitors for every 5 visitors.

Books, pamphlets, and maps received during the year numbered 12,148. Of these 6,780 were not already in the library and were added to it. The others were disposed of in various ways, some being placed in the rental collection, some presented to libraries, and the rest placed in the duplicate collection for sale or exchange. Sales of duplicates amounted to $660.39. Current issues of 1,358 periodicals were received and 437 books worth approximately $1,700 were received for review.

Gifts numbered 10,795 items. Among these were gifts from the estates of F. A. Halsey, H. de B. Parsons, C. M. Weld, and H. H. Wolff. Valuable gifts also were received from A. W. Berresford (A'04, F'14, past-president), F. W. Doolittle, George B. Holderer, David Landau, Donald D. Liddell, Sanford A. Moss, James L. Pitcher, F. F. Sharpless, Horace Wemple, Haverford College, Lafayette College, and the McGraw-Hill Book Company.

During the year a legacy of $1,500 was received from the estate of the late H. de B. Parsons and was added to the endowment fund. Through the generosity of W. S. Barstow (A'94, F'12, Life Member) $1,500 was devoted to the repair and rebinding of many rare books.

On October 1, 1935, the library contained 136,464 volumes, 7,064 maps, and 4,259 manuscript bibliographies. On September 30, 1936, the collection numbered 138,742 volumes, 7,246 maps, and 4,298 bibliographies. In addition 3,646 pamphlets had been incorporated in volumes previously counted. The lending collection contained 620 volumes at the close of the year, and there were approximately 14,000 volumes and pamphlets in the duplicate collection. The new acquisitions have been cataloged day by day as received, and substantial progress has been made upon the recataloging of the Wheeler collection.

Work has progressed as rapidly as possible upon the classified index to periodicals, 20,000 cards having been added during the year. The index now contains references to 150,000 articles published since the year 1927 and is being used more and more every day. More indexers are urgently needed, as the work required for indexing the volume of printed matter is beyond the capacity of the present staff.

The budget for general operations during the year was $42,800. Of this sum $33,000 was appropriated by the Founder Societies as follows:

American Society of Civil Engineers	$8,922.10
American Institute of Mining and Metallurgical Engineers	6,821.30
American Society of Mechanical Engineers	8,497.00
American Institute of Electrical Engineers	8,759.60

Expenditures amounted to $1,365.80, of which $7,743.79 was spent for books and other equipment which increased the assets of the library. The service bureau received $8,416.23 and expended $5,623.25.

Letters to the Editor

CONTRIBUTIONS to these columns are invited from Institute members and subscribers. They should be concise and may deal with technical papers, articles published in previous issues, or other subjects of some general interest and professional importance. ELECTRICAL ENGINEERING will endeavor to publish as many letters as possible, but of necessity reserves the right to publish them in whole or in part, or reject them entirely.

ALL letters submitted for consideration should be the original typewritten copy, double spaced. Any illustrations submitted should be in duplicate, one copy to be an inked drawing but without lettering, and other to be lettered. Captions should be furnished for all illustrations.

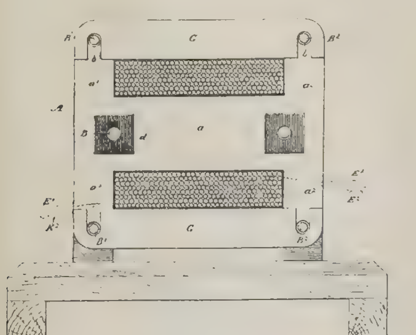
STATEMENTS in these letters are expressly understood to be made by the writers; publication here in no wise constitutes endorsement or recognition by the American Institute of Electrical Engineers.

Mechanical Development of Transformer by Westinghouse

To the Editor:

In this letter I offer a few comments on the mechanical development of the transformer by George Westinghouse, which was referred to in the article "Early History of the A-C System in America," by C. C. Chesney and the writer, published in the March 1936 issue of ELECTRICAL ENGINEERING (pages 228–35), but which was not elaborated. I offer a few comments also on the Stanley alternator.

When the Gaulard and Gibbs "secondary generator" arrived in America in 1885, it soon underwent some radical changes. Westinghouse, mechanical genius, and Stanley, electrical expert, applied their complementary abilities to the transformer prob-

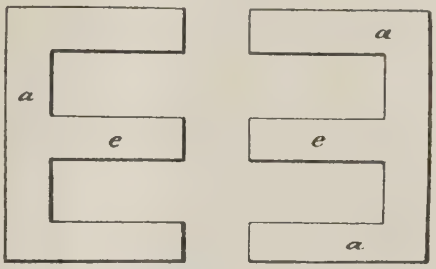


Westinghouse H-plate

The punchings are held by 2 bolts surrounded by square insulating washers. The iron core is placed in a lathe and the coils are wound around the horizontal bar of the "H." Afterward horizontal iron strips are placed across the top and the bottom and are held by insulated bolts at the corners

lem. Stanley was concerned with parallel connection, counter electromotive force, and the magnetic circuit. His patent on a core of sheet-iron rings, applied for in November 1885, shows an ideal type of completely closed magnetic circuit; his Great Barrington (Mass.) investigations dealt with electrical and magnetic relations, length of wire, and thickness and weight of iron, as well as a practical demonstration of the transformer system. His important patent deals with windings and counter electromotive force.

Westinghouse, on the other hand, was concerned with mechanical design and simplification of manufacturing methods. From an impractical construction he evolved the modern type of transformer. He said that the coils should be of wire wound on a lathe and not composed of flat copper rings, slotted and soldered together in a spiral as in the Gaulard and Gibbs construction, nor should it be necessary to thread the wire through iron rings. He therefore proposed to wind the coils first and then to build the core with strips of sheet iron placed around and through the coils, requiring 5 pieces. Then he devised an H-shaped plate; the plates were piled up, bolted together, placed in a lathe, and the coils wound around the horizontal bar. Two straight strips were added across the top



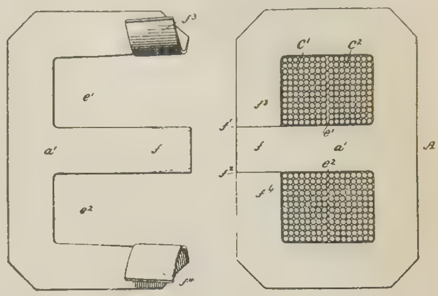
Stanley E-plate

The plates are built up by slipping the middle part through the prewound coils, first from one side and then from the other. Parts of the magnetic circuit where the plates do not overlap have half the cross section of iron

and bottom for completely surrounding the coils. The transformers used in the Great Barrington tests were wound on H-plates. The interleaving of the horizontal strips and the vertical ends is clearly seen in the published photographs. Later Stanley devised the E-plate for prewound coils. This was later improved by Albert Schmid who used a single complete plate.

Applications for patents were made in 1886: for the Westinghouse H-plate in February, for the Stanley E-plate in November, and for the Schmid complete plate in December. The latter continued in use for several years. The several plates are shown in the accompanying illustrations reproduced from patent drawings.

Let us go back to December 1885. Belfield, the Gaulard and Gibbs expert, had arrived in Pittsburgh, and there were tests and conferences. He endorsed Stanley's ideas as to parallel operation and a closed magnetic circuit, as they were in accord with his own views. Regarding the construction he says, "I was in position to give G. W. the electrical data he needed, and I was surprised when later on he told me of his idea of turning the secondary generator inside out as it were, by winding the coils on a lathe, and building the iron core, in the shape of plates around them; this was a revelation to me. . . . Gradually the invention developed; we all had a part in the discussion as to how it should be applied, G. W., Pope, Stanley, Shallenberger, and I, also doubtless Schmid." Affairs crystallized rapidly; on December 23 application



Schmid complete plate

Extensions on the tips of the "E" are bent to allow the tongue to be slipped through the coil. The final core is of uniform cross section

was made for the charter for a company and on the next day a new contract was made with Stanley for continuing his services as electrical expert and including Stanley's project for transferring activities to Great Barrington where he would prepare and operate a demonstration plant. The plant operated from March 20, 1886 for 3 months. It was convincing and Westinghouse proceeded actively. An alternator from original drawings by Stanley and transformers suited for commercial service were made in Pittsburgh, and proved successful in the Lawrenceville test. Commercial service at Buffalo began a few months later on Thanksgiving Day, 1886.

The Stanley alternator was very different from the Siemens alternator brought from England. In the latter the field coils produced magnetic fields parallel with the shaft. The armature, which had no iron, was of flat disk shape. If one were to draw a large circle with 16 small circles equally spaced near the circumference to represent coils and a circle at the center for the shaft, he will have a picture of the Siemens armature. Stanley placed his armature coils on the surface of a cylinder built up of circular iron disks. The field magnets, as is usual today, were radial. Stanley stated, if I recall correctly, that his alternator was the first to employ iron in its armature.

Thus, both transformer and alternator were radically changed in type within a few months after arriving in America; Westinghouse gave the transformer a practical form by making the coils of wire and winding them on a lathe, discarding copper punchings, while iron punchings replaced a core of iron wires; in the Stanley alternator the magnetic poles were turned 90 degrees so that they were radial instead of parallel with the shaft. The essential elements of the new designs have persisted through a half century of development.

Very truly yours,

CHAS. F. SCOTT (A'92, F'25, HM'29, past-president)

Professor of Electrical Engineering Emeritus, Yale University, New Haven, Conn.

Personal Items

L. W. W. MORROW (A'13, F'25, director) has resigned as editor of *Electrical World*, New York, N. Y., to become general manager of the newly created fiber products division of the Corning (N. Y.) Glass Works. Mr. Morrow is a native (1888) of Hammond, W. Va., and was graduated from Marshall College in 1907 and Cornell University in 1911. During the scholastic year following his graduation in 1911 he served as an instructor at Cornell University, and in 1913 was appointed assistant professor of electrical engineering at the University of Oklahoma. During the year 1917–18 he served as professor of electrical engineering and director of the school of electrical engineering, before becoming assistant director of the U.S. Signal Corps school at Yale University. He served concurrently as an assistant professor of electrical engineering on the faculty of Yale University, and, following the World War, was retained in that position. In 1922 Mr. Morrow accepted a position as associate editor of *Electrical World*, and for the past 10 years has been editor of that publication. He has been active in the committee work and other affairs of the Institute, is at present chairman of the committee on co-ordination of Institute affairs, as well as a member of several other committees, and is vice-president of the Thomas Alva Edison Foundation and the Institute's representative on the Engineers' Council for Professional Development. He is a member of The American Society of Mechanical Engineers, American Electrochemical Society, Epsilon Xi, and Sigma Xi.

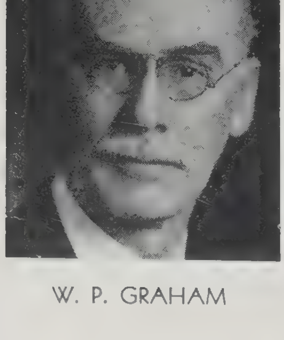
S. K. BARRETT (A'11, M'17) director of evening engineering division and professor of electrical engineering, New York University, New York, N. Y., has been appointed assistant dean of engineering, in charge of the evening engineering division. Dean Barrett was born at Saratoga Springs, N. Y., in 1886, and was graduated from the Polytechnic Institute of Brooklyn, N. Y., in 1910, with the degree of electrical engineer. He accepted an appointment, following his graduation, as an instructor in electrical engineering at the Polytechnical Institute of Brooklyn; later he was appointed assistant professor of electrical engineering, and continued to serve in that capacity until he was appointed to the faculty of New York University in 1919. In addition to his regular teaching and administrative duties, Dean Barrett serves as consulting engineer on illumination for several companies. He was a member of the Institute's committee on production and application of light during 1933–34. He is a member of the Society for the Promotion of Engineering Education, American Association for the Advancement of Science, The American Society of Mechanical Engineers, Tau Beta Pi, and the Illuminating Engineering Society.

J. M. FERNALD (A'20, M'23) general manager of the Baker Ice Machine Company, Inc., Omaha, Nebr., recently was elected president of the Refrigerating Machinery Association at the annual meeting of that association. Mr. Fernald was born at Newtonville, Mass., in 1890, and received his formal engineering education at the Hawley School of Engineering. Following a brief service with the Boston (Mass.) Elevated Railway Company, he became electrical superintendent of the Metz Motor Car Company, Waltham, Mass., in 1911, remaining with that company until 1915, when be became a sales engineer for the V. V. Fittings Company, Philadelphia, Pa. During the period 1917–19 he served with the U.S. Army Corps of Engineers, and following the World War became a sales engineer for the Cutler-Hammer Manufacturing Company, Boston, Mass. In 1926 Mr. Fernald became associated with the Electric Refrigeration Corporation, later the Kelvinator Corporation, as assistant director of sales, and 2 years later was appointed head of the commercial refrigeration division of the Kelvinator Corporation; however, he left that position in 1932 to become general manager of the Baker Ice Machine Company.

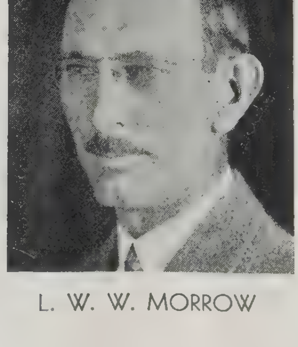
W. P. GRAHAM (A'02, F'23) for the past 15 years vice-chancellor of Syracuse University, Syracuse, N. Y., has been appointed acting chancellor. Doctor Graham was born at Oswego, N. Y., in 1871, and received the degree of bachelor of science at Syracuse University in 1893. Pursuing an early interest in physics and mathematics, he remained at the university another year doing advanced work in physics. From 1894 to 1898 he lived continuously in Europe, studying for 3 years at the University of Berlin, from which he received the degree of doctor of philosophy in 1897, and one year at the Technische Hochschule, Darmstadt. Upon returning to the United States, Doctor Graham was appointed associate professor of electrical engineering in the college of liberal arts of Syracuse University, and in 1901, when the college of applied science was established, he was appointed head of the department of electrical engineering. In 1911 Doctor Graham became dean of applied science, and continued in that capacity for 10 years before being appointed vice-chancellor in 1921. He is a member of the American Association for the Advancement of Science, American Astronomical Society, Phi Beta Kappa, Sigma Xi, and Tau Beta Pi, and has been active in the work of the Institute's Syracuse Section.

D. S. JACOBUS (A'03, member for life) advisory engineer, The Babcock & Wilcox Co., New York, N. Y., has received the Morehead Medal of the International Acetylene Association for the year 1935 "for his outstanding leadership in the formulation of codes and procedures which have made fusion welding acceptable." Doctor Jacobus was born January 29, 1862, at Ridgefield, N. J., and was graduated from Stevens Institute of Technology in 1884. He received the degree of doctor of engineering from the same institution in 1906. Upon his graduation in 1884, he was appointed to the faculty of Stevens Institute of Technology as an instructor in the department of experimental mechanics; later he was appointed assistant professor and, in 1894, professor of experimental mechanics. In 1906 Doctor Jacobus resigned his professorship to become advisory engineer for The Babcock & Wilcox Co., where he has remained for 30 years, and has become head of the engineering department of that company. He is the author of many technical papers and is active in several engineering and scientific societies. He is a past-president of The American Society of Mechanical Engineers, American Welding Society, and American Society of Refrigerating Engineers, and is a member of

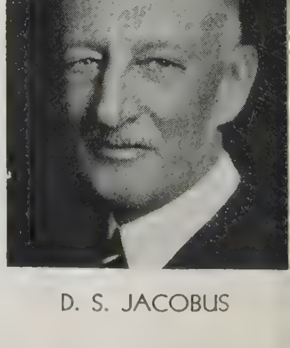
J. M. FERNALD



W. P. GRAHAM



L. W. W. MORROW



D. S. JACOBUS

N. W. STORER

FREDERICK KRUG

J. W. WHITE

T. F. PETERSON

the Society of Naval Architects and Marine Engineers, American Institute of Mining and Metallurgical Engineers, American Mathematical Society, Society for the Promotion of Engineering Education, Franklin Institute, and the American Association for the Advancement of Science. Doctor Jacobus is also a member of the welding research committee of the Engineering Foundation.

N. W. Storer (A'95, F'13, member for life) consulting railway engineer, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa., has retired from active service. Mr. Storer was born January 11, 1868, at Orangeville, Ohio, and was graduated from Ohio State University with the degree of mechanical engineer in electrical engineering in 1891. Immediately following his graduation, he was employed by the Westinghouse Electric & Manufacturing Company, and has remained in the services of that company continuously. In 1893 he began work as an electrical designer under the supervision of the late B. G. Lamme (A'03, M'03, Edison Medallist '18) and in 1895 was placed in charge of the design of d-c machinery. Mr. Storer was transferred to the railway division as general engineer, in 1904, later being placed in charge of all railway development work for the Westinghouse company. He was appointed consulting railway engineer in 1926, and in that capacity assisted in the electrification of several of the large railway systems in the United States. Mr. Storer has been active in Institute affairs, having been manager, 1911–14, and vice-president for 2 terms, 1914–16 and 1921–23; in addition, he has been a member of many of the technical committees, a representative on the International Electrotechnical Commission, and author of several papers presented before the Institute. He is a member of The American Society of Mechanical Engineers.

T. F. Peterson (A'25, M'32) consulting cable engineer, American Steel & Wire Company, Worcester, Mass., has been appointed director of the electrical cable work of that company. Mr. Peterson was born at Brooklyn, N. Y., in 1902, and after receiving the bachelor of science degree at Cooper Institute of Technology, attended Stanford University and graduated with the degree of engineer in electrical engineering. Following his graduation, he was placed in charge of cable research for the Brooklyn (N. Y.) Edison Company in 1924, and in the following year was made cable engineer, while serving at the same time as an instructor in advanced electrical measurements at Cooper Union. In 1927 Mr. Peterson became associated with the American Steel & Wire Company as cable engineer and remained in that capacity until he was appointed consulting engineer for that company in 1931. He served the Institute as a member of the committee on power transmission and distribution during the period 1928–32, and was active as chairman of the subcommittee on cable development. Mr. Peterson has been a frequent contributor to the technical press, having presented several papers before the Institute, and was associate editor of the "Underground Systems Reference Book" of the National Electric Light Association. He is a member of Sigma Xi.

J. W. White (A'29) formerly managing director, Cia. Westinghouse Electric International, Buenos Aires, Argentina, has been appointed general manager of the Westinghouse Electric International Company, with headquarters at New York, N. Y. Mr. White was born at Indianapolis, Ind., in 1889, and received his early education at Randolph-Macon Academy; later, while engaged with the Westinghouse company at East Pittsburgh, Pa., he attended Carnegie Institute of Technology. He joined the Westinghouse Electric & Manufacturing Company, East Pittsburgh, in 1905, and continued at the main works until 1912. In 1917 Mr. White filled the position of manager of the central station and transportation divisions of the Detroit, Mich., offices of that company. His first connection with the export business was in 1918, when he was assigned the managership of the Westinghouse offices at Havana, Cuba. In 1925 he was made managing director of the Westinghouse Company of Japan, with his staff offices at Tokyo. Since 1931 Mr. White has been managing director of Cia. Westinghouse Electric International in Argentina.

R. D. Miller (M'32) former assistant vice-president, Pacific Telephone and Telegraph Company, San Francisco, Calif., recently was appointed chief engineer of the company. Mr. Miller was born at Washington, Iowa, in 1899, and received the degree of bachelor of science in electrical engineering at the University of California in 1921. Immediately following his graduation he entered the employ of the Pacific Telephone and Telegraph Company as a student engineer; later he was made an engineer and remained with that company until he became affiliated with the Southern California Telephone Company, Los Angeles, in 1924. In 1925 Mr. Miller was appointed fundamental plan engineer of that company; in 1928, plant extension engineer. He returned to the Pacific Telephone and Telegraph Company in 1930, as chief engineer of the Oregon area, and has since remained with that company continuously. He was transferred to the San Francisco offices in 1934.

Frederick Krug (A'17, F'36) formerly vice-president and general manager of the Porto Rico Railway, Light, and Power Company, San Juan, has been appointed supervisor of southern properties in the Montreal (Canada) Engineering Company, with headquarters at Montreal. Mr. Krug was born October 12, 1893, at New York, N. Y., and was graduated in electrical engineering from Cooper Union, New York, in 1916. He also attended the New Mexico School of Mines (1916–17) and Massachusetts Institute of Technology (1925). In 1917 he became assistant superintendent of the New York and Honduras Rosario Mining Company, San Jacinto, Honduras; later he became superintendent of the electrical department, which position he held until he returned to the United States in 1918 as a war instructor at Carnegie Institute of Technology, Pittsburgh, Pa. Mr. Krug joined the Porto Rico Railway, Light, and Power Company in 1922 as superintendent of the Comerio (Porto Rico) plants, becoming successively superintendent of power production (1924) and assistant to the president (1926). He was appointed vice-president and general manager in 1927.

C. J. Fechheimer (A'05, F'14) consulting engineer, Fechheimer, Kisa, and Associates, Milwaukee, Wis., recently joined the engineering staff of The Louis Allis Company, Milwaukee, Wis., in the capacity of consulting engineer. Mr. Fechheimer was born at Cincinnati, Ohio, in 1882, and received the degrees of bachelor of science in electrical engineering (1904) and mechanical engineer (1905) at Purdue University and

Cornell University, respectively. Following his graduation from Cornell, he was engaged by the Bullock Electric Company, Cincinnati, as an electrical designer; later, when he became associated with the Allis-Chalmers Manufacturing Company, Milwaukee, he was placed in charge of the design of the turbogenerators manufactured by that company. Mr. Fechheimer accepted a position with the Crocker-Wheeler Company, Ampere, N. J., as engineer in charge of a-c design, in 1910, in which position he remained until 1915, when he became a power engineer for the Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa. He was associated with the Westinghouse company continuously until 1931, serving successively as design engineer, research engineer, and section engineer. In 1931 Mr. Fechheimer established his own consulting engineering offices in Milwaukee. He served the Institute as a member of the committee on electrical machinery during 1926-27, and is the author of several papers presented before the Institute. He is a member of The American Society of Mechanical Engineers.

A. V. Karpov (M'22) designing engineer, Aluminum Company of America, Pittsburgh, Pa., has been awarded the Thomas Fitch Rowland Prize of the American Society of Civil Engineers as co-author of a paper "Model of the Calderwood Arch Dam." Mr. Karpov is a native (1886) of Kursk, Russia, and was graduated from the Darmstadt (Germany) Technical College and the Kharkov (Russia) Technical Institute. After serving as engineer in charge of several engineering projects in Russia and Germany, he came to the United States in 1920 and secured employment as an engineer and designer for the Standard Arch Company, New York, N. Y. He has been associated with the Aluminum Company of American since 1928. The Thomas Fitch Rowland Prize is awarded annually for a paper that describes in detail some accomplished work of construction.

M. R. Gowing (M'28) assistant manager of the power utilities department, Ohio Brass Company, Mansfield, has been transferred to the Chicago, Ill., offices of that company as district sales manager. Mr. Gowing was born at Toledo, Ohio, in 1894, and received the degrees of bachelor of electrical engineering (1917) and electrical engineer (1928) at Ohio State University. After serving brief periods with the U.S. Army (1917-19) and the Western Electric Company, New York, N. Y. (1920-22), he was engaged by the Ohio Brass Company as a sales-development engineer in the railway division. In 1925 he was transferred to the power utilities department, and later was made assistant manager of that department.

Alex Dow (A'93, F'12, member for life) president, Detroit (Mich.) Edison Company, recently was elected to honorary membership in the American Society of Civil Engineers and The American Society of Mechanical Engineers. Doctor Dow has been president of the Detroit Edison Company since 1912 and is well known for various enterprises in the field of power generation and distribution. He is active in other technical societies, and is a past-president of The American Society of Mechanical Engineers. A biographical sketch of Doctor Dow was published in the July 1936 issue of Electrical Engineering, page 846.

D. W. Mead (A'11, F'13) consulting engineer, Madison, Wis., has been awarded the Norman Medal of the American Society of Civil Engineers for his paper "Water-Power Development of the St. Lawrence River." Doctor Mead is president and an honorary member of that society. The Norman Medal has been awarded annually since 1872 to the authors of original papers that are considered to be particularly notable contributions to engineering literature. A biographical sketch of Doctor Mead appeared in the March 1936 issue of Electrical Engineering, page 314.

P. C. Cromwell (A'28) instructor in electrical engineering, New York University, New York, N. Y., has been promoted to assistant professor of electrical engineering. Professor Cromwell is a native (1902) of Beverly, W. Va., and a 1924 electrical engineering graduate of Carnegie Institute of Technology. Before joining the faculty of New York University he was affiliated with the Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa., the Duquesne Light Company, Pittsburgh, and Arthur E. Andersen and Company, Chicago, Ill.

P. L. Warren (A'26) has resigned as district sales manager of the Ohio Brass Company, Chicago, Ill., to become affiliated with the Royal Electric Manufacturing Company, Chicago. Mr. Warren, a native (1897) of Toledo, Ohio, attended the University of Missouri and the Tri-State College of Engineering. Following a brief training period, he entered the commercial engineering department of the Ohio Brass Company in 1923. He became district sales manager in 1928.

Robert MacDonald (A'28) formerly with The English Electric Company, Ltd., London, now is employed by Ericsson Telephones, Ltd., Beeston, Notts, England.

J. O. Tucker (A'11, M'22) division engineer, Illinois Light and Power Corporation, Champaign, has been transferred to the Ottawa offices of that company.

D. J. De Boer (A'34) formerly transmission engineer, Harza Engineering Company, Columbus, Nebr., now is with Sargent and Lundy, Inc., Chicago, Ill.

L. J. Nartker (A'32) formerly an electrical motor designer, Delco Products Corporation, Dayton, Ohio, now is with the Sunlight Electric Company, Warren, Ohio.

W. A. McWhorten, Jr. (A'36) now is employed by the Goodman Manufacturing Company, Chicago, Ill.

Obituary

Robert A. Ross (A'92, M'93, F'12, member for life) consulting engineer, R. A. Ross and Company, Montreal, Que., Canada, died September 23, 1936. Doctor Ross was born August 29, 1865, at Woodstock, Ont., Canada, and received his formal engineering education at the University of Toronto, graduating from the school of practical science in 1890. In 1896 he was granted the professional degree of electrical engineer by the University of Toronto; in 1921 the same institution bestowed upon him the honorary degree of doctor of science. Following his graduation in 1890 he was employed by the Canadian General Electric Company at the Sherbrooke, Que., and Peterborough, Ont., plants until 1893, when he was appointed chief electrical and mechanical engineer of the Royal Electric Company, Montreal. In 1896 Doctor Ross established his own consulting engineering practice in Montreal. As a consulting engineer he was responsible for the design of many electrical and power projects, acted at various times as advisor to most of the large cities and municipalities in Canada, and worked on engineering enterprises in Europe and Asia. In 1923 Doctor Ross was appointed a member of a commission instituted by the provincial government of Ontario to investigate and report upon the affairs of the Hydro-Electric Power Commission of Ontario, and in 1932 was a member of a board of engineers that carried out an important investigation into power rates in the province of Quebec. He was a senior member and past-president of the Engineering Institute of Canada, and in 1934 received the highest distinction of that organization—the award of the Sir John Kennedy Medal for outstanding merit in the profession of engineering.

Louis Charles Nichols (A'02, M'11, F'13) engineer in charge of transformer design, Allis-Chalmers Manufacturing Company, Milwaukee, Wis., died November 3, 1936. Mr. Nichols was born December 28, 1877, at Westminster, Mass., and was graduated from Worcester Polytechnic in 1900 with the degree of bachelor of science. Following his graduation he entered the employ of the General Electric Company, Schenectady, N. Y., as an apprentice, but later he joined the engineering staff of the Converse Transformer Company, Pittsburgh, Pa., where he remained until 1903. While with that company his work consisted of general transformer design, and when he accepted a position with the Bullock Electric Company, East Norwood, Ohio, in 1903, he was placed in charge of that company's transformer engineering and design. When the Bullock Electric Company was absorbed by the Allis-Chalmers Manufacturing Company in 1904 Mr. Nichols was retained in charge of the transformer department, and in 1911 was made chief engineer of the Bullock division of the Allis-Chalmers company. In 1915 the Allis-Chalmers company transferred its transformer division to Milwaukee, and Mr. Nichols was placed in charge of trans-

former design for the entire company, which position he retained for almost 21 years. He was a member of the Institute's committee on electrical machinery, 1926–27, and the Edison Medal committee, 1934–36. He was also active in the National Electrical Manufacturers Association, being chairman of the committee on transformers and a member of the board of governors of that organization.

Membership

Applications for Election

Applications have been received at headquarters from the following candidates for election to membership in the Institute. If the applicant has applied for direct admission to a grade higher than Associate, the grade follows immediately after the name. Any member objecting to the election of any of these candidates should so inform the national secretary before December 31, 1936, or February 28, 1937, if the applicant resides outside of the United States or Canada.

Aita, F. S., Consumers Power Company, Jackson, Mich.
Anderson, A. E. (Member), Jamestown (N. Y.) Telephone Corporation.
Armstrong, J. W. F., 3444 Rhoda Ave., Oakland, Calif.
Barlow, D. S., Y.M.C.A., Albuquerque, N. M.
Barnes, E. M., New York and Queens Electric Light and Power Company, Long Island City, N. Y.
Barney, J., Fort San, Sask., Canada
Barry, T. J., Interborough Rapid Transit Company, New York, N. Y.
Bayha, W. G., Meyers Safety Switch Company, Inc., San Francisco, Calif.
Berry, M. D., Leland Electric Co., Toronto, Ont., Canada.
Bland, C. H., West Kootenay Power and Light Co., Bonnington, B. C., Canada.
Bollbach, H. E. (Member), Transit Commission, New York, N. Y.
Brewer, W. M. Jr., Memphis (Tenn.) Power and Light Company.
Breyfogel, A. W. (Member), Western Union Telegraph Co., New York, N. Y.
Bricker, G. W. Jr., (Member), c/o O'Hare-Lewis, Boston, Mass.
Burke, B. S., Westinghouse Electric & Manufacturing Company, St. Louis, Mo.
Cave, J. S., Jr., Ohio Bell Telephone Company, Zanesville.
Catching, W. R. (Member), Le Carbone Company, Inc., New York, N. Y.
Christensen, J. H., Tennessee Valley Authority, Florence, Ala.
Clarke, J. W., Eggers Pole and Supply Company, Chicago, Ill.
Colman, S. A., H. P. Foley Company, Washington, D. C.
Colton, R. M., Brooklyn (N. Y.) Edison Company, Inc.
Coyne, M. G. (Miss), Tennessee Valley Authority, Wilson Dam, Ala.
Davis, J. L. (Member), American Telephone and Telegraph Company, New York, N. Y.
Edgar, W. S. W., Jr., Western Union Telegraph Company, New York, N. Y.
Enns, W. E., Portland (Ore.) General Electric Company.
Ericson, R. C., Northern Indiana Public Service Company, Hammond, Ind.
Erwin, R. A., Department of Water and Power, Los Angeles, Calif.
Farquhar, W. A., Underwriter's Laboratories, Inc., New York, N. Y.
Fox, A. (Member), New York Edison Company, Inc., New York.
Fulton, C. R. (Member), 2327 South 19 St., Lincoln, Neb.
Galdabini, E. J., Globe Union Inc., Milwaukee, Wis.
Goldfarb, B. J., West Penn Power Company, Charleroi, Pa.
Greene, N. J., National Electric Coil Company, Columbus, Ohio.
Grimm, G. A., Indiana Service Corporation, Ft. Wayne, Ind.
Gueffroy, R. S., Northwestern Electric Company, Portland, Ore.
Havens, E. G., New York and Queens Elec. Light and Power Company, Flushing, N. Y.
Heinbach, P. R., American Potash & Chemical Corp., Trona, Calif.
Hewitt, G. W., Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.
Hiller, J., 924 West End Ave., New York, N. Y.
Holland, J. W., 122 Spruce St., Toronto, Ont., Canada.
Horne, M. R., I-T-E Circuit Breaker Company, Dearborn, Mich.
Horsfall, W. H. D., Canada Wire and Cable Company, Ltd., Toronto, Ont., Canada.
Howarth, G. A., Utah Power and Light Company, Salt Lake City.
Howe, C. F., Jr., Independent System Subway, New York, N. Y.
Ireland, L. G. (Member), New Orleans (La.) Public Service Inc.
Jessel, J. J.-A., Federal Power Commission, San Francisco, Calif.
Kilian, J. M., Westinghouse Electric & Manufacturing Company, Kansas City, Mo.
Kodama, G. T., Globe Union Inc., Milwaukee, Wis.
Kushler, C. A., Champion Spark Plug Company, Detroit, Mich.
Langhus, A., Commonwealth Edison Company, Chicago, Ill.
Lohr, F. T., Public Service Commission, New York, N. Y.
Lowy, P. M., Y.M.C.A., Schenectady, N. Y.
Mathews, J. A., 518 Nichols St., Clearfield, Pa.
McClure, E. LeR., Milwaukee (Wis.) Electric Railway and Light Company.
Menut, J. G., Securities & Exchange Commission, Washington, D. C.
Meyers, R. S., American District Telegraph Company, New York, N. Y.
Montgomery, M. (Member), English Electric Company of Canada, Ltd., Vancouver, B. C.
Moore, H. S., Rockbestos Products Corp., New Haven, Conn.
Newell, E. L. (Member), Western Union Telegraph Company, New York, N. Y.
Nichols, W. A., Northern Elec. Company, Ltd., Montreal, P. Q., Canada.
O'Hanlon, J. R., Potomac Electric Power Company, Washington, D. C.
O'Leary, R. E., Ohio Power Company, Coshocton.
Powers, F. B. (Member), Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.
Preve, F. P., 608 W. Elm St., Urbana, Ill.
Rands, O. O., Bureau of Power and Light, Los Angeles, Calif.
Richter, W. (Member), A. O. Smith Corporation, Milwaukee, Wis.
Rickman, T. B. (Member), Mountain States Telephone and Telegraph Company, Denver, Colo.
Rosing, T. F., Cutler-Hammer, Inc., Milwaukee, Wis.
Ruoff, G. M., Locke Insulator Corporation, Baltimore, Md.
Schwartz, E. B., Globe Union, Inc., Milwaukee, Wis.
Simrall, H. C. F., Mississippi State College, State College.
Smith, R. E., General Electric Company, Cleveland, Ohio.
Smith, W. M. (Member), Vapor Car Heating Company, Chicago, Ill.
Snavely, A. B. (Member), Hershey Chocolate Corporation, Hershey, Pa.
Speller, G. V., Toronto Hydro-Electric System, Ont., Canada.
Teale, E. P. (Member), American Telephone and Telegraph Company, Richmond, Va.
Thompson, H. A., General Electric Company, Schenectady, N. Y.
Timmerman, M. C., New York Central System, Albany, N. Y.
Treece, C. C., 337 Buchanan Hall, Fayetteville, Ark.
Turner, R. C., Jr., Connecticut State College, Storrs.
von Hippel, A. (Member), Massachusetts Institute of Technology, Cambridge.
Walmsley, A. W., Electro Metallurgical Company, Alloy, W. Va.
Wells, C. J. (Member), Interior U.S. Irrigation Service, Coolidge, Ariz.
Weygandt, C. N., University of Pennsylvania, Philadelphia.
Willcutt, F. W. (Member), Potomac Electric Power Company, Washington, D. C.
Winkler, M. R., General Electric X-Ray Corporation, Chicago, Ill.
Wise, D. M. (Member), American Telephone and Telegraph Company, Philadelphia, Pa.
Wise, J. C., Rockbestos Products Corporation, Cleveland, Ohio.
Wolf, F. E., Mountain States Telephone and Telegraph Company, Denver, Colo.
Xenis, C. P. (Member), New York (N. Y.) Edison Company.
Young, G. C., Abington Electric Company, Clarks Summit, Pa.

91 Domestic

Foreign

Carey, F. M., Hydro Elec. Comm., Shannon, Tasmania.
Iqbal, M. M., Power House, Qaziwara, Ambala City, India.
Keinath, G. (Member), Siemens & Halske, Berlin, Germany.
King, J. L., Hong Kong (China) Electric Company.
Newman, S. F., Taikoo Sugar Refinery, Quarry Bay, Hong Kong, China
Tulloch, M. M., Kuala Kampar Tin Fields Ltd., Malim Nawar, Perak, Fed. Malay States.
Vaughan, C. W. (Member), Eastern Trading Company, Ltd., Sydney, Australia.
Whitney, W. H. (Member), Mildura City Council, Victoria, Australia.

8 Foreign

Engineering Literature

New Books in the Societies Library

Among the new books received at the Engineering Societies Library, New York, recently, are the following which have been selected because of their possible interest to the electrical engineer. Unless otherwise specified, books listed have been presented gratis by the publishers. The Institute assumes no responsibility for statements made in the following outlines, information for which is taken from the preface of the book in question.

SHORT WAVE WIRELESS COMMUNICATION. By A. W. Ladner and C. R. Stoner. 3 ed. N. Y., John Wiley & Sons, 1936. 453 p., illus., 9x6 in., cloth, $4.50. A practical presentation of principles and practice, intended for radio engineers and operators.

Die TECHNIK SELBSTTÄTIGER STEUERUNGEN und ANLAGEN. By G. Meiners. Munich and Berlin, R. Oldenbourg, 1936. 225 p., illus., 10x7 in., cloth, 12.50 rm. An account of modern developments in automatic switchgear and power plants, which describes modern equipment and processes, and their uses for automatic control from a practical point of view.

THERMIONIC EMISSION. By T. J. Jones. Lond., Methuen & Co., Ltd., 1936. 108 p., illus., 7x4 in., cloth, 3s. A survey of the fundamental principles to meet the needs of those contemplating experimental research.

UNIVERSAL STRESS SAG CHART for Power Line Computations. By J. T. Hattingh. Lond. and Glasgow, Blackie & Son, 1936. 74 p., illus., 9x6 in., cloth, 12s. 6d. A method for determining sags and tensions in electric transmission lines, which has been used since 1926 by the Electricity Supply Commission of South Africa. The chart has proved sufficiently accurate for practical purposes for spans up to 800 to 1,000 feet and is adapted to all types of materials used in conductor and overhead cable design.

WICKLUNGEN ELEKTRISCHER MASCHINEN und IHRE HERSTELLUNG. By F. Heiles. Berlin, Julius Springer, 1936. 185 p., illus., 9x6 in., cloth, 15.60 rm. Covers the winding of electrical machinery with emphasis upon the practical side.

PLANE TRIGONOMETRY. By E. S. Allen. N. Y. and Lond., McGraw-Hill Book Co., 1936. 152 p., diagrs., tables, 8x6 in., cloth, $2.25. Emphasizes the subject of projections and presents a new treatment of complex numbers. A set of 6-place tables is bound with the text.

Great Britain, Mines Department. REPORTS of H.M. INSPECTORS of MINES under the Coal Mines Act, 1911, for the YEAR 1935. 1. Scotland Division. 2. Northern Division. 3. Yorkshire Division. 4. North Midland Division. 5. North Western Division. 6. Cardiff and Forest of Dean Division. 7. Swansea Division. 8. Midland and Southern Division. Lond. His Majesty's Stationery Office, 1936. Illus., 10x6 in., paper, 1s. each part (Obtainable from British Library of Information, N. Y., $0.35 each). Reports statistics of employment and output, and includes discussions of mine lighting, and ventilation.

Der WERT der WÄRMEERSPARNIS, erläutert an der elektrowirtschaftlichen Gesamtstatistik Deutschlands und der Vereinigten Staaten von Amerika 1912–1934; ein betriebswirtschaftlicher Beitrag zur Kostendynamik. By F. zur Nedden. Munich and Berlin, R. Oldenbourg, 1936. 163 p., illus., 10x7 in., paper, 8 rm. Through an analysis of statistics of German and American electric power industries, a method is derived for determining the point at which the investment of further capital to effect savings in fuel becomes unprofitable.

HANDBOOK of ENGINEERING FUNDAMENTALS. (Wiley Engineering Handbook Series, v. 1). Ed. by O. W. Eshbach. N. Y. and Lond., John Wiley and Sons, 1936. 1081 p., illus., 9x6 in., lea., $5.00. The first volume in a proposed new engineering handbook series. Includes a summary of the principles of mathematics, physics, and chemistry, properties of materials, mechanics, and mathematical tables.

1936 Index—Electrical Engineering and Transactions

This multientry annual reference index covers comprehensively the entire text content of the 12 issues of ELECTRICAL ENGINEERING published during 1936 and of the identical content of the 1936 AIEE TRANSACTIONS, volume 55.

Discussions have received particular attention and a special effort has been made to provide effective correlation between references to technical papers and to all published discussions of those papers. In this connection it is of importance to note that discussions of many technical papers published during the latter half of 1935 were published in the early part of 1936, and hence appear in this current index.

Likewise, many discussions of the later 1936 papers will be published early in 1937, and consequently will not be found among the references contained in this current index.

For convenience in use, this index is subdivided into the following general divisions:

1. Technical subjects.
2. Authors, including the writers of discussions.
3. News items pertaining to Institute activities.
4. News items of a general nature.
5. Biographical (personal and obituary) items.

Division I—Technical Subject Index

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Division II—Author Index

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Division III—News of AIEE Activities

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Division V— Biographical Index

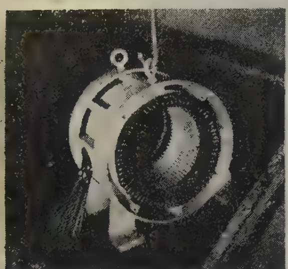
Includes All Personal and Obituary Items.

NOTE—An asterisk (*) before a page number denotes publication of individual's picture on that page.

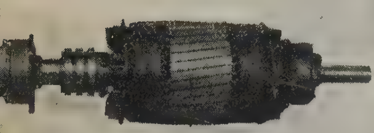
B



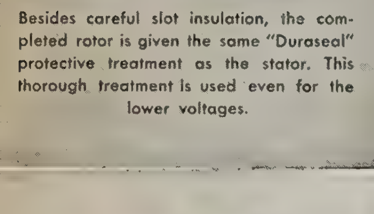
Bar-steel frames, electrically welded, provide a substantial enclosure for the stator core. There's a lot of strength to stand jolts and overloads.



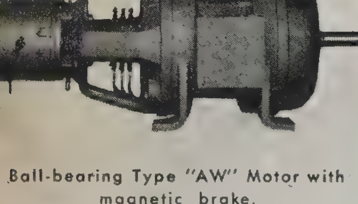
Stator windings are effectively protected by the Reliance "Duraseal" Insulation Process to put up greater than usual resistance against dust, moisture and fumes.



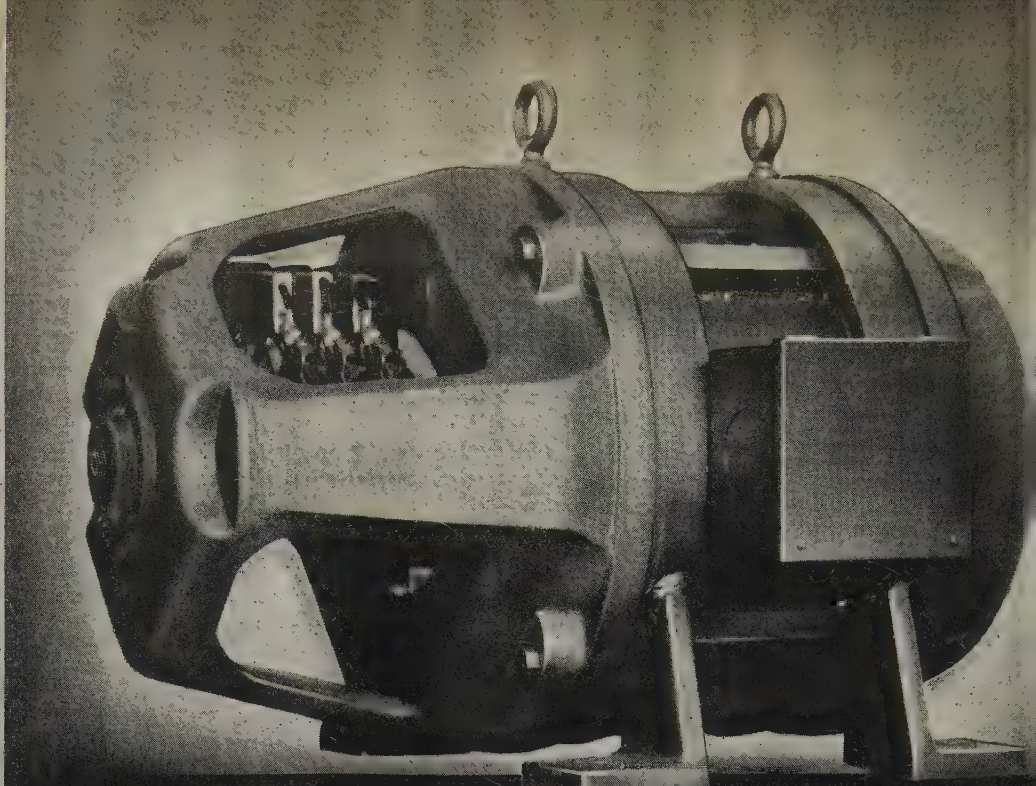
Besides careful slot insulation, the completed rotor is given the same "Duraseal" protective treatment as the stator. This thorough treatment is used even for the lower voltages.



Ball-bearing Type "AW" Motor with magnetic brake.



Three crane-type wound-rotor motors are used on this Cleveland Crane in a large metal-working plant.



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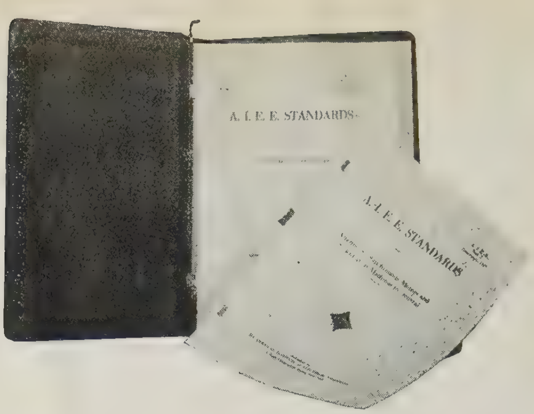

The Standards of the American Institute of Electrical Engineers now consists of forty sections on electrical machinery and apparatus. They are chiefly devoted to defining terms, conditions, and limits which characterize behavior, with special reference to acceptance tests, and many of them are recognized officially as American Standards.

The Standards available are listed below, together with prices. A discount of 50% is allowed to Institute members. Such discount is not applicable on extra copies unless ordered for other members. Numbers of the Standards Sections should be given when ordering.
A binder (illustrated above) for standards is available. Long wearing stiff covers, resembling leather, lettered in gold. Price $1.75 net.

[Figures in Parentheses Give Dates of Latest Editions.]

No. 1	General Principles Upon Which Temperature Limits Are Based in the Rating of Electrical Machinery (4-25)	$0.20
4	Measurement of Test Voltage in Dielectric Tests (5-28)	.30
*C50	Rotating Electrical Machinery (This American Standard supersedes the former A.I.E.E. Standards Nos. 5, 7, 8, 9 and 10, Direct Current Generators and Motors, Alternators, Synchronous Converters, Induction Motors and Fractional H-P Motors, respectively) (4-36)	1.30
*11	Railway Motors (7-25)	.30
*12	Constant Current Transformers (1-34)	.30
13	Transformers, Induction Regulators and Reactors (5-30)	.40
*14	Instrument Transformers (3-25)	.30
*15	Industrial Control Apparatus (5-28)	.40
*16	Railway Control Apparatus (1-33)	.40
*17f	Mathematical Symbols (2-28)	.30
*17g1	Letter Symbols for Electrical Quantities (11-28)	.20
*17g2	Graphical Symbols for Electric Power and Wiring (1-34)	.20
*17g3	Graphical Symbols for Radio (1-34)	.20
*17g5	Graphical Symbols for Electric Traction Including Railway Signaling (1-34)	.40
*17g6	Graphical Symbols for Telephone and Telegraph Use (3-29)	.20
*18	Capacitors (6-34)	.20
19	Oil Circuit Breakers (7-25)	.30
20	Air Circuit Breakers (5-30)	.30
22	Disconnecting and Horn Gap Switches (7-25)	.30
26	Automatic Stations (5-30)	.30
27	Switchboards and Switching Equipment for Power and Light (10-30)	.30
*28	Lightning Arresters (3-36)	.30
*30	Wires and Cables (9-32)	.40
33	Electrical Measuring Instruments (1-27)	.30
*36	Storage Batteries (2-28)	.20
*38	Electric Arc Welding Apparatus (1-34)	.40
*39	Electric Resistance Welding Apparatus (1-34)	.30
*41	Insulators (3-30)	.30
45	Recommended Practice for Electrical Installations on Shipboard (10-30)	1.50
*46	Hard Drawn Aluminum Conductors (6-27)	.20
*60	Specifications for Tinned Soft or Annealed Copper Wire. (See No. 61 for price.)	
*61	Specifications for Soft or Annealed Copper Wire. (Nos. 60 and 61 published as one pamphlet) (9-28)	.30
*63	Specifications for 30 Per Cent Rubber Insulation for Wire and Cable for General Purposes (9-28)	.30
*69	Specifications for Cotton Covered Round Copper Magnet Wire. (See No. 71 for price.)	
*70	Specifications for Silk Covered Round Copper Magnet Wire. (See No. 71 for price.)	
*71	Specifications for Enameled Round Copper Magnet Wire. (Nos. 69, 70, and 71 published as one pamphlet) (9-28)	.30
*72	Specifications for Weatherproof Wires and Cables. (See No. 73 for price.)	
*73	Specifications for Heat-Resisting Wire and Cables. (Nos. 72 and 73 published as one pamphlet) (9-32)	.20
100	Recommendations for the Operation of Transformers (6-30)	.20
	Total Cost of Complete Set	$12.30

* *Approved as American Standard.*

Sections in Preparation

Eight such sections are now available in report form for purpose of criticism, and copies will be sent without charge upon request, with the exception of No. 2 for which there is a non-member charge of $1.00, with usual 50% discount to members. These sections are as follows:

- No. 2 (Aug. 1932) Electrical Definitions. $1.00
- 6 Mercury Arc Rectifiers.
- 19A Oil Circuit Breakers (Proposed revision of No. 19)
- 23 Relays.
- 25 Fuses Above 600 Volts
- 40 Electrical Recording Instruments.
- — Test Code for Transformers.
- — Test Code for Polyphase Induction Machines.

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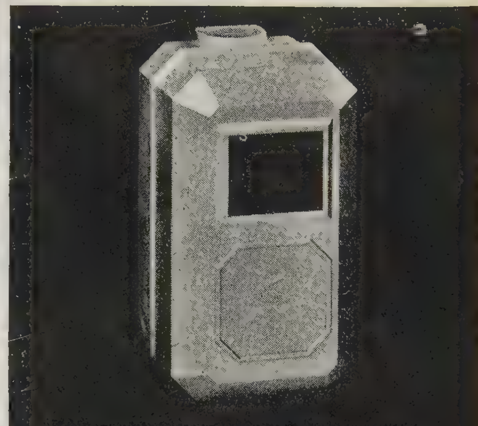
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Employment Bulletin

Engineering Societies Employment Service

MAINTAINED for their members by the national societies of civil, mining, mechanical, and electrical engineers, in co-operation with other organizations. An inquiry to any of the three offices will bring full information.

A weekly bulletin of engineering positions open is available to members of the co-operating societies at a subscription of $3 per quarter or $10 per annum, payable in advance.

In the interest of effective service, it is essential that members using the employment service keep the bureau office serving them advised at reasonable intervals concerning their availability for employment, concerning any change in status, and immediately upon acceptance of any employment.

Employers interested in the following announcements should address replies to the key numbers indicated, and mail to the New York Office.

Men Available

A.B., S.B. in E.E., 37; 5 yrs with mfr; 9 yrs util indus heating, new business mgr, asst to pres, analyzing oprtg and mgmt problems. Desires analytical or consulting pos. D-5318.

E.E., Des, constr, util; estimating, draftg, contractors; pwr plants, distr systems, indus and commercial bldg wiring; 14 yrs exper; best of ref. D-5478.

E.E. GRAD, 34, 10 yrs exper pwr station, substation des; 3 yrs valuation, appraisal. Exper with largest util East, South, Middlewest. Desires contact where varied exper would be of service. D-3149.

TRANSM ENGR, E.E., 35; 10 yrs devpmt, des, engg, constr steel tower, wood pole pwr lines, distr systems, substations. Prepared estimates, specifications, contracts for new constr and rebuild for larger capacities. C-3564.

E.E., Cornell, G.E. test, 14 yrs asst engr, pub util, high and low tension substations, switch structures, estimating, des, constr distr systems, investigations, economic studies; 2 yrs elec des, indus concern. B-9614.

E.E., M.I.T., '27. Instructing, testg, purchasing, des, prod, business exper, languages. Interested in prod, elec devpmt, patents, des, research, tech promotion. Salary secondary. Available on short notice. C-3612.

B.S., E.E., M.A. (Physics). Prof license, 29, single. Exper pumping, testg gas-filled electronic tubes, small transf des, radio research. Good personality. Desires des, teaching, research, exec asst. Location NYC. D-5527.

TECH GRAD, elec, 29; 10 yrs exper draftg, valuation-appraisal procedure; elec constr, pwr plant, distr, in charge; licensed E.E. Seeks permanent connection East on cost, inventory systems; constr, maintenance. D-5457.

ELEC ENGR, 38; 15 yrs exper, 7 as exe Pwr plant, RR and sales promotion exper. Sp cialties pub relations and sales research. Wish connection pub util or mfr. D-5555.

E.E., B.S., 21, single, high honors Newark C of Engg 1936. Desires pos affording engg exp and advancement. Interested in des and co Exper: Machine shop and elec testg. D-5487.

E.E., B.S., Univ of S. C., 1936, single, 21. D sires chance for exper and advancement in elec fiel Prefers pos in pwr transm or distr. Location i material, salary secondary. D-5511.

RECENT GRAD E.E., 1936, desires a pos junior sales engr with future. D-5104.

GRAD 1934, E.E., B.S., single; 1½ yrs mai tenance of pwr plant. Exper in spot welding, i spec and calibration of thermostats. Assembly air conditioning units. Bldg maintenance. Loc tion immaterial. D-3719.

YOUNG ELEC GRAD, B.S. in E.E., 193 academic honors, indus elec and radio tube te set maintenance and instal exper. Location pr ferred, N Y City or vicinity. C-9496.

GRAD E.E., 30, married; 2 yrs banking, o yr GE test, 5 yrs marine engg modern turbo-el ships, charge engine and boiler rooms. Practic elec and steam machy exper. C-9958.

PUB UTIL ACCOUNTANT and engr, B.A B.S., M.B.A.; 11 yrs exper in oprtg util, one with regulatory commission. Pos desired with p util or pub util consulting firm. B-2470.

RESEARCH, DEVPMT ENGR, marrie 43, E.E., B.S., M.S. in physics; 10 yrs exper tel, timers, H F oscillators, amplifiers, tran rectifiers, switches, relays; also des, cost estima ing, mfg, supervision, inspec, testg. C-2677.

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Engineering Societies Library

A reference library for engineers—contains 150,000 volumes—receives over 1,300 technical journals and periodicals on all branches of engineering.

Searches are made upon engineering subjects, translations from foreign publications, photostats made, etc.

Books may be borrowed by members of the A.I.E.E. For information address, The Engineering Societies Library, 33 West 39th St., New York.

At Your Service

WANTED

COPIES of the January (1936) issue of ELECTRICAL ENGINEERING are required.

If you care to dispose of your copy please mail it (parcel post) to American Institute of Electrical Engineers, 33 West 39th St., New York, printing your name and address upon the enclosing wrapper. Twenty-five cents, plus postage, will be paid for each copy returned.

TRANSACTIONS

FOR 1937

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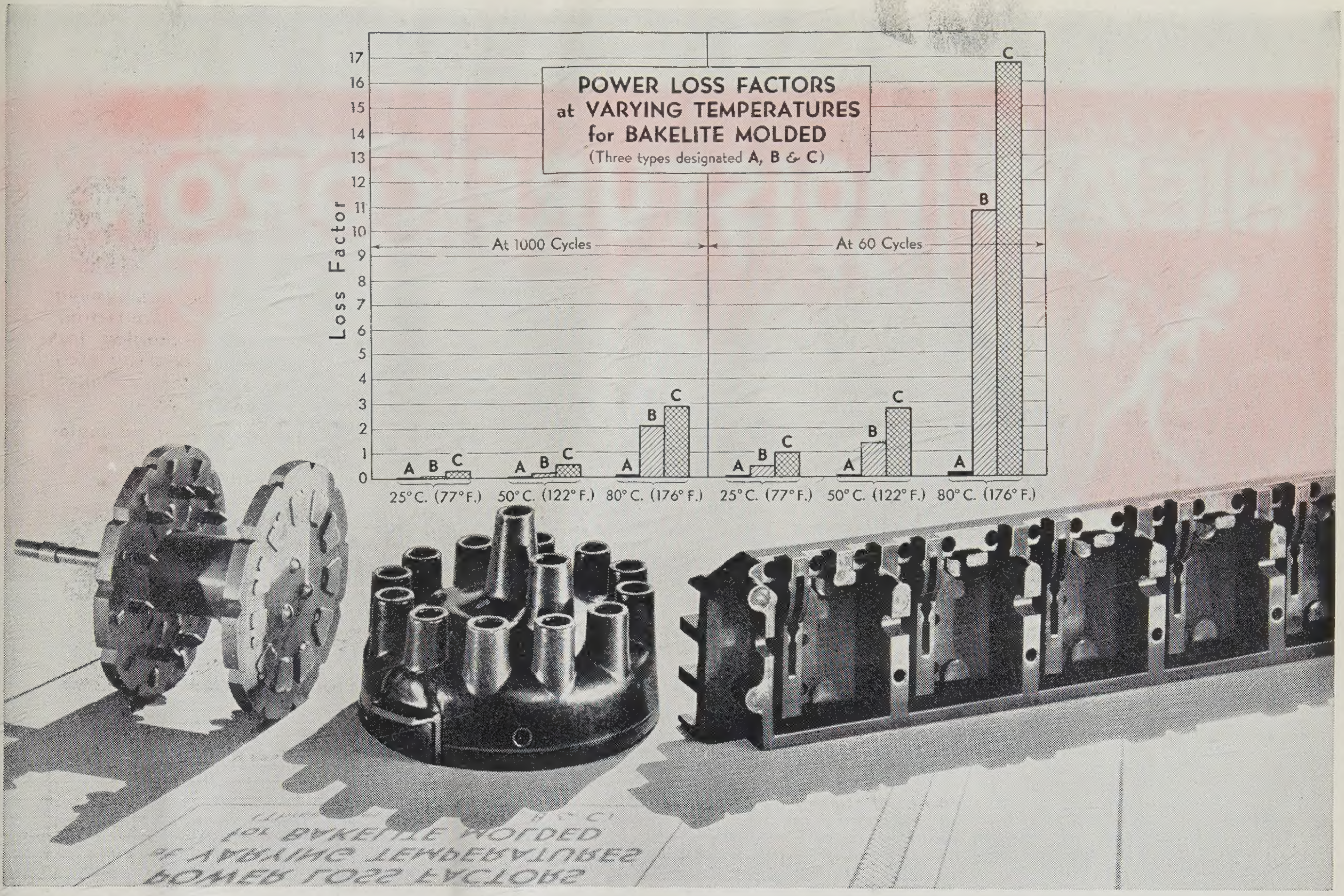

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